14.121 Final Exam

Professor Susan Athey **Fall 1996**

Instructions: Answer all questions. If you need to make *any* assumptions beyond those given in the problem, state them clearly and state why you need them. You may invoke mathematical results and theorems without proving them, so long as you are clear about which result you are using and why it applies.

Question 1 - 35 minutes

Consider a standard, utility-maximizing graduate student who consumes three goods: Refresher Course coffee (x_1), Au Bon Pain Arizona Chicken Sandwiches (x_2), and Rebecca's Croissants (x_3) (since she sleeps in her office in E52, she saves on rent). Prices are (p_1,p_2,p_3). Her stipend is Y.

The first part of this question is distinct from the rest of the question!

(i) Suppose that she works on her PhD in economics from 1996 until 1999, during which time her stipend stays fixed. In 1996 and 1997, the prices for the three goods were (2,4,3) in each year. In 1998, they changed to (1,3,5). In 1999, they changed again, to (3,5,1). Her happiness stayed fixed from 1998 to 1999. Was the student able to achieve the same happiness over the course of 1998 and 1999 as she experienced during 1996 and 1997? (Compare the sum of utilities in 1996 and 1997 to the sum of utilities in 1998 and 1999).

Suppose for the rest of the question that the student goes on a diet and cuts out the Rebecca's croissants.

(ii) Another graduate student convinces the Refresher Course and Au Bon Pain to change prices around for a couple of weeks so that he can estimate the first student's demand for Refresher Course coffee and pass his econometrics paper requirement. He estimates the demand curve to be $x_1(p_1,p_2,Y) = Y/(2\, p_1)$. Is this a valid demand curve? Why?

Now assume that the student got an *A* on his econometrics paper (since Jerry Hausman grades these, it must be right).

(iii) What must the demand $x_2(p_1,p_2,Y)$ for Au Bon Pain sandwiches be (assume that the student doesn't cheat on her diet)?

(iv) Suppose that prices for the two goods are fixed at **p**=(2,3) and that the stipend is $10,000. Let the first student's maximum attainable utility given these parameters be $\bar{u}$. If the student wishes to find the cheapest way to achieve utility $\bar{u}$, how much coffee will she buy?

(v) Derive the Slutsky substitution matrix. How does the compensated (Hicksian) demand for x_1 change with p_2?

Question 2 - 25 minutes

Consider a risk averse individual whose preferences for lotteries over money can be represented using the VN-M utility function $u(w)$. Suppose that the individual has initial wealth w_0, and further faces a risk that his basement will be flooded. With probability π, a Nor'easter strikes, creating losses Δ. With probability $(1-\pi)$, there is no storm. Insurance is available for $\$P$ per dollar of insurance. Thus, if the individual buys I units of insurance, then the total cost (paid for sure) is $I \cdot P$, and the individual receives I if the storm strikes and nothing otherwise. Insurance can only be purchased up to the amount of the potential loss, that is, $I \leq \Delta$.

(i) Suppose that $\$P = \pi$. Characterize the amount of insurance purchased by the consumer. (Prove your answer.)

(ii) Now suppose that $\$P > \pi$. Does the optimal amount of insurance purchased increase or decrease (weakly) in response to a change in π or can you tell? What about Δ? Interpret.

(iii) How does the consumer's (maximized) utility change with P?

<u>Question 3 - 30 minutes</u>

Consider a monopolist who has three choice variables: q, the quantity to sell; "marketing," m, and new equipment, e.

Production costs: The cost of producing q units of output is given by $c \cdot q / e$, where c is a constant.

Other input costs: Equipment is purchased on a competitive market at price r. The cost of m units of marketing is given by $h(m;\theta)$, where θ is a parameter which increases the incremental cost of marketing (such as advertising fees). That is, h satisfies increasing differences in $(m;\theta)$.

Demand is described by $P(q;m)$, so that total revenue is given by $q \cdot P(q;m)$. Assume that $P(q;m)$ is differentiable as many times and in as many ways as you would like.

(i) Write down conditions on the function P which guarantee that marketing increases marginal revenue.

Assume that your conditions from (i) hold throughout the rest of the problem. Note that the firm's profit function is given as follows:

$$q \cdot P(q;m) - c \cdot q / e - r \cdot e - h(m;\theta)$$

(ii) Suppose that in the short run, equipment is fixed. How do the firm's short run choices of quantity and marketing change with the parameter θ? With the fixed level of equipment?

(iii) How does the firm's long run choice of equipment change with θ?

(iv) In response to a 10% increase in θ, will the choices of quantity and marketing change by more in the short run or in the long run? Describe the intuition. For full credit, sketch a proof of your answer.

(v) How do the firm's long run profits change with r? Does your answer change with the parameter θ? If so, in what direction? Interpret.