Lecture Notes on Monotone Comparative Statics and Producer Theory

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These notes summarize the material from the monograph which is most important for this course. It is not presented in the same order as the monograph, so I have not preserved theorem numbers, etc., but I don't think it will be too hard to find the relevant sections. Much of the material is drawn from the end of Chapter 2. These notes are pretty rough so if in doubt refer to the monograph.

These notes, as well as the monograph, may have many typos and suffer from "copy-and-paste" mistakes as well as notational inconsistencies. If you find mistakes in the monograph, please send me email, and also let me know what parts you found confusing.

I. Comparative Statics and Producer Theory

Consider the two-input profit-maximization problem. For the moment, let consider the cost-minimization part of this problem. The firm's problem is as follows:

$$\min_{\substack{k,l \\ s.t.\, F(k.l)=q}} \quad rk + wl$$

In the case where output is strictly increasing in each input and the production function F is suitable well-behaved, we can define the isoquant function $L(k,q)$ according to the following implicit function:

$$F(k,L(k,q)) = q.$$

Thus, we can rewrite the firm's problem as follows:

$$\max_k - rk - wL(k,q) \qquad \text{(L1)}$$

Question: How does the firm's choice of capital (k) change with the input prices and the level of output required (q)?

Notice that we in general want conditions which will allow us to answer these questions independent of the particular parameter values. In other words, we do not rule out *ex ante* any particular values of q, r, and w. Thus, we want the solution to be robust to all positive prices and quantities, or at least all prices and quantities within a given range.

To answer the comparative statics questions posed above, and others, we will develop a general theory of univariate comparative statics in problems with an additive structure such as the problem (L1).

II. Univariate Comparative Statics: The General Setup

The general form of the problem is as follows:

$$\arg\max_{x \in S} f(x,\theta) + g(x) \qquad \text{(L2)}$$

The choice variable is x, the parameter of interest for comparative statics is θ, and g belongs to a family of functions (G), and we want the result to hold for all $g \in G$.

To be concrete, in the producer theory problem, if we wanted to ask:

> How does capital change with the rental price r?
> Then: x is capital, θ is the rental price, $f(x,\theta) = -\theta\, x$.
> typical $g(x) = -w \cdot L(x,q)$
>
> $G = \{-w \cdot L(x,q) \mid w, q \in \Re_+,\ L \text{ is "allowable" isoquant}\}$

The family of functions G corresponds to different wages, different quantities, and different production functions (we might want to specify some allowable class of production functions w/ corresponding isoquants).

Likewise, if we ask:

> How does capital change with output q?
> Then: x is capital, θ is output quantity, $f(x,\theta) = -w \cdot L(x,\theta)$.
> typical $g(x) = -r\, x$
>
> $G = \{ -r\, x \mid r \in \Re_+\}$.

We might be interested in the family of functions G which corresponds to different rental prices r. This leads us to a special case we will consider explicitly:

$$\arg\max_{x \in S} f(x,\theta) + t \cdot x$$

This just formalizes the idea that we will especially interested in problems where the family of functions G contains only linear functions of the form $t \cdot x$.

III. Non-Convexities, Multiple Optima, and the Implicit Function Theorem

Recall the implicit function theorem for strictly quasi-concave, differentiable objectives with an interior optimum. If $x \in \Re$, we have

> $x^*(\theta) = \arg\max_{x \in S} f(x,\theta)$ is nondecreasing in θ iff $f_{x\theta}(x^*(\theta),\theta) \geq 0$.

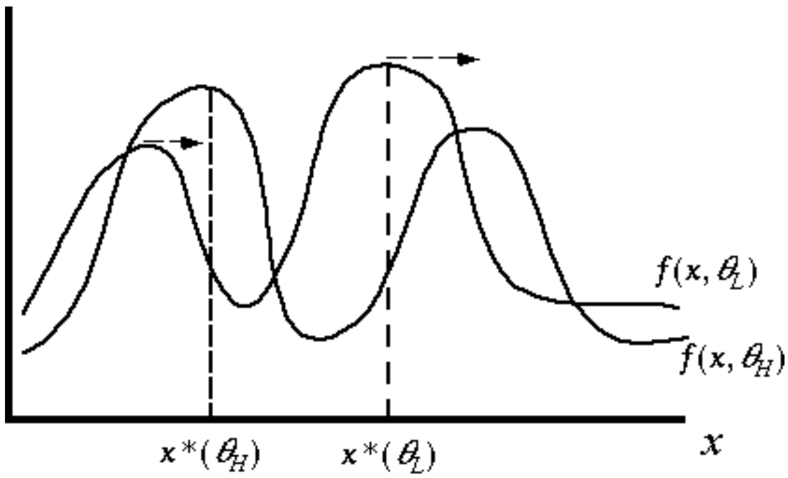
That is, $f_{x\theta}(x,\theta) \geq 0$ whenever $f_x(x,\theta) = 0$.

If $\mathbf{x} \in \Re^2$, then

$$x_1{}^*(\theta) = \arg\max_{\mathbf{x}\in S} f(\mathbf{x},\theta) \text{ is nondecreasing in } \theta \text{ iff } f_{1\theta}(\mathbf{x},\theta) + \frac{f_{2\theta}(\mathbf{x},\theta)\cdot f_{12}(\mathbf{x},\theta)}{\left|f_{22}(\mathbf{x},\theta)\right|} \geq 0$$

evaluated at $\mathbf{x} = \mathbf{x}^*(\theta)$.

When we relax our usual quasi-concavity assumptions on an objective function, we must deal with the possibility of multiple optima. Since the implicit function is based on the first order conditions for maximization (which do not uniquely determine the optimum in non-concave problems), it will not have anything to say about what happens to a global optimum when parameters change.



In the figure, the global optimum is decreasing in θ. However, each local optimizer is nondecreasing in the parameter θ, which would be consistent with the condition that $f_{x\theta}(x,\theta) \geq 0$ whenever $f_x(x,\theta)=0$.

IV. Multiple Optima: The Strong Set Order in $\Re$

We wish to have a theory which can handle multiple optima. We will consider two possible approaches. The first is to choose extremal optimizers and analyze those: that is, ask what happens to the *lowest* and *highest* optimizers (or, more generally, the greatest lower bound and least upper bound). The second approach is to define what we mean for a *set* of optimizers to increase with a parameter, and to do comparative statics on the set of optimizers.

It turns out that the content of our theorems is the same for either approach. So, if you find any of the discussion of orders over sets confusing, you will not lose that much by thinking in terms of lowest and highest optimizers.

For the moment, we will consider only the case where our choice variable is a real number, x, and the objective function is $\pi(x,\theta)$. Then, we define:

$$x^{\mathrm{H}}(\theta) \equiv \sup_x \left\{ \arg\max_{x \in S} \pi(x,\theta) \right\}$$
$$x^{L}(\theta) \equiv \inf_x \left\{ \arg\max_{x \in S} \pi(x,\theta) \right\}.$$

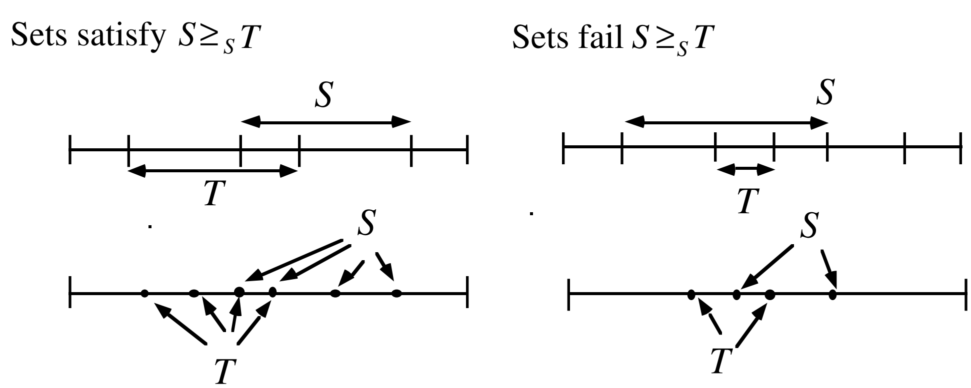
For simplicity these notes will treat $x^{\mathrm{H}}(\theta)$. Further, define $X^*(\theta) \equiv \arg\max_{x \in S} \pi(x,\theta)$.

Now consider what it might mean for one subset of $\Re$ to be higher than another. The following definition treats the *strong set order*, attributed to Pete Veinott.

Definition: A set $S \subseteq \Re$ *is* ***as high as*** *another set* $T \subseteq \Re$*, written* $S \geq_S T$*, if for every* $x \in S$ *and* $y \in T$*,* $y \geq x$ *implies both* $x \in S \cap T$ and $y \in S \cap T$.

Definition: A set-valued function $H: \Re \to 2^{\Re}$ *is* ***nondecreasing*** *if for* $x > y$*,* $H(x) \geq_S H(y)$.

The strong set order is illustrated in the following figure.



Note that the empty set as high as itself. Thus, we can state comparative statics theorems and existence theorems separately. See the monograph for further discussion.

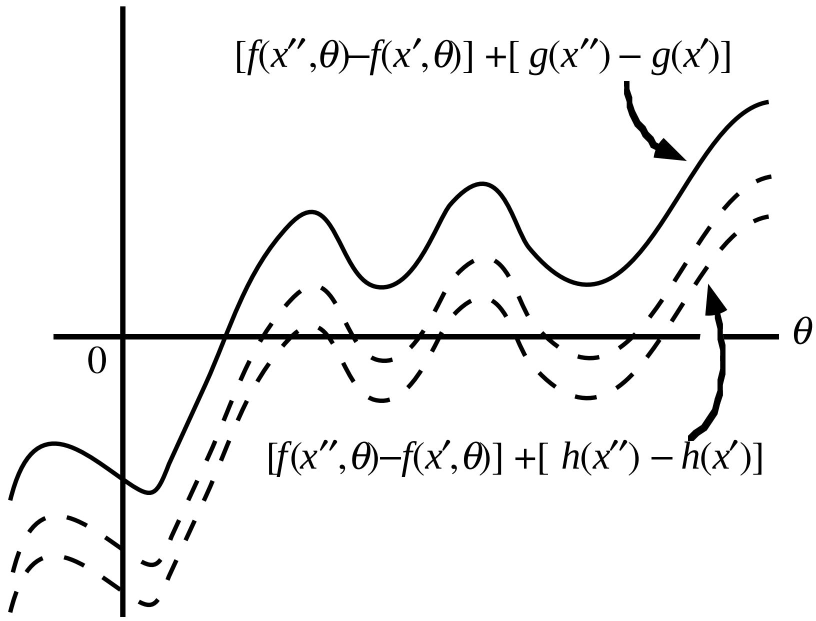
If a set is nondecreasing in the strong set order, so its supremum and its infimum will be increasing as well. It turns out that our comparative statics results are the same when we analyze comparative statics questions about the greatest upper bound of optimizers, $x^H(\theta)$, or the entire set of optimizers, $X^*(\theta)$. For simplicity we'll stick to x^H in these notes.

V. Increasing Differences and Comparative Statics

To begin, consider the choice between two values of x, $x'' > x'$, when the agent's objective is $\pi(x,\theta) = f(x,\theta) + g(x)$. The agent looks at the sign of the following expression, which show the returns to changing from x' to x'':

$$\begin{aligned} &f(x'',\theta) + g(x'') - [f(x',\theta) + g(x')] \\ &= [f(x'',\theta) - f(x',\theta)] + [\, g(x'') - g(x')] \end{aligned}$$

When this in positive, the agent prefers the higher choice, while when it is negative, the agent prefers the lower choice. Notice that the first bracketed expression depend on θ, while the second bracketed expression is constant in θ. The following figure shows how the returns to the higher choice might change with θ.



With the original function g, the returns changing from x' to x'' change sign once as a function of θ, from negative to positive. So increasing θ can't possibly cause the agent to switch from x'' to x'. This is a comparative statics result: increasing θ can't make x shift down if $\{x',x''\}$ are the only two choices.

However, this result is not robust to the specification of g. Note that changing g is equivalent to shifting the curve in the figure up or down by a constant, since g doesn't depend on θ. Thus, for an alternative function h, increasing θ could cause the agent to increase *or* decrease x, depending on the particular values of θ chosen.

Clearly, the only way to ensure that the comparative statics result holds for any function g is to require that the incremental returns $f(x'',\theta)-f(x',\theta)$ are nondecreasing in θ. If this condition holds for all $x'' > x'$, we say that the function f satisfies *increasing differences.*

The following result, proved using the fundamental theorem of calculus, characterizes the property increasing differences.

Theorem . Let $f(x,\theta){:}\Re^2 \to \Re$.

(1) If f is twice continuously differentiable, then f has increasing differences if and only if for all (x,θ), $\frac{\partial^2}{\partial x \partial\theta} f(x,\theta) \geq 0$.

(2) If, for all θ, $f(x,\theta)$ *is continuously differentiable in x, then f has increasing differences if and only if for all x,* $\frac{\partial}{\partial x} f(x,\theta)$ *is nondecreasing in* θ.

(3) If, for all x, $f(x,\theta)$ *is continuously differentiable in* θ, *then f has increasing differences if and only if for all* θ, $\frac{\partial}{\partial\theta} f(x,\theta)$ *is nondecreasing in x.*

Now, we have the following comparative statics theorem on the lowest and highest optimizers of the objective:

$$\arg\max_{x \in S} f(x,\theta) + g(x)$$

Theorem. Assume the following:

$S \subseteq \Re$.

No restrictions are imposed on f.

No restrictions are imposed on g. $(G = \{g{:}\Re \to \Re\})$.

Then, $x^H(\theta;g)$ *is nondecreasing in* θ *for all functions g if and only if f has increasing differences. The same conclusion holds for the set* $X^*(\theta,g)$.

Next, we ask if adding convexity and differentiability assumptions buys us anything. The answer is no.

Theorem. Assume the following:

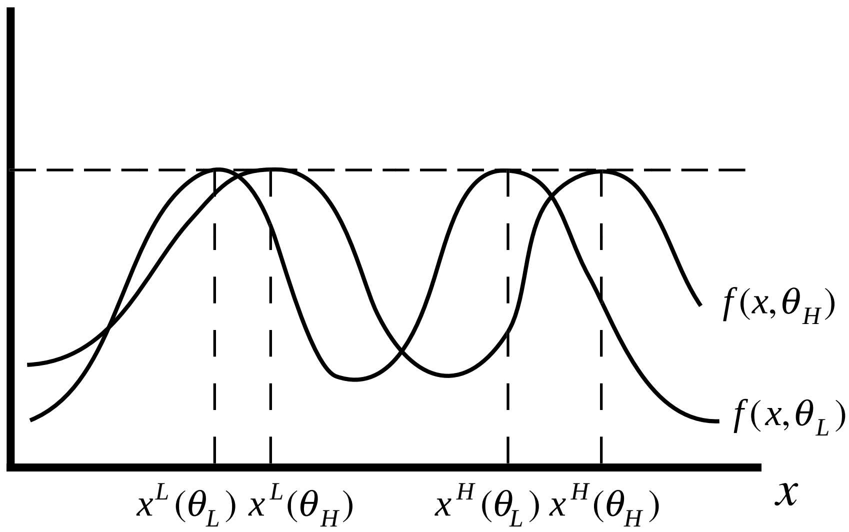
$S \subseteq \Re$ *is convex and compact.*

The function $f(x,\theta)$ *is twice continuously differentiable in* (x,θ) *and strictly concave in x.*

The function $g_t(x) = t \cdot x$. $(G = \{g_t{:}g_t(x) = t\,x\})$.

Then $x^H(\theta;f,g_t)$ *is nondecreasing in* θ *for all t, if and only if f has increasing differences. The same conclusion holds for the set* $X^*(\theta,g)$.

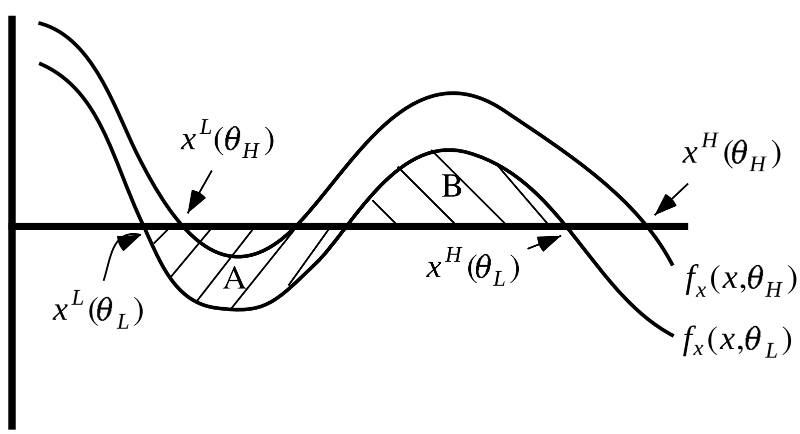
Changes in Sets



Two optima which both increase in θ. We will show that this f cannot satisfy increasing differences.

Another possibility which is ruled out by the theorem is illustrated in above. In this illustration, there are two global maxima, and both shift up when the parameter value increases.

To see why this cannot happen, consider the case where f is differentiable in x. The figure below graphs f_x as a function of x. By the definition of increasing differences, moving from θ_L to θ_H increases this curve. If we are indifferent between $x^L(\theta_L)$ and $x^H(\theta_L)$, then the area marked A must be equal to the area marked B (this follows since $f(x^L(\theta_L),\theta_L) = f(x^H(\theta_L),\theta_L)$ if and only if $\int_{x^L(\theta_L)}^{x^H(\theta_L)} f_x(x,\theta_L)dx = 0$, by the fundamental theorem of calculus). But, increasing θ to θ_H must decrease A and increase B, so that $x^H(\theta_H)$ is strictly preferred to $x^L(\theta_H)$.



Increasing differences rules out the case of two optima which both increase in θ.

VI. Applications: Comparative Statics in Producer Theory

Now we return to our motivating problems.

Effects of changes in an input's own price on input demand

How does capital change with the rental price r?

> Then: x is capital, θ is the rental price, $f(x,\theta) = -\theta\, x$.
> $g(x) = -w \cdot L(x,q)$.

Applying our theorems, the answer is immediate. $f(x,\theta)$ has increasing differences in $(x,-\theta)$, and so:

> In problem L1, the highest choice of capital (k) (alternatively, the set of optimal chioces of capital) is nonincreasing in the rental price (r). This result does not rely on *any* assumptions except the setup of the objective function.

(Though the statement has more content if there exists an optimal choice of capital in the relevant region.) Further, this condition would continue to hold for any specification of the input costs, i.e., allowing for monopsony power or scarcity in the input market, *so long as the parameter r increases the incremental or marginal costs of capital.*

Effects of changes in output on input demand

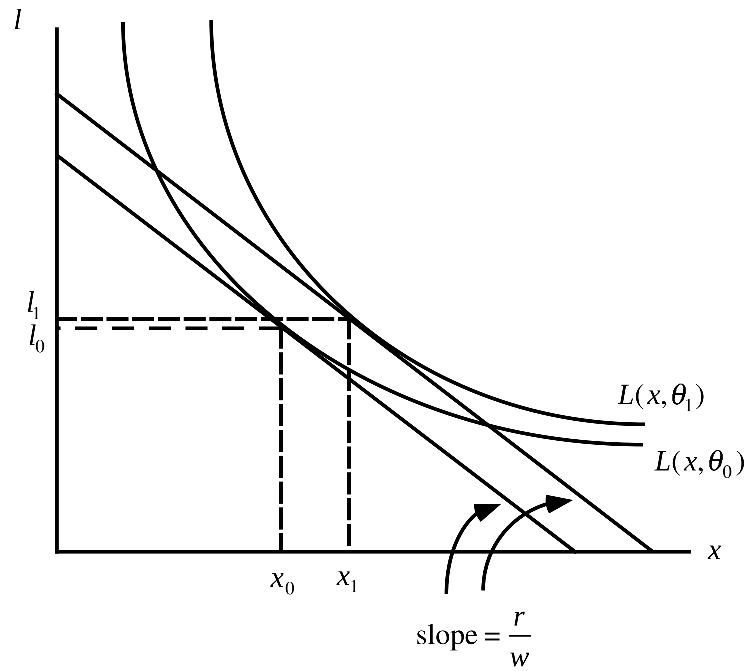
How does capital change with output q?

> Then: x is capital, θ is output quantity, $f(x,\theta) = -w \cdot L(x,\theta)$.
> $g(x) = -r\, x$.

Applying our theorems, we see that the critical sufficient condition for capital to increase with output (i.e. labor is a normal input) is that $L(x,\theta)$ satisfies increasing differences.

How do we interpret this? Well, if the isoquant function is appropriately differentiable, we are looking for the condition $L_{x\theta} \leq 0$. That is, if increasing the level of output required makes the isoquant steeper at a fixed level of capital, then the demand for capital will increase with output.

Recall that an isoquant function satisfies $F(x,L(x,\theta)) = \theta$. Thus, if F is continuously differentiable and the isoquant is well-defined, we conclude that $-L_x(x,\theta) = F_x(x,l)/F_1(x,l)\big|_{l=L(x,\theta)}$, which will be increasing in θ if $F_x(x,l)/F_l(x,l)$ is increasing in labor for a given level of capital. This is the standard condition on the marginal rate of technical substitution from price theory.



When the isoquant $L(x,\theta)$ becomes steeper when output (θ) increases, the choice of capital (x) increases with output.

Is there a weaker condition which will yield this conclusion? Our theorems tell us not, irrespective of convexity and differentiability assumptions. Capital is a normal input *if and only if* the isoquant function satisfies increasing differences in $(x,-\theta)$ that is, if and only if $F_x(x,l)/F_l(x,l)$ is increasing in l for any given level x of capital.

A typical intuitive argument based on the standard theory might go as follows: If θ_0 is the initial output level, the optimal capital-labor combination (x_0,l_0) must satisfy $r/w = F_x(x_0,l_0)/F_l(x_0,l_0)$. Further, convexity implies that $F_x(x,l)/F_l(x,l)$ is diminishing in x along an isoquant. Since moving to a higher isoquant (θ=θ_1) leads to an *increase* in marginal rate of substitution at the old choice of capital, then to get to a new optimum, capital must be increased to $x_1 > x_0$ in order to diminish the marginal rate of substitution so that the new optimal point, (x_1,l_1), satisfies $r/w = F_x(x_1,l_1)/F_l(x_1,l_1)$.

However, this cannot be a correct intuition for the robust comparative statics conclusion, since we know increasing differences is the critical sufficient condition *even in the absence of convexity assumptions.*

Cross-Price Effects on Input Demands

To consider cross-price effects, it is helpful to study the firm's profit maximization problem, given as follows:

$$\max_{k,l} \pi(k,l;r,w) \equiv pF(k,l) - rk - wl \qquad \text{(L3)}$$

We break the firm's maximization problem into two stages, as follows:

$$\hat{l}(k;r,w) = \sup_{l}\left\{\arg\max_{l} \pi(k,l;r,w)\right\} \tag{L4}$$

$$\hat{k}(r,w) = \sup_{k}\left\{\arg\max_{k} \pi(k,\hat{l}(k;r,w);r,w)\right\} \tag{L5}$$

With this notation in place, we can begin to use the comparative statics theorems to answer comparative statics questions about input demands.

Our first observation concerns the function $\hat{l}(\cdot)$. Notice that

$$\arg\max_{l} \pi(k,l;r,w) = \arg\max_{l} pF(k,l) - rk - wl = \arg\max_{l} pF(k,l) - wl$$

so that the choice of labor in this sub-problem is independent of the rental price of capital (since capital is fixed, its price does not affect the optimization). Thus, we write $\hat{l}(k;r,w) = \hat{l}(k;w)$.

A standard price-theoretic definition holds that capital and labor are substitutes or complements depending on whether an increase in the price of capital increases or reduces the optimal quantity of labor. We shall use a different definition here – one that is not so closely tied to price theory. The new definition is mathematically equivalent to this older definition in the two input case if we limit attention to firms with strictly concave production functions.

Specifically, we define two inputs to be *complements* if an exogenous increase in one input increases the returns to using more of the other input; two inputs are *substitutes* if such an exogenous increase in one input *decreases* the returns to using more of the other.

For the problem at hand, then, capital and labor are complements in this sense if $F(k,l)$ satisfies increasing differences in (k,l), while capital and labor are substitutes if $F(k,l)$ satisfies increasing differences in $(-k,l)$. (In this latter case, we will say that F satisfies *decreasing differences* in (k,l)).

For the moment, let us fix w and supress it in our notation. Then, a pair that solves problem (L3) can be written as $(\hat{k}(r),\hat{l}(\hat{k}(r)))$. But then:

> F has increasing differences implies that $\hat{l}(k)$ is nondecreasing.
>
> Then, $k(r)$ nonincreasing implies that $\hat{l}(\hat{k}(r))$ is nonincreasing.

Proposition. Let $k^*(r,w) \equiv \hat{k}(r,w)$*, and let* $l^*(r,w) \equiv \hat{l}(\hat{k}(r,w);w)$.

(i) Then (with no restrictions on $F(k,l)$*),* $k^*(r,w)$ *is nonincreasing in r, and* $l^*(r,w)$ *is nonincreasing in w.*

(ii) If $F(k,l)$ *satsifies increasing differences in* (k,l)*,* $k^*(r,w)$ *and* $l^*(r,w)$ *are nonincreasing in both arguments; in contrast, if* $F(k,l)$ *satsifies decreasing differences in* (k,l)*,* $k^*(r,w)$ *is nondecreasing in w and* $l^*(r,w)$ *is nondecreasing in r.*

(iii) Results (i) and (ii) continue to hold if the objective function in (P4) is replaced by $\hat{\pi}(k,l;r,w) \equiv p\,F(k,l) - h^k(k;r) - h^l(w;l)$*, so long as the functions* $h^k(k;r)$ *and* $h^l(l;w)$ *satisfy increasing differences.*

Using the Envelope Theorem: Effects of Changes in Input Prices on Maximum Profits and Output

Here is a general version of the envelope theorem.

Theorem ***(Envelope Theorem)****. Suppose that* $S \subseteq \Re^N$ *is a compact set, that* $h(\mathbf{x},\theta)$ *is differentiable in* θ *and continuous in* $\mathbf{x}$*, and that the derivative* h_θ *is bounded. Let* $H(\theta) = \max_{\mathbf{x}\in S} h(\mathbf{x},\theta)$*. Then H is absolutely continuous and differentiable almost everywhere, and for all* θ *such that* $H'(\theta)$ *exists,* $H'(\theta) = h_\theta(\mathbf{x},\theta)$ *for* $\mathbf{x} \in \mathbf{x}^*(\theta)$*.*

Now, if we ask how the firm's maximum profits from solving (L3) change with input prices, it is immediately that almost everywhere, the effect on profits of a small increase in the wage rate is simply a decrease in profits equal to the optimal choice of labor. [For the mathematically curious: what do you know about the set of parameter values for which there can be multiple optima in this problem?]

Next, we ask how total output chosen changes with input prices. We will now let our choice variable be total output, and we denote the minimum cost way of producing x be as follows:

$$C(x,w) = \begin{array}{l} \min_{k,l} rk + wl \\ \text{subject to } F(k,l) = x \end{array}.$$

Further, let

$$l^{**}(x,w) = \left\{ l \,\middle|\, (\exists k)\ (k,l) \in \operatorname*{arg\,min}_{k,l\ s.t.\ F(k,l)=x} rk + wl \right\}.$$

The firm's problem can be restated as follows (where we now allow the price of output to vary):

$$\max_x px - C(x,w)$$

So, according to our theorems, if $-C$ is smooth, decreasing and concave (C is smooth, increasing and convex), then the critical sufficient condition for monotone comparative statics in this problem is that $-C$ has increasing differences. Moreover, this condition is still sufficient even if C is not convex.

Then, we are left with the final question, when does $-C$ satisfy increasing differences? According to the Envelope Theorem, $C_w(x,w)=l^{**}(x,w)$ when this set is a singleton. Moreover, by Theorem 2.2, $-C$ has increasing differences exactly when $C_w(x,w)$ is nonincreasing in x, that is, when $l^{**}(x,w)$ is nondecreasing in x. By definition, this means that the critical sufficient condition for total output to decrease with the wage rate is that labor be a "normal input," that is, an input whose use grows with the level of output. But, we showed above that a critical sufficient condition for labor to be a normal input is that isoquants become steeper as output increases.

VII. The LeChatlier Principle

As above, let $\hat{k}(r,w)$ denote the (long-run) optimal quantity of capital to employ at each r and w, and let $\hat{l}(k;w)$ denote the short run optimal choice of labor. Since we are holding r fixed, we will suppress it in our notation. For a given wage w, the long-run demand for labor will be denoted

$$l_{LR}(w) \equiv \hat{l}(\hat{k}(w);w).$$

According to the usual treatment of these issues in textbook analyses, if the relevant functions are differentiable at w, then $l_{LR}'(w) \le \frac{\partial}{\partial w}\hat{l}(k,w)\big|_{k=\hat{k}(w)}$. To see this, note that:

$$\pi(\hat{k}(w),l_{LR}(w)) - \pi(k,\hat{l}(k;w)) \ge 0$$

since the long-run optimal choices of capital and labor must be better than what can be done with short-run best responses to an arbitrary amount of capital. But this must equal zero at $k = \hat{k}(w)$, and thus the two functions must be tangent at that point (the difference between the profit functions is minimized at $\hat{k}(w)$ so the derivatives must be the same). This confirms that the short and long-run net supplies must be equal at $k = \hat{k}(w)$, i.e.:

$$-l_{LR}(w) - [-\hat{l}(\hat{k}(w);w)] = 0.$$

But further, since the difference between the profit functions is at a *minimum*, it must be locally convex, and thus $l_{LR}'(w) \le \frac{\partial}{\partial w}\hat{l}(k,w)\big|_{k=\hat{k}(w)}$.

This is supposed to formalize the idea that demand varies more in the long-run than in the short-run. Strikingly, *it requires no assumptions beyond differentiability of the factor demand functions*. However, the intuition has no qualifications about very small changes, while the formal argument does.

Now consider an alternative approach based on our comparative statics theorems.

Suppose first that capital and labor are complements, that is, that F has increasing differences. Recall first that this implies:

$\hat{l}(k;w)$, is nondecreasing in k and nonincreasing in w.

(Intuitively, this is because capital increases the marginal product of labor, while higher w makes labor more expensive.) Also, recall that:

$\hat{k}(w)$ is nonincreasing

(since labor will fall in response to higher wages and lower labor decreases the returns to increasing capital).

Now, consider an initial wage w_0. Then:

For any increase in the wage to $w>w_0$, $\hat{k}(w_0) > \hat{k}(w)$,
and we have

$$\hat{l}(\hat{k}(w_0);w) \geq \hat{l}(\hat{k}(w);w) = l_{LR}(w).$$

The quantity of labor demanded falls more in response to a price increase in the long run than in the short run. The intuition is simple: the higher short-run level of labor will be chosen when the marginal product of labor is higher (due to the higher fixed capital stock).

On the other hand, when F has decreasing differences so that capital and labor are substitutes, then:

$\hat{l}(k;w)$ is nonincreasing in both arguments and,

$\hat{k}(w)$ is nondecreasing.

So, again, for all $w>w_0$,

$$\hat{l}(\hat{k}(w_0);w) \geq \hat{l}(\hat{k}(w);w) = l_{LR}(w)$$

the quantity of labor demanded falls more in the long run than in the short run.

Note well that this is a global argument using only the assumptions of increasing or decreasing differences. It applies to both concave and non-concave production functions F and works for price changes of any size.

Its logic is essentially the same as the verbal logic. Suppose that oil and other inputs are substitutes, and the price of oil rises. The initial fall in the quantity of oil demanded leads to a higher marginal product of the substitute input, resulting in increased use of the substitute. That change, in turn, further reduces the marginal product of oil and encourages additional reductions in oil consumption. Complementarity between oil and other inputs leads to a similar amplification of the effect of the initial price change, as the initial reduction in the use oil leads to long-run reductions in the use of complementary inputs (whose marginal products fall as oil usage falls), which in turn encourages further reductions in oil usage. This intuitive idea is reflected exactly in the mathematics.

Assumption of substitutes or complements is not dispensable

The new treatment of the LeChatelier principle involves an extra assumption compared to the treatment using traditional methods, namely, the assumption that *F* has either increasing or decreasing differences. Is this assumption dispensable? It is not. Here is a counterexample.

		Output 0	1
New Capital	0	0	$56\text{-}2p$
	1	–24	$32–p$

Net revenues when the price of oil is *p*.

Output: chosen from {0,1}

Capital: fixed in short run at $k = 0$. Long run: increase by {0,1}.

Price on increasing capital: $24.

Oil usage is variable.

Production function: at k=0, takes 2 units of oil for 1 unit output.
at $k = 1$, takes 1 unit of oil for 1 unit output.

The price of the output is fixed at $56.

Effects of change in price of oil:
Initial price of oil: p = \$20. LR Optimum is $q = 1$, $k = 0$, $oil = 2$. Profits \$56–2p = \$16.

Oil price rises to p =\$30. SR Optimum: $q = 0$, profits = 0.
LR Optimum: $q = 1$, $k = 1$, $oil = 1$. Profits \$32 - p = \$2.

Oil use adjusts by more in the short run.

Intuitively, fuel efficient equipment can be a substitute for oil at low oil prices, because its use reduces oil consumption at the current level of output, but also be a complement for oil at high oil prices, when oil might not be used at all but for the availability of the efficient equipment. There is nothing strange or unconventional or non-robust about this possibility.

Why the difference with the conventional analysis: for a local change, inputs are either substitutes or complements but can't change.