Microeconomic Theory I *14.121* *Fall 1996*

Problem Set II

Charles Morcom E52-243C
Tel: x3-8712, email: chasm@mit.edu

Office Hours:
Tuesdays & Wednesdays 11:00-12:00

---

1. A consumer in a three good world faces initial prices $p_1 = p_2 = p_3 = 1$. He or she buys $x_1 = x_2 = x_3 = 2$. Prices change, and in the following year, the consumer faces $p_1 = 4$, $p_2 = 2$, and $p_3 = 4$. He or she buys $x_1 = 1$, $x_2 = 2$, $x_3 = 10$.
   (a) Construct Paasche and Laspeyres price indices.
   (b) What can you say about the consumer in view of the statement that "the Laspeyres price index bounds the true cost of living index from above, while the Paasche bounds it from below"?
   (c) Given the possible answers to (b), what can you say about whether the consumer is better or worse off in the second period than the first?

2. A consumer has Cobb-Douglas utility function $u(\boldsymbol{x}) = x_1^{1/2}x_2^{1/2}$. Prices are $\boldsymbol{p}$. Income is Y.
   a) Find the expenditure function $E(\boldsymbol{p},u)$.

   The government now introduces taxes $\boldsymbol{t} = (t_1, t_2)$, so that the prices now become $\boldsymbol{q} = \boldsymbol{t} + \boldsymbol{p}$.
   b) How much money does the tax raise if the consumer is not given any additional income (This is the *uncompensated* tax yield)?
   c) How much money would have to be given to the consumer to make him or her as happy at the new prices $\boldsymbol{q}$ as he or she was before the introduction of the taxes?
   d) How much money does the tax raise if the consumer is given enough money to be as happy as he or she was without the taxes. (This is the *compensated* tax yield, as analyzed in class.)
   e) What is the dead-weight loss of the compensated tax (again, as analyzed in class)? Show that it is always positive for positive taxes. Illustrate your answers to (d) and (e) with a graph.
   f) Suppose now that the tax on the first good is zero. Show that the dead-weight loss is strictly increasing in the tax rate on the second good, $\tau_2 = t_2 / p_2$.
   g) Even aware as it is that taxes are inefficient, the government wants to use a consumption tax to raise revenue R to build a statue of Paul Samuelson in the market place. How should it choose tax rates on goods 1 and 2 so as to do this in the most efficient way? That is, find the tax rates which minimize the dead-weight loss associated with raising R. Set up (*i.e.* a Lagrangian *and* first-order conditions) the problem, but do not solve it (unless you want to). Interpret your results.