Output changing with input prices -- correcting sign confusion!

Next, we ask how total output chosen changes with input prices. We will now let our choice variable be total output, and we denote the minimum cost way of producing x be as follows:

$$C(x,w) = \begin{array}{c} \min\limits_{k,l} rk + wl \\ \text{subject to } F(k,l) = x \end{array}.$$

Further, let the minimum-cost choice of labor be:

$$l^{**}(x,w) = \left\{ l \middle| (\exists k)\ (k,l) \in \underset{k,l\ s.t.\ F(k,l)=x}{\arg\min}\ rk + wl \right\}.$$

The firm's problem can be restated as follows (where we now allow the price of output to vary):

$$\max_x px - C(x,w)$$

Now, let us look for conditions under which the choice of output goes down when the wage rises. That is, when does x increase with $-w$? According to our theorems, if $-C$ is smooth, decreasing and concave (C is smooth, increasing and convex), then the critical sufficient condition for monotone comparative statics in this problem is that C has increasing differences. Moreover, this condition is still sufficient even if C is not convex.

Then, we are left with the final question, when does C satisfy increasing differences? According to the Envelope Theorem, $C_w(x,w)=l^{**}(x,w)$ when this set is a singleton. Moreover, by Theorem 2.2, C has increasing differences exactly when $C_w(x,w)$ is nondecreasing in x, that is, when $l^{**}(x,w)$ is nondecreasing in x. By definition, this means that the critical sufficient condition for total output to decrease with the wage rate is that labor be a "normal input," that is, an input whose use grows with the level of output.

But, we showed above that a critical sufficient condition for labor to be a normal input is that isoquants (now, of $K(l,q)$) become steeper as output increases.