14.03 Final - Spring 1996
Part II Solutions

Part II - 49 pts.

Short Answer

1. [10 pts.] Suppose that there are two goods in the economy, steak and chicken. A consumer has income \$1 in 1990. Prices change between 1990 and 1996. The government constructs a Paasche price index. Suppose that the consumer's income increases in proportion to the Paasche index, and denote the new income by X. Let Z be the amount that the consumer would need to be equally happy in 1996 as in 1990. Is X greater or less than Z? Prove your answer mathematically and with a diagram.

Solution: The formula for the Paasche index is as follows: $X = \dfrac{P_{96}^S \cdot S_{96} + P_{96}^C \cdot C_{96}}{P_{90}^S \cdot S_{96} + P_{90}^C \cdot C_{96}}$.

The formula for the ideal index is as follow: $Z = \dfrac{E(\overline{U} | P_{96}^S, P_{96}^C)}{E(\overline{U} | P_{90}^S, P_{90}^C)}$, where $\overline{U}$ is some reference utility, for example $\overline{U} = U(C_{96}, S_{96})$. Not that the ideal index is *not* the ratio of the expenditures needed to buy each bundle! The ideal index is designed to keep utility constant, not expenditure. It is the ratio of the minimum expenditures needed to keep utility constant, at 1996 versus 1990 prices.

If we let $\overline{U} = U(C_{96}, S_{96})$, then we can compare the two indices directly: the numerator of both expressions is equal by duality (under the assumption that (C_{96}, S_{96}) was the most preferred bundle given an expenditure of $I_{96} = P_{96}^S \cdot S_{96} + P_{96}^C \cdot C_{96}$). To see this, observe that $U(C_{96}, S_{96})$ is the maximum utility which can be achieved with I_{96}, while I_{96} is the minimum expenditure required to achieve $U(C_{96}, S_{96})$.

On the other hand, the amount of money needed to give the consumer utility $U(C_{96}, S_{96})$ at 1990 prices, denoted $E(\overline{U} | P_{90}^S, P_{90}^C)$, is less than $P_{90}^S \cdot S_{96} + P_{90}^C \cdot C_{96}$, since (C_{96}, S_{96}) is *not* necessarily the best bundle the consumer could buy with I_{96} given 1990 prices. Thus, we have shown that

$$\frac{E(\overline{U} | P_{96}^S, P_{96}^C)}{E(\overline{U} | P_{90}^S, P_{90}^C)} > \frac{P_{96}^S \cdot S_{96} + P_{96}^C \cdot C_{96}}{P_{90}^S \cdot S_{96} + P_{90}^C \cdot C_{96}},$$

or $Z > X$.

The reason that the Paasche index understates inflation is that it does not properly account for the substitution effect. It doesn't take into account the fact that consumers could have achieved $\overline{U}$ for less money in 1990 by making choices better suited to the prevailing 1990 prices. Since the 1990 expenditure is evaluated at choices which reflect 1996 prices, the Paasche estimate of the 1990 expenditure is too high, and thus the Paasche estimate of the change from 1990 to 1996 is too low.

Constructing the graph is a bit tricky, but it involves noting that the Paasche index calculates the change in expenditure using a budget line which runs through (C_{96}, S_{96}) at a 1990 price ratio. However, utility is not constant for the 1996 budget line through (C_{96}, S_{96}) and the 1996 budget line through (C_{96}, S_{96})! Utility is lower on the 1996 budget line through (C_{96}, S_{96}). Thus, if the Paasche index is applied, the consumer is worse off in 1990.

To see it another way, we can draw a second graph (which doesn't match the algebra as well) which takes the 1990 utility as the reference utility. Again, what is required is a graph of two budget lines through (C_{96}, S_{96}), at the two different price ratios. This graph illustrates that the consumer is worse off in 1996 than in 1990 if she is given the Paasche income.

2. [12 pts.] Assume that we observe the choices of 1000 firms about organizational design decisions. In particular, we observe their choices about discretion afforded managers (d) and investments in computing equipment (c) for the managers which can be used to maintain a database. Firm i chooses c_i and d_i to maximize the following expression, taking P_{ci} and P_{di} as given: $g(c_i,d_i) - P_{ci}\, c_i - P_{di}\, d_i$.

(a) Suppose that we cannot observe P_{ci} and P_{di} for each firm, but we have independent evidence which shows that P_{ci} is statistically indepedent of P_{di}. Yet, we find statistically significant evidence that c and d are positively correlated in the firms we observed. What, if anything, can we infer from this information about whether or not c and d are complementary?

Solution: This problem requires two logical steps of analysis. The first step is to notice that, *if* c and d were complements, then by the increasing differences theorem, for each firm $c_i^*(P_{ci},P_{di})$ and $d_i^*(P_{ci},P_{di})$ would be nondecreasing functions.

Consider for a moment an individual firm's problem. Of course, if c and d are complements, we can only be certain that c_i^* and d_i^* move together in response to a change in prices when both prices move together. What is important to realize is that this problem asks not about one firm's problem, but about what we can learn by observing many different firms' choices (where each firm faces different prices).

In our case, the logic works like this: suppose that P_{di} was fixed for all firms, and only P_{ci} varied across firms. This satisfies the condition that prices are independent. If c and d are complements, then when P_{ci} is low, *both* c_i^* and d_i^* will be high! Likewise, when P_{ci} is high, *both* c_i^* and d_i^* will be low. This *induces* a positive correlation between c^* and d^* in the population, even though the prices were independent. Constrast this to the case where c and d do not interact in the function g, so that g is additively separable. Then, the organizational choices do not depend on each others' prices, so that $c_i^*(P_{ci},P_{di})=c_i^*(P_{ci})$ and $d_i^*(P_{ci},P_{di})=d_i^*(P_{di})$. In that case, independent prices would lead to a finding that (c^*,d^*) were independent in the population! The organizational choices would just vary with the prices, independently of one another.

Therefore, observing a positive covariation of (c^*,d^*) is evidence of some other force besides prices which makes the choices move together. Complementarity is just such a force.

In general, the conceptual point is this: if you have *any* two independent random variables X and Y, and you observe *any* two **nondecreasing** functions of those random variables, $Z = f(X,Y)$ and $W = h(X,Y)$, then $\mathrm{cov}(Z,W)$ will be positive. Likewise, evidence that $\mathrm{cov}(Z,W) \geq 0$ is consistent with the hypothesis that f and h are nondecreasing, given the assumption that X and Y are independent.

(b) How would your answer change if we knew that P_{ci} and P_{di} were positively correlated? negatively correlated?

Solution: This part builds on the answer to part (a). As we discussed in class, there are two possible explanations for (c,d) to covary positively in the population of firms. The first is that they are complements. The second is that their prices covary positively.

Again, it is useful to use the case of organizational choices which do not interact with one another in a firms' profit function, so that $c_i^*(P_{ci},P_{di})=c_i^*(P_{ci})$ and $d_i^*(P_{ci},P_{di})=d_i^*(P_{di})$. In that case, positive correlation of the prices might lead to positive correlation of (c,d), while negative correlation of the prices might lead to negative correlation of (c,d). In the absense of complementarities or substitutability between choices, each choice just responds to its own price, without regard to the other price.

Now, let us consider the case of negative correlation of the prices. Without complementarities between (c,d), the previous paragraph's analysis implies that we would expect to see a *negative* covariance between (c,d) in the population. Thus, our finding of a *positive* covariance provides very strong support for complementarities! Firms were likely to find c expensive at the same time they found the d cheap. Yet, in spite of that, they were likely to choose high levels of both c and d together, or low levels of both choices together. Clearly, there must be some force leading firms to make these choices together, and complementarity is such a reason.

On the other hand, when prices are positively correlated, it is difficult to distinguish between competing hypotheses. It could have been the positive correlation in the prices which lead to the positive covariation of (c,d) and not complementarities at all! Firms could have just adjusted the individual choices in response to price changes, and since firms tended to find both inexpensive together or both cheap together, their choices c and d would covary positively. On the other hand, complementarities also would lead to positive correlation of (c,d). Thus, it is impossible to sort out what is causing the covariation of c and d.

Of course, for an individual firm, if c and d are complements, we can only make statements about how that firm's choices change in response to a change in prices if both prices move together. But this question is about covariation in a large sample of firms.

This question was tricky because the logic works exactly opposite of the logic of when we can make predictions. The question asked you to think backwards from observations, not forward from assumptions about the profit function. When one price goes up, and the other down, it is hard to predict what the firm will do. This is the logic many people in the class remembered, and that logic is correct. But you have to take that logic one step farther: if, in spite of the fact that one price goes up and the other down, *both* choices increase together, this is indeed evidence of complementarity.

3. [12 pts.]

(a) Assume that all of the assumptions for perfectly competitive markets hold. Further, assume that a benevolent central planner can read minds, know production functions, and dictate actions of its citizens, and it can costlessly process any amount of information. In this case, can the centrally planned economy lead to a Pareto improvement over the decentralized free market?

Solution: Under the assumptions of the perfectly competitive markets, free markets lead to Pareto efficiency. This is the beauty of the price system: each agent needs only to know what prices are and maximize his or her own objective function, and efficiency obtains. They don't need to know information about anyone else in the economy. Thus, the central planner cannot improve on free markets in terms of the Pareto criteria, although the central planner might be able to do better according to some *other* criteria, such as equity.

Only when some of the assumptions of perfect competition are relaxed, such as information about prices being costly to obtain, or externalities, etc., can central planning improve up on the market.

(b) Name two assumptions which can be relaxed which increase the desirability of free markets relative to central planning, and briefly explain why. [Note: In b and c, the criteria for desirability is Pareto efficiency.]

Solution:

1- Central planner can read minds. If this is relaxed, the central planner will have to rely on reports from consumers to know their utility functions, making it difficult to know how much of different goods to produce and how to allocate resources. People may have incentives to misrepresent their skills and knowledge in order to get jobs which they like more.

2- Central planner knows production functions. If this is relaxed, the central planner will need to rely on managers of firms to report information about their production functions, and again managers may have incentives to misreport if they don't like working hard (recall the ratchet effect from the MR articles).

3 - Costlessly process information. If not, many resources may be used up trying to figure out what to do. The problem is so complicated that even if information processing is very cheap on a per unit basis, the information processing task may blow up very quickly. It will also cause delays: in a changing economy, the planner may never do better than knowing what would have been optimal yesterday.

(c) Repeat the exercise of (b) for two assumptions which increase the desirability of central planning relative to free markets.

Solution:

Any of the assumptions of perfect competition are important for the efficiency of perfect competition. Some examples:

1- Externalities. If people's actions affect each other in ways which are not captured in the price system, then central planning can in principle improve upon free markets. Taxes and regulations can force people to internalize their externalities.

2- Price taking. Monopolies and oligopolies do not allocate resources efficiently. Regulation can force firms to sell higher quantities, hire more workers, or locate in the efficient locations.

3- Knowledge of prices. If it is expensive to learn prices, resources may not be allocated efficiently.

4. [15 pts.] Krugman argued that competition between countries was different from competition between companies. For each of the following statements, support the statement with a graph and a brief argument, *where you use the tools of the course* to show explicitly that the statement is true in one of the models we analyzed. (For example, use things like production functions and indifference curves and diagrams of

equilibrium determination). [Hint: once you have figured out which model matches the quote, your answers should be short and might be almost completely graphical.]

(a) Countries cannot increase their market share in all industries at once.

Solution: Since countries are more or less closed systems with a fixed amount of resources, increasing output in one industry will necessarily come at the cost of a decrease in output in another industry, since resources will be diverted away from one industry and towards another. This is formalized in the Edgeworth box diagram of productive efficiency, which is used to derive the production possibilities frontier (PPF).

(b) Trade is not a zero-sum game.

Solution: As used in the readings, the term "zero sum game" refers to a situation where two agents are competing over a pie of fixed size, so that one agent's gain is the other's loss. In the case of trade, countries can both become better off through trade.

The basic Edgeworth box model of exchange shows that trade between two agents can make both better off. In a production economy, the theory of comparative advantage reaches the same conclusion. Trade allows *both* countries to consume goods on a higher indifference curve than can be attained on the PPF.

(c) Unlike price wars between firms (where firms undercut each others' prices to the benefit of consumers), trade wars between countries hurt consumers.

> **Solution:** Trade wars occur between large countries in part due to the fact that each country has a unilateral incentive to impose a tariff in order to improve the terms of trade. In contrast to price wars, where the incentive of the company is to lower prices to the benefit of consumers, the incentive of countries is to impose ever-higher tariffs. While the terms of trade may or may not improve in the end, one certain result is that the volume of trade is diminished. This is bad for consumers in both countries, as it pushes both countries back towards what they could achieve in isolation, their own PPF.