

Cheat Sheet: This is what you need to know!!

Cost minimization:

$$\max_k -rk - wL(k, q) \quad (\text{L1})$$

How does k change with r ? Own-price effects negative!

Verbal logic: when the rental price of capital rises, each unit of capital is more expensive, so the choice of capital must fall.

Formal logic: The objective in (L1) satisfies increasing differences in $(k, -r)$. So capital is nonincreasing in its own price.

Notice the verbal logic applies for anything which makes increasing capital by a unit more expensive: likewise, so does the formal logic.

How does k change with q ? When is capital a normal input?

Verbal logic: Consider the rate of technical substitution, which is the rate at which labor can be decreased in response to a unit increase in capital, while still producing q . If *higher* quantities lead to a situation where a unit increase in capital allows a savings of *more* labor than before, then the firm wants to substitute capital for labor when q increases. Then, capital increases with q .

Formal logic: The objective function in (L1) satisfies increasing differences in (k, q) if $-L(k, q)$ satisfies increasing differences in (k, q) . We can interpret this directly as a rate of technical substitution (try it!). If L is differentiable, we know that $-L_k(k, q) = F_k(k, l) / F_l(k, l) \Big|_{l=L(k, q)}$, which is nondecreasing in q if the rate of technical substitution increases as we move to higher isoquants (higher q).

Two step problem: minimize costs and then maximize profits.

$$l^{**}(x, w) = \left\{ l \mid (\exists k) (k, l) \in \arg \min_{k, l, s.t. F(k, l) = x} rk + wl \right\}.$$

The second step is:

$$\max_x px - C(x, w)$$

When does the choice of quantity (x) decrease with w ? If labor is a normal input!

Verbal logic: Higher wages lead to higher marginal costs of output (and thus lower choices of output) exactly when labor is a normal input. That is, if producing more

output leads you to use more labor, then higher wages make it more expensive to produce output.

Formal logic: $px - C(x,w)$ has decreasing differences in (x,w) when $C(x,w)$ has increasing differences. Since the marginal effect of w on $C(x,w)$ is $l^{**}(x,w)$ by the envelope theorem, w increases the marginal cost of output when $l^{**}(x,w)$ is increasing in x .

Profit Maximization (all in 1 step):

$$\max_{k,l} \pi(k,l;r,w) \equiv pF(k,l) - rk - wl \tag{L3}$$

We break the firm's maximization problem into two stages, as follows:

$$\hat{l}(k;r,w) = \sup_l \left\{ \arg \max_l \pi(k,l;r,w) \right\} \tag{L4}$$

$$\hat{k}(r,w) = \sup_k \left\{ \arg \max_k \pi(k,\hat{l}(k;r,w);r,w) \right\} \tag{L5}$$

Fact: $\hat{l}(k;r,w) = \hat{l}(k;w)$, since capital is fixed when l is chosen.

Own price effects:

Verbal logic: An increase in r makes each unit of k more expensive, so the firm chooses less k .

Formal logic: Choice of capital solves $\max_k pF(k,\hat{l}(k;w)) - rk - w\hat{l}(k;w)$. This satisfies increasing differences in $(k,-r)$. Thus k is nonincreasing in r .

Cross price effects:

Definition: k and l are complements if $F(k,l)$ satisfies increasing differences in (k,l) . Substitutes if $F(k,l)$ satisfies decreasing differences in (k,l) .

Verbal logic: An increase in r always leads to a (weak) decrease in k . But if capital and labor are complements, a decrease in k always leads to a decrease in l , since capital increases the incremental returns to labor. Thus, when capital and labor are complements, an increase in r will lead to a (weak) decrease in *both* k and l .

In contrast, when capital and labor are substitutes, a decrease in k leads to a weak *increase* in l , and so capital and labor will move in opposite directions. Thus, an increase in r will lead to a (weak) decrease in k and a (weak) increase in l .

Formal logic: Applying the comparative statics theorems, if $F(k,l)$ satisfies increasing differences, then $\hat{l}(k;w)$ is nondecreasing in k ; if $F(k,l)$ satisfies decreasing differences, then $\hat{l}(k;w)$ is nonincreasing in k .

Changes in input prices on maximum profits:

Verbal logic: When the price of capital goes up by a small amount, a maximizing firm's profits change according to the amount of capital which was currently in use, since given a smooth profit function, the adjustments to capital and labor in response to a small price change are second order.

Formal logic: The objective is differentiable in r . If in addition, the production function is continuous in k , the envelope theorem tells us that $\frac{\partial}{\partial r} \pi^*(r,w)$ exists almost everywhere, and further that $\frac{\partial}{\partial r} \pi^*(r,w) = -\hat{k}(r,w)$ where it exists.

LeChatlier

Consider the profit maximization problem above.

Verbal logic (case of complements): In the short run, an increase in the wage leads to a decrease in the choice of labor. However, labor is chosen at the old (long-run) choice of capital. In the long run, capital can adjust, and since capital and labor are *complements*, the higher wage will lead to lower levels of *both* capital and labor. When comparing the short and long run choices of labor, note that the short run decision was made at a *higher* level of capital: since they are *complements*, the optimal choice of labor was higher in the short run. Thus, labor adjusts (goes down) by more in the long run than in the short run.

Formal logic: By the comparative statics theorems, if F has increasing differences, our above results tell us that: $\hat{l}(k;w)$ is nondecreasing in k and nonincreasing in w and, $\hat{k}(w)$ is nonincreasing. Thus, for all $w > w_0$, $\hat{k}(w_0) > \hat{k}(w)$, and $\hat{l}(\hat{k}(w_0);w) \geq \hat{l}(\hat{k}(w);w) = l_{LR}(w)$.

Verbal logic (case of substitutes): In the short run, an increase in the wage leads to a decrease in the choice of labor. However, labor is chosen at the old (long-run) choice of capital. In the long run, capital can adjust, and since capital and labor are *substitutes*, the higher wage will lead to lower levels of labor and *higher* levels of capital. When comparing the short and long run choices of labor, note that the short run decision was made at a *lower* level of capital: since they are *substitutes*, the optimal choice of labor was higher in the short run. Thus, labor adjusts (goes down) by more in the long run than in the short run.

Formal logic: By the comparative statics theorems, if F has increasing differences, our above results tell us that: $\hat{l}(k; w)$ is nonincreasing in both arguments and, $\hat{k}(w)$ is nondecreasing. Thus, for all $w > w_0$, $\hat{k}(w_0) < \hat{k}(w)$, and $\hat{l}(\hat{k}(w_0); w) \geq \hat{l}(\hat{k}(w); w) = l_{LR}(w)$.

Finally: note that without the complements or substitutes assumption, a discrete change in w need not lead to a larger long run than short run adjustment. In our counterexample, an increase in the price of oil caused the firm to shut down in the short run, but in the long run new fuel-efficient capital allowed the firm to operate using less oil.

Good luck -- don't stress!!