Problems on Comparative Statics

1. Let $g : \Re \to \Re$ be an increasing function. Show that if $f : \Re^2 \to \Re$ has increasing differences, then so does the composite function defined by $h(x,y) \equiv f(g(x),y))$.
2. Let $g : \Re \to \Re$ be continuously differentiable. Prove that the function $f(x,\theta) = g(x - \theta)$ has increasing differences if and only if g is concave. [Extra: Prove the same result under the weaker assumption that g is continuous but not necessarily differentiable. Can one prove the same conclusion without the assumption that g is continuous?]
3. Here is a variation on one of the implications of the increasing differences comparative statics theorem. Suppose that $f(x,\theta)$ has strictly increasing differences, that is, the incremental returns to increasing x are *strictly* increasing in θ. Show that if $\theta'' > \theta'$, then for all $x'' \in x^*(\theta'')$ and all $x' \in x^*(\theta')$, $x'' \geq x'$. [Hint: Write down inequalities which must be satisfied, by definition, in order for the two choices to be optimal at their respective parameter values. In particular, the objective must be at least as large at an optimal value of x as it is at any other value of x. Then try to combine the inequalities in a way that you can use the definition of increasing differences. *I will not put a question like this on the exam.*]
4. Consider a firm that chooses its product price p to maximize profits, which are given by $(p-c)Q(p,\theta)$, where Q is a demand function with parameter θ which is decreasing in p. Let $\partial \ln(Q(p,\theta)) / \partial \ln(p) = \varepsilon(p) + \theta$ be the elasticity of demand at various prices, so that an increase in θ simply makes demand less elastic at each price. Show that $p^*(\theta,c)$ is nondecreasing in each argument. [Hint: maximizing $\pi(x)$ is the same as maximizing $\ln(\pi(x))$. Use this fact to simplify the objective function.]