14.121 Fall 1999, Problem Set 2

Professor Susan Athey

Handout exercises due Friday, October 1

Remaining exercises due Monday, October 4

1. Exercises 1-7 on the "Handout on Consumer Theory."

2. In a three good world, a consumer has the following demands:

p_1	p_2	p_3	Y	x_1	x_2	x_3
1	1	1	20	10	5	5
3	1	1	20	3	5	6
1	2	2	25	13	3	3
1	1	2	20	15	3	1

   where p_i is the price of good i, Y is income, and x_i is demand for good i. Are these demands consistent with the maximization of a locally nonsatiated utility function?

3. Derive a consumer's direct utility function if the indirect utility function is given by $V(\mathbf{p}, Y) = Y p_1^{\alpha} p_2^{\beta}$.

4. A consumer in a three good economy (goods and prices denoted x_i and p_i respectively) with wealth $w > 0$ has demand functions for goods 1 and 2 given by:

$$\begin{aligned} x_1 &= 100 - 5\frac{p_1}{p_3} + \beta\frac{p_2}{p_3} + \delta\frac{w}{p_3} \\ x_2 &= \alpha + \beta\frac{p_1}{p_3} + \gamma\frac{p_2}{p_3} + \delta\frac{w}{p_3}, \end{aligned}$$

   where the Greek letters are constants.

   (a) Explain how to find the demand for good 3. (Don't actually do it.)

   (b) Are the demands for goods 1 and 2 appropriately homogeneous?

   (c) Do the adding-up theorems restrict the values of α, β, γ, or δ?

   (d) What restrictions on the coefficients are implied by utility maximization?

5. (OPTIONAL-DO NOT HAND IN SOLUTION) A consumer has utility function $U(x_1, x_2) = (x_1^{\rho} + x_2^{\rho})^{1/\rho}$.

   (a) Write down the first order conditions for utility maximaization, given the budget constraint $\mathbf{xp} = Y$, where Y is income.

   (b) Derive the Marshallian demand functions and compute the indirect utility function.

   (c) Write down the optimization program which leads to the Hicksian demand functions, and write down the first order conditions.