14.121 Waiver Exam | Professor Susan Athey

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Instructions. Relax. Your grade on this exam does not matter for anything except the decision about whether you need to take (or re-take) the class.

You have two hours to complete the exam.

In the following questions, you are asked to "prove" or "disprove" certain statements. In so doing, you may rely on any results from mathematics you like, but you should clearly state the steps of your argument and the theorems you reference, if appropriate. Results from economics should in be proven unless otherwise noted. Partial credit will be generously awarded for clear reasoning and careful logic. If you are asked to explain or give an intuition, please do so in a few sentences.

1. Suppose that there are two goods. Let $\mathbf{D}(\mathbf{p};Y)$ ($\mathbf{D}:\mathbb{R}^3_+ \rightarrow \mathbb{R}^2_+$) represent the consumer's choice vector as a function of prices and income. Suppose that this choice function is budget-balanced: the consumer exhausts her income for every price vector.

   (a) Show that if $\mathbf{D}(\mathbf{p};Y)$ is homogeneous of degree zero in $(\mathbf{p};Y)$, then the Slutsky matrix associated with $\mathbf{D}(\mathbf{p};Y)$ is symmetric. [**NOTE TO FUTURE STUDENTS: THIS PROBLEM TURNED OUT TO BE MORE ALGEBRA-INTENSIVE THAN I INTENDED. THIS IS NOT A GOOD INDICATION OF FUTURE QUESTIONS.**]

   (b) State the Weak Axiom of Revealed Preference (WARP).

   (c) Define the preference relation $\mathbf{R}$ by $\mathbf{x}\ \mathbf{R}\ \mathbf{y}$ if and only if $\mathbf{x}$ is revealed preferred to $\mathbf{y}$ using the choice function $\mathbf{D}(\mathbf{p};Y)$. Show that if $\mathbf{D}(\mathbf{p};Y)$ satisfies WARP, then $\mathbf{R}$ has no intransitive cycles.

2. Consider two distinct problems. In both problems, inputs/goods are purchased (and sold, in the case of the firm) on competitive markets with linear prices, in the standard way. (i) A consumer with income Y maximizes utility subject to a budget constraint, and the resulting demand function is $\mathbf{D}(\mathbf{p};Y)$. (ii) A firm with production function $f(k,l)$ chooses capital and labor inputs to maximize profits.

   (a) In problem (i), what assumptions (if any) are required for you to conclude that when the price of good 1 goes up, the consumer buys less of good 1? More of good 2?

   (b) In problem (ii), what assumptions (if any) are required for you to conclude that when the rental price of capital goes up, the firm uses less capital? More labor?

   (c) Carefully contrast your answers to parts (a) and (b). Are there any differences? If so, explain.

   (d) Does your answer to part (b) depend on any simplifying assumptions, i.e., what if the production technology is not convex, or capital is chosen in lumpy units?

(e) Derive an expression for the marginal effect of an increase in the rental price of capital on firm profits. How would your answer change if capital is chosen from the set $\{1, 2, .., n\}$, representing the number of machines?

(f) Now suppose that the machines are sold by Microsoft Machine Company, the local monopolist on machines, and that nonlinear pricing is used. Under this arrangement, the *total cost* of k machines ($k \in \{1, 2, .., n\}$) is given by the function $m(k;\theta)$, where m is increasing in k and θ and differentiable in θ (θ might be related to the level current scrutiny by the Justice Department!). Under what conditions on the production technology and the function m will the firms use less capital in response to an increase in θ? More labor?

3. Consider a consumer with preferences over two goods. The preferences are lexicographic: $(x_1, x_2)\mathbf{R}(y_1, y_2)$ whenever $x_1 > y_1$; if $x_1 = y_1$, $(x_1, x_2)\mathbf{R}(y_1, y_2)$ if $x_2 \geq y_2$.

(a) For a given $\mathbf{x}$, sketch the set of bundles weakly preferred to $\mathbf{x}$.

(b) Are the preferences complete? Reflexive? Transitive? Weakly monotonic? Locally non-satiated? Are the better-than sets closed? Prove your answers.

(c) Can these preferences be represented by a continuous utility function? (Sketch a proof of your answer).