14.121 Final Exam — Professor Susan Athey

Fall 1998

Instructions. Relax. You have two hours to complete the exam. The three questions will be weighted equally (but different parts of each question will be weighted according to difficulty). Partial credit will be generously awarded for clear reasoning and careful logic. If you are asked to explain or give an intuition, please do so in a few sentences. The word "qualitatively" is used to ask you to compare whether one quantity is bigger than another, or in what direction it changes, without necessarily giving exact numbers or solutions (i.e., "the use of labor goes up because...").

1. Suppose that there are $4n$ identical consumers living in three different cities. The first two cities have n consumers each, while the last city has $2n$ consumers. In 1997, prices in all three cities were given by $\mathbf{p}^O$, and each consumer chose the bundle $\mathbf{x}^O$. In 1998, prices changed in each of the three cities, to $\mathbf{p}^{N1}$, $\mathbf{p}^{N2}$, and $\mathbf{p}^{N3}$, in cities 1, 2, and 3 respectively, and the consumers chose $\mathbf{x}^{N1}$, $\mathbf{x}^{N2}$, and $\mathbf{x}^{N3}$, in cities 1, 2, and 3. An economics graduate student observes all of the prices and bundles just described. Suppose that $\mathbf{p}^{N1} = (10, 5, 8)$, $\mathbf{p}^{N2} = (14, 1, 16)$, and $\mathbf{p}^{N3} = (12, 3, 12)$.

   (a) State a formula for the Laspeyres price index for each city. How does the average (over consumers) Laspeyres index in city 3 relate to the average (over consumers) index in cities 1 and 2? (You should know what the Laspeyres index is. If you have forgotten, define a sensible price index and try to answer the question – for a hint, see part (b)).

   (b) Now state a formula for the "true" cost of living index, taking the utility at the "old" prices as the reference utility, so as to be comparable to the Laspeyres index. Can the graduate student compute the value of this index with the observed data?

   (c) Qualitatively, how does the average (over consumers) of the true cost of living index in cities 1 and 2 compare to the average (over consumers) of the true cost of living index in city 3? Prove your answer from first principles (that is, you may cite definitions, but any theorems must be proven).

   (d) (i) How does the Laspeyres index for city j change in response to a small increase in p_i^{Nj}, the price of good i in city j in 1998? (ii) How does the true cost of living index for city j change? (iii) Do your answers (for (i) and (ii)) vary by city? (iv) Describe (precisely) how, with a potentially larger dataset of observed prices and bundles, the graduate student could estimate the effects of such price changes on the true cost of living index.

2. In 1998, a firm with production function $F(k, l)$ purchased inputs (computers and software programmers) on perfectly competitive input markets at prices r and w, respectively. Then, in 1999, computer suppliers began offering nonlinear pricing. Thus, the total cost of buying k computers was given by $G(k) \cdot k$, where G is differentiable and strictly *decreasing*. Further, with software programmers in short supply, the cost of labor changes to $W(l) \cdot l$, where W is differentiable and strictly *increasing*. Finally, suppose that k and l are *substitutes* (F satisfies decreasing differences in (k, l)).

(a) How will the profit-maximizing choices of computers and labor change in 1999, if $G(0) = r$ and $W(0) = w$?

(b) Let k^{99} and l^{99} be the profit-maximizing choices in 1999. Will $F_k(k^{99}, l^{99}) = G(k^{99})$ and will $F_l(k^{99}, l^{99}) = W(l^{99})$? If not, in what direction will each choice be "distorted" (relative to the choices that satisfy the latter equalities)? Carefully interpret your answer, explaining any differences between k and l in this example.

(c) Qualitatively, how would your answer to (a) change if $G(0) > r$? [Just describe the possibilities and give a brief logic.]

(d) Qualitatively, how would your answer to (a) change if k and l were complements instead of substitutes? [Just describe the possibilities and give a brief logic.]

(e) Now change the assumptions of the problem a bit. Suppose that the Justice department blocks the use of nonlinear pricing for computers, and the input cost of computers (r) stays fixed from 1998 to 1999. Further, suppose it takes a year to install new computers, while software programmers can be adjusted immediately. Maintain the assumptions that capital and labor are substitutes, and that $W(0) = w$. Will the profit-maximizing choice of l change by more in the short run (just after the cost of labor changes to $W(l) \cdot l$), or the long run (when computers can be adjusted)? Sketch a proof of your answer.

3. Let X be a finite subset of $\Re$ (for example, $X = \{1, 2, .., 10\}$), and let $\mathcal{P}$ be the set of lotteries over X. Suppose that a consumer has a preference relation over lotteries $\mathbf{p} \in \mathcal{P}$ which satisfy the axioms for the existence of a VonNeumann-Morgenstern expected utility representation. Let $u : X \to \Re$ be such a VN-M utility function.

(a) Show that if for each x, $v(x) = au(x) + b$ for $a > 0$, $b \in \Re$, this v represents the same preference relation over lotteries. [Hint: start by writing out what it means for a VN-M utility function $u(x)$ to represent preferences over lotteries.]

(b) Show that if, instead, $v(x) = g(u(x))$, where g is increasing but not linear, this v will not have the same preferences over lotteries. To keep things simple, you may simply give the illustration where g is quadratic ($g(y) = ay + by^2$, $a > 0$), and show in this case that preferences might differ. [See the hint to part (a).]

(c) Suppose that the consumer with VN-M utility function u is risk averse. Suppose she considers a lottery $\mathbf{p} \in \mathcal{P}$. Let CE($\mathbf{p}$,$u$) be the certainty equivalent wealth for lottery $\mathbf{p}$ for this consumer. Write the expression which implicitly defines the certainty equivalent, and show that CE($\mathbf{p}$,u)$< \sum_{i=1}^{n} p_i x_i$. [You may appeal to a result from mathematics but show exactly how you are applying it.]

(d) Suppose that an insurance company sells insurance against the gamble $\mathbf{p}$, offering a certainty equivalent wealth of CE($\mathbf{p}$,u) in exchange for the gamble. Show that another consumer whose VN-M utility function is given by $v(x) = g(u(x))$, where g is strictly increasing and strictly concave, will be better off from buying this insurance. [Hint: proceed in a manner *similar* to part (c).]