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HANDOUT: Price Indices and the Expenditure Function

The goal of a price index is to measure the change in a consumer's well-being as a result of changes in prices. The "ideal" measure of the consumer's change in well-being can be expressed as follows:

$$I_{true} = \frac{E(P_x^N, P_y^N, \overline{U})}{E(P_x^O, P_y^O, \overline{U})}.$$

Unfortunately, we don't in general know the consumer's expenditure function. Assume that we can't observe people's utility, just their choices and the prices at which they made those choices. We want to know whether a consumer would have to spend more to achieve a given utility level under (i) old prices, or (ii) new prices.

It is typical to use new prices in numerator, old prices in denominator (this is in contrast to Nicholson's treatment of consumer surplus). A generic price index takes the following form:

$$\text{Generic price index: } \frac{P_x^N W_x + P_y^N W_y}{P_x^O W_x + P_y^O W_y}$$

The two most commonly used indices are Laspeyres and Paasche. The difference between Laspeyres and Paasche is the weights on the prices. The weights are given as follows:

Laspeyres: Observed quantities at old prices. $W_x = X^O$, $W_y = Y^O$.

Paasche: Observed quantities at new prices. $W_x = X^N$, $W_y = Y^N$.

Laspeyres overstates change in cost of living: We can argue this using revealed preference. First, we show that when the Laspeyres index shows no change in the cost of living ($I_L = 1$), the consumer is better off. When $P_x^N X^O + P_y^N Y^O = P_x^O X^O + P_y^O Y^O$, then a consumer who can afford bundle (X^O, Y^O) under the old prices can still afford (X^O, Y^O) under the new prices. So, at the new prices there might be some other bundle which costs the same as (X^O, Y^O), but which yields higher utility.

Likewise, if $P_x^N X^O + P_y^N Y^O < P_x^O X^O + P_y^O Y^O$, the same argument holds: at the new prices, the cost of buying the old bundle (and thus staying on the old indifference curve) goes down. This corresponds to Laspeyres index less than one. You can similarly show that we can't say anything about the change in welfare when Laspeyres is greater than one.

We can also show that Laspeyres overstates the change in the cost of living using the expenditure function. But, there is a sticky issue which we glossed over in the last paragraph: what to take as the reference utility? We wish to compare the following quantities:

$$\frac{P_x^N X^O + P_y^N Y^O}{P_x^O X^O + P_y^O Y^O} \text{ and } \frac{E(P_x^N, P_y^N, \overline{U})}{E(P_x^O, P_y^O, \overline{U})}$$

Notice that, if we take $\overline{U}_0 \equiv V(P_x^O, P_y^O, Y) = U(X^O, Y^O)$, then $E(P_x^O, P_y^O, \overline{U}_O) = P_x^O X^O + P_y^O Y^O$, which gives us something to work with for comparisons: the denominator will be the same. How about the numerator? Well, it is straightforward to see that $E(P_x^N, P_y^N, \overline{U}_O) \leq P_x^N X^O + P_y^N Y^O$. Clearly, buying the old bundle is one way to keep utility constant at the old level, $U(X^O, Y^O)$; but, the expenditure function tells us the *least-cost* way to keep utility constant at a given price vector, and there is no reason to believe that the old bundle is the least-cost way of getting to the old indifference curve at the new prices. Thus, we have shown that

$$\frac{P_x^N X^O + P_y^N Y^O}{P_x^O X^O + P_y^O Y^O} \geq \frac{E(P_x^N, P_y^N, \overline{U}_O)}{E(P_x^O, P_y^O, \overline{U}_O)}$$

Paasche understates the change in the cost of living: When the Paasche index is one (I_P = 1), the consumer is worse off. That is, if $P_x^N X^N + P_y^N Y^N = P_x^O X^N + P_y^O Y^N$, a consumer who can afford bundle (X^N, Y^N) under the old prices can also afford (X^N, Y^N) under the new prices. But, (X^N, Y^N) isn't necessarily what the consumer would choose given that she can afford to buy (X^N, Y^N) at the old prices; there might be some other bundle at the old prices which costs the same as (X^N, Y^N), but that makes the consumer better off. Thus, if the prices are such that the new bundle costs the same in both periods, the consumer must have been better off under old prices, where she can do even better than (X^N, Y^N) at the same cost.

Similar to the case of Laspeyres, the algebraic argument proceeds by showing that:

$$\frac{P_x^N X^N + P_y^N Y^N}{P_x^O X^N + P_y^O Y^N} \leq \frac{E(P_x^N, P_y^N, \overline{U}_N)}{E(P_x^O, P_y^O, \overline{U}_N)}$$

where $\overline{U}_N \equiv V(P_x^N, P_y^N, Y) = U(X^N, Y^N)$. The numerators are the same, by definition; the demonimator of the Paasche index is greater than that of the ideal index, since the expenditure function gives the least cost way of achieving $U(X^N, Y^N)$.