Problem Set 2

Due 2/26/96 in class or by 5 p.m. to Michael Kreutz, E52-251

Theory

1. Nicholson, Problems 3.2, 3.4, 4.4, 4.5, and 5.8.
2. Let $U(X,Y)= X^{1/4}Y^{3/4}$. Suppose that prices are P_x and P_y, and income is I. Calculate the utility-maximizing choices of X and Y, that is, the Marshallian demand functions $d_x(P_x,P_y,I)$ and $d_y(P_x,P_y,I)$.

   (a) Calculate "indirect utility," or the utility at the optimal choices, $V(P_x,P_y,I)$.

   (b) For a given utility level U_0, solve the dual expenditure-minimization problem, and compute the optimal choices of X and Y (the "compensated demand functions," $h_x(P_x,P_y,U_0)$ and $h_y(P_x,P_y,U_0)$).

   (c) Calculate the minimum expenditure function $E(P_x,P_y,U_0)$. Show that the expenditure and indirect utility functions you have calculated are inverses of one another.

   (d) Consider two famous applications of the envelope theorem in consumer theory, Shephard's Lemma and Roy's Identity ((E5.1) and (E5.2) in Nicholson). State a verbal interpretation of each of these results.

   (e) Verify directly that Shephard's Lemma and Roy's Identity hold in this problem.

Application ("Irish Potato" article)

1. Draw a diagram illustrating Giffen's paradox. Your diagram should illustrate a consumer's 2-good utility maximization problem for two different budget constraints, where P_X is higher under the second constraint.
2. Consider Figure 1 of Dwyer and Lindsay. Explain why the top diagram is consistent with the your diagram from question (1). Why is the lower diagram a better description of the Irish potato famine?
3. Dwyer and Lindsay make two claims about Giffen goods: "For a good to be Giffen, some normal good must be displaced by the inferior good as the price rise lowers real income." "Inferiority is necessary for a good to be Giffen." Formally prove these two statements, and then briefly state a reason why each is unlikely to hold for the case of the Irish potato famine.
4. We are interested in measuring the effects of a price change on the amount demanded of potatoes. Graph the budget constraint facing a person with income I and the ability to purchase potatoes and bread if the price of bread is \$1.00 and the cost of potatoes is P. Draw a standard set of indifference curves and illustrate the optimal bundle.

Show how the budget constraint and optimal bundle change when the price rises from $P=P_1$ to $P=P_2$, $P_1<P_2$, for each of the following assumptions.

a) I is independent of the price.

b) $I = S \cdot P$, where S is fixed.

c) $I = S \cdot P$, where we first consider $P=P_1$ and $S=S_1$, and then we consider $P=P_2$ and $S=S_2$, where $S_2<S_1$ (price rises and supply falls).

Explain how the trade-off between potatoes and bread changes in (a)-(c). What does this say about the interpretation of the Irish potato famine as an example of the kind of experiment which might be used to determine whether a good is Giffen?

5. What is an alternative example of a Giffen good (in addition to the one suggested in the article)? Why is this a more plausible scenario?