Problem Set II: Solutions

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1.

(a) The Laspeyres index is based on last period's quantities:
L = $\mathbf{x_0.p_1/x_0.p_0}$ = (2x4 +2x2 + 2x4) / (2 + 2 + 2) = 20/6 = 3 1/3
The Paasche is based on this period's quantities:
P = $\mathbf{x_1.p_1/x_1.p_0}$ = (1x4 + 2x2 + 10x4) / (1 + 2 + 10) = 48/13 = 3 9/13

(b) The answer is "nothing". The trap you are supposed to fall into is to get confused because P < cost of living, and L > cost of living where P > L, but the cost of living indices referred to are different.

(c) The consumer in the second period spent 48. If he or she had bought the first period bundle, he or she would have had to have spent 20. So by revealed preference, you can say that the consumer is better off, since he or she didn't buy the previous period's bundle even though it was feasible. You may only say this if you assume that preferences haven't changed. Imagine that the consumer's boss had committed a particularly bad fashion error in period 1. Even a pay rise mightn't wipe out the dire effects on utility of a particularly tasteless tie. In fact even a consumer with the same indifference maps, but where the curves have merely been renumbered could be better or worse off.

2 This is almost the same as the log-utility example from last week, if you did it.

(a) $E(\mathbf{p},u) = 2u(p_1 p_2)^{1/2}$

(b) The Marshallian demands are $x_i = Y / 2p_i$, so we easily find the uncompensated tax yield to be:

$$T_u = \frac{Y}{2}\left(\frac{\tau_1}{1+\tau_1} + \frac{\tau_2}{1+\tau_2}\right),$$ where τ_i are the *ad valorem* tax rates $\tau_i = t_i / p_i$.

(c) The amount of money is just the difference between the expenditure at the taxed prices and the expenditure at the untaxed prices, at the same utility:

$$\Delta E = E(q,u) - E(p,u) = 2u(p_1 p_2)\left[\sqrt{(1+\tau_1)(1+\tau_2)} - 1\right].$$

(d) We just want the sum of the taxes times the Hicksian demands at the taxed prices **q**:

$$T_c = t_1 u\sqrt{\frac{q_2}{q_1}} + t_2 u\sqrt{\frac{q_1}{q_2}} = u\sqrt{p_1 p_2}\left[\tau_1\sqrt{\frac{1+\tau_2}{1+\tau_1}} + \tau_2\sqrt{\frac{1+\tau_1}{1+\tau_2}}\right].$$

(e) The dead-weight loss of the compensated tax is the difference between the amount by which the consumer is worse off, and the compensated tax raised:

$$L_c = \Delta E - T_c = u\sqrt{p_1 p_2}\,\frac{\left[\sqrt{1+\tau_1} - \sqrt{1+\tau_2}\right]^2}{\sqrt{1+\tau_1}\sqrt{1+\tau_2}}.$$

You'll notice that this is always positive. Interestingly, it is zero if the tax rate on both goods is the same. Why? Because of the nature of the demand functions, equal *ad valorem* taxes don't distort consumer choice. In general, this will not be the case. I'll try to draw graphs in class.

(f) We are now restricted to tax only the second good. This means that the DWL is:

$$L_c(\tau_2) = u\sqrt{p_1 p_2}\,\frac{\left[\sqrt{1+\tau_2} - 1\right]^2}{\sqrt{1+\tau_2}}.$$ Differentiating this,

$$\frac{\partial L}{\partial \tau_2} = \frac{u\sqrt{p_1 p_2}}{2\sqrt{1+\tau_2}}\left(1 - \frac{1}{1+\tau_2}\right).$$ This is strictly positive.

(g) In this problem, because of the demand functions, the government can raise as much as it likes with zero DWL by making sure that the *ad valorem* rates on the two goods are equal.