14.121 Waiver Exam | Professor Susan Athey

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Instructions. Relax. Your grade on this exam does not matter for anything except the decision about whether you need to take (or re-take) the class.

You have two hours to complete the exam.

In the following questions, you are asked to "prove" or "disprove" certain statements. In so doing, you may rely on any results from mathematics you like, but you should clearly state the steps of your argument and the theorems you reference, if appropriate. Results from economics should in be proven unless otherwise noted. Partial credit will be generously awarded for clear reasoning and careful logic. If you are asked to explain or give an intuition, please do so in a few sentences.

1. Consider a continuous utility function $u : \Re^n_+ \to \Re$ which represents a locally nonsatiated, strictly convex preference relation. Denote by $V(\mathbf{p}; Y)$ the indirect utility function, and let $D_i(\mathbf{p}; Y)$ be the Walrasian demand for good i; let $E(\mathbf{p}; \bar{u})$ be the expenditure function, and let $h_i(\mathbf{p}; \bar{u})$ be the Hicksian demand for good i.

   (a) Does $\frac{\partial}{\partial p_j} D_i(\mathbf{p}; Y) = \frac{\partial}{\partial p_i} D_j(\mathbf{p}; Y)$? Does $\frac{\partial}{\partial p_j} h_i(\mathbf{p}; \bar{u}) = \frac{\partial}{\partial p_i} h_j(\mathbf{p}; \bar{u})$? Sketch a proof of your answer (you may rely on formulas you know from economics without proving them, if you state and reference them appropriately). Provide a short interpretation of any differences you find between the two types of demand.

   (b) Show that $E(\mathbf{p}; \bar{u})$ is concave in $\mathbf{p}$. (If you like, you may use a graphical approach with two goods, so long as you carefully label and explain your diagram.)

   (c) Prove that each good has one net substitute, that is, for each i, there exists a $j \neq i$ such that $\frac{\partial}{\partial p_j} h_i(\mathbf{p}; \bar{u}) > 0$.

2. Consider a differentiable production function $F(k, l)$, where the ouput depends on the capital (k) and labor (l) inputs chosen by the firm. Let r and w be the (per-unit) input prices.

   (a) Suppose that the firm chooses inputs to minimize the cost of producing a target quantity q. Qualitatively, how does the choice of capital change with r and q? Prove your answer. Might your answer change if the isoquants are not convex? Explain.

   (b) How does the minimum production cost change with the cost of capital, r? Derive an expression for this. How would your answer change if capital were a "lumpy" choice (as in the case where the firm can buy machines in discrete units at a cost of r per machine)?

3. Consider a risk-averse consumer with wealth W who is exposed to the risk of a loss, represented by the random variable $x \sim F(x)$, where the support of x is $[0, \bar{x}]$. Suppose the there are two kinds of insurance available. Type A insures a portion of the risk, α, so that the consumer is still exposed to $(1 - \alpha)x$. Type B offers a deductible, so that the consumer experiences the loss $\min(x, D)$, where D is the amount of the deductible.

(a) Assume the consumer can buy only one type of insurance. Derive expressions which determine the consumer's willingness to pay for type A insurance ($P(\alpha)$) and type B insurance ($P(D)$).

(b) Derive expressions showing how the consumers' willingness to pay for each type varies with the amount of insurance offered (α and D).

(c) Suppose that an actuarily fair premium is charged. Which type of insurance is more valuable in very small amounts?