Microeconomic Theory I *14.121* *Fall 1996*

Problem Set I

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1. (From Frank's lecture notes). Prove that the following are true, using only the axioms given in class: ($\wedge$, by the way, is standard logical notation for "and")

   (a) $\forall\ \boldsymbol{x}, \boldsymbol{x}\ \mathrm{I}\ \boldsymbol{x}$
   (b) $\forall\ \boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$:
   $(\boldsymbol{x}\ \mathrm{P}\ \boldsymbol{y}) \wedge (\boldsymbol{y}\ \mathrm{P}\ \boldsymbol{z}) \Rightarrow (\boldsymbol{x}\ \mathrm{P}\ \boldsymbol{z})$
   $(\boldsymbol{x}\ \mathrm{I}\ \boldsymbol{y}) \wedge (\boldsymbol{y}\ \mathrm{I}\ \boldsymbol{z}) \Rightarrow (\boldsymbol{x}\ \mathrm{I}\ \boldsymbol{z})$
   $(\boldsymbol{x}\ \mathrm{P}\ \boldsymbol{y}) \wedge (\boldsymbol{y}\ \mathrm{I}\ \boldsymbol{z}) \Rightarrow (\boldsymbol{x}\ \mathrm{P}\ \boldsymbol{z})$
   $(\boldsymbol{x}\ \mathrm{I}\ \boldsymbol{y}) \wedge (\boldsymbol{y}\ \mathrm{P}\ \boldsymbol{z}) \Rightarrow (\boldsymbol{x}\ \mathrm{P}\ \boldsymbol{z})$
   (c) If u is a utility function (defined, remember, as $u(\boldsymbol{x}) \geq u(\boldsymbol{y})$ if and only if $\boldsymbol{x}\mathrm{R}\boldsymbol{y}$, so this question isn't quite tautological), prove that:
   $u(\boldsymbol{x}) > u(\boldsymbol{y})$ if and only if $\boldsymbol{x}\mathrm{P}\boldsymbol{y}$, and
   $u(\boldsymbol{x}) = u(\boldsymbol{y})$ if and only if $\boldsymbol{x}\mathrm{I}\boldsymbol{y}$.
   (d) Let v be a monotonic transformation of u (*i.e.* $v = F_\circ u$, where F: $\Re \rightarrow \Re$ is strictly increasing. Show that v is also a utility function.

2. (from Fall 1993 Final Examination) A consumer has a strictly quasi-concave, differentiable utility function given by $U(\ b + x_1, x_2, \ldots, x_n\ )$. Here x_k is the amount of the kth good consumed, and b is a parameter. This question studies the effects of changes in the parameter b.

   (a) Does an increase in b (with all the x_i held constant) increase or decrease the consumer's marginal utility of the first good?
   (b) Write down the Lagrangian for minimising expenditure ($\mathbf{p}\cdot\mathbf{x}$) subject to a utility constraint. What is the derivative of minimised expenditure with respect to b? Use the first-order conditions (for expenditure minimisation) to express that derivative in terms of directly observable variables.
   (c) Using the answer to part (b), what is the value of $\partial x_k/\partial b$ with utility held constant for $k > 1$? For $k = 1$?

3. In a three good world, a consumer has the Marshallian demands given in the table. Are these choices consistent with the usual model of a locally insatiable, utility maximising consumer?

Prices			Income	Demand		
p_1	p_2	p_3	y	x_1	x_2	x_3
1	1	1	20	10	5	5
3	1	1	20	3	5	6
1	2	2	25	13	3	3
1	1	2	20	15	3	1

4. A consumer has utility function $U(\boldsymbol{x}) = [\alpha_1 x_1^{\rho} + \alpha_2 x_2^{\rho}]^{1/\rho}$. Where the x_i are her consumption of the three goods, α_i and ρ are parameters, and $\boldsymbol{x} = (x_1, x_2,)$.

(a) Write down the first order conditions for utility maximisation, given the budget constraint: $\mathbf{x}\cdot\mathbf{p} = Y$, where Y is income and $\mathbf{p} = (p_1, p_2)$ is the vector of prices.

(b) Derive the Marshallian demand functions (the optimal choices of $\mathbf{x}$) and compute the indirect utility function (that is, the maximum level of utility attainable given prices and income).

(c) Verify the adding-up theorems

(d) Write down the optimisation conditions which define the expenditure function and find the first-order conditions.

(f) Derive the expenditure function and the Hicksian demand functions.

(g) Verify Slutsky's Theorem.

(h) Verify that substitution terms are symmetric, that own substitution terms are negative, and that the substitution matrix has the appropriate rank and is negative semi-definite.

5. A consumer in a three good economy (goods and prices denoted x_i, and p_i respectively) with wealth $w > 0$ has demand functions for goods 1 and 2 given by:

$$x_1 = 100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \delta\frac{w}{p_3}$$

$$x_2 = \alpha + \beta\frac{p_1}{p_3} + \gamma\frac{p_2}{p_3} + \delta\frac{w}{p_3}$$

where greek letters are nonzero constants.

(a) Indicate how to calculate the demand for good 3 (but don't actually do it).

(b) Are the demands for goods one and two appropriately homogeneous?

(c) Calculate the restrictions on the numerical values of α, β, γ, and δ implied by utility maximisation.

(d) Given your results in part (c), for a fixed level of x_3 draw the consumer's indifference curve in the x_1, x_2 plane.

(e) What does your answer to (d) imply about the form of the consumer's utility function $u(x_1, x_2, x_3)$?