Cheat Sheet: This is what you need to know!!

Cost minimization:

$$\max_k -rk - wL(k,q) \qquad \text{(L1)}$$

How does *k* change with *r*? Own-price effects negative!

Verbal logic: when the rental price of capital rises, each unit of capital is more expensive, so the choice of capital must fall.

Formal logic: The objective in (L1) satisfies increasing differences in $(k,-r)$. So capital is nonincreasing in its own price.

Notice the verbal logic applies for anything which makes increasing capital by a unit more expensive: likewise, so does the formal logic.

How does *k* change with *q*? When is capital a normal input?

Verbal logic: Consider the rate of technical substitution, which is the rate at which labor can be decreased in response to a unit increase in capital, while still producing q. If *higher* quantities lead to a situation where a unit increase in capital allows a savings of *more* labor than before, then the firm wants to substitute capital for labor when q increases. Then, capital increases with q.

Formal logic: The objective function in (L1) satisfies increasing differences in (k,q) if $-L(k,q)$ satisfies increasing differences in (k,q). We can interpret this directly as a rate of technical substitution (try it!). If L is differentiable, we know that $-L_k(k,q)=F_k(k,l)/F_l(k,l)\Big|_{l=L(k,q)}$, which is nondecreasing in q if the rate of technical substitution increases as we move to higher isoquants (higher q).

Two step problem: minimize costs and then maximize profits.

$$l^{**}(x,w) = \left\{ l \,\middle|\, (\exists k)\ (k,l) \in \underset{k,l\ s.t.\ F(k,l)=x}{\arg\min}\ rk + wl \right\}.$$

The second step is:

$$\max_x px - C(x,w)$$

How does the choice of quantity (x) decrease with *w*? If labor is a normal input!

Verbal logic: Higher wages lead to higher marginal costs of output (and thus lower choices of output) exactly when labor is a normal input. That is, if producing more output leads you to use more labor, then higher wages make it more expensive to produce output.

Formal logic: $px - C(x,w)$ has *de*creasing differences in (x,w) when $C(x,w)$ has increasing differences. Since the marginal effect of w on $C(x,w)$ is $l^{**}(x,w)$ by the envelope theorem, w increases the marginal cost of output when $l^{**}(x,w)$ is increasing in x.

Profit Maximization (all in 1 step):

$$\max_{k,l} \pi(k,l;r,w) \equiv pF(k,l) - rk - wl \tag{L3}$$

We break the firm's maximization problem into two stages, as follows:

$$\hat{l}(k;r,w) = \sup_{l}\left\{\arg\max_{l} \pi(k,l;r,w)\right\} \tag{L4}$$

$$\hat{k}(r,w) = \sup_{k}\left\{\arg\max_{k} \pi(k,\hat{l}(k;r,w);r,w)\right\} \tag{L5}$$

Fact: $\hat{l}(k;r,w) = \hat{l}(k;w)$, since capital is fixed when l is chosen.

Own price effects:

Verbal logic: An increase in r makes each unit of k more expensive, so the firm chooses less k.

Formal logic: Choice of capital solves $\max_{k} pF(k,\hat{l}(k;w)) - rk - w\hat{l}(k;w)$. This satisfies increasing differences in $(k,-r)$. Thus k is nonincreasing in r.

Cross price effects:

Definition: k and l are complements if $F(k,l)$ satisfies increasing differences in (k,l). Substitutes if $F(k,l)$ satisfies decreasing differences in (k,l).

Verbal logic: An increase in r always leads to a (weak) decrease in k. But if capital and labor are complements, a decrease in k always leads to a decrease in l, since capital increases the incremental returns to labor. Thus, when capital and labor are complements, an increase in r will lead to a (weak) decrease in *both* k and l.

In contrast, when capital and labor are substitutes, a decrease in k leads to a weak *increase* in l, and so capital and labor will move in opposite directions. Thus, an increase in r will lead to a (weak) decrease in k and a (weak) increase in l.

Changes in input prices on maximum profits:

Verbal logic: When the price of capital goes up by a small amount, a maximizing firm's profits change according to the amount of capital which was currently in use, since given a smooth profit function, the adjustments to capital and labor in response to a small price change are second order.

Formal logic: The objective is differentiable in r. If in addition, the production function is continuous in k, the envelope theorem tells us that $\frac{\partial}{\partial r}\pi^*(r,w)$ exists almost everywhere, and further that $\frac{\partial}{\partial r}\pi^*(r,w) = -\hat{k}(r,w)$ where it exists.

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Consider the profit maximization problem above.

Verbal logic (case of complements): In the short run, an increase in the wage leads to a decrease in the choice of labor. However, labor is chosen at the old (long-run) choice of capital. In the long run, capital can adjust, and since capital and labor are *complements*, the higher wage will lead to lower levels of *both* capital and labor. When comparing the short and long run choices of labor, note that the short run decision was made at a *higher* level of capital: since they are *complements*, the optimal choice of labor was higher in the short run. Thus, labor adjusts (goes down) by more in the long run than in the short run.

Formal logic: By the comparative statics theorems, if F has increasing differences, our above results tell us that: $\hat{l}(k;w)$ is nondecreasing in k and nonincreasing in w and, $\hat{k}(w)$ is nonincreasing. Thus, for all $w>w_0$, $\hat{k}(w_0) > \hat{k}(w)$, and $\hat{l}(\hat{k}(w_0);w) \geq \hat{l}(\hat{k}(w);w) = l_{LR}(w)$.

Verbal logic (case of substitutes): In the short run, an increase in the wage leads to a decrease in the choice of labor. However, labor is chosen at the old (long-run) choice of capital. In the long run, capital can adjust, and since capital and labor are *substitutes*, the higher wage will lead to lower levels of labor and *higher* levels of capital. When comparing the short and long run choices of labor, note that the short run decision was made at a *lower* level of capital: since they are *substitutes*, the optimal choice of labor was higher in the short run. Thus, labor adjusts (goes down) by more in the long run than in the short run.

Formal logic: By the comparative statics theorems, if F has increasing differences, our above results tell us that: $\hat{l}(k;w)$ is nonincreasing in both arguments and, $\hat{k}(w)$ is nondecreasing. Thus, for all $w>w_0$, $\hat{k}(w_0) < \hat{k}(w)$, and $\hat{l}(\hat{k}(w_0);w) \geq \hat{l}(\hat{k}(w);w) = l_{LR}(w)$.

Hope this helps... Don't Stress!!