Consumer Theory

(CP) - Consumer Problem

n goods, x is a bundle, $x \in \mathbb{R}^n_+$, price vector $p \in \mathbb{R}^n_+$, Y is income.

Let u represent a preference relation **R** which satisfies axioms A1-A6

Define

$$x^*(p, Y) = \quad \arg\max_x \ u(x)$$

$$\text{subject to } p \bullet x \leq Y, \ x_i \geq 0, \ \sum p_i x_i \leq Y$$

The form of the budget constraint follows from the assumption that consumption goods can be purchased on a perfectly competitive input market, at linear prices.

- **Revealed Preference**
  - Setup
    - n goods, x is a bundle, $x \in \mathbb{R}^n_+$.
    - Observe: time $1, ..., T$; $\left(p^1, x^1\right), ..., \left(p^T, x^T\right)$
      - p^t is price vector in period t, x^t is choice vector in period t.
    - Can formalize this as a "choice function" $c(p)$. A choice function simply tells you which bundles were chosen for each price vector.

        · Note: Can allow multiple bundles chosen, so that c maps price vectors to sets of bundles.

    * Critical assumption: preferences do not change over time.

  - Question: How can we tell if $(p^1, x^1), ..., (p^T, x^T)$ are consistent with CP for some $\mathbf{R}$?
    * Are observed choices "rationalizable"?
  - First: the budget is exhausted.
    * If $\mathbf{R}$ satisfies A1-A6, more is better and $px^* = Y \; \forall p, Y$.
    * If consumer maximizes, can infer Y^t from (p^t, x^t).

- Question: Can I reject optimizing behavior from these observations?
  - Suppose that bundle x^1 is chosen in first period, bundle x^2 in second.
  - If x^1 is the solution to CP at date 1, then it must be preferred to anything else in the consumer's budget set given that the prices and income at date 1.
  - If x^2 is the solution to CP at date 2, then it must be preferred to anything else in the consumer's budget set given that the prices and income at date 2.
  - So: if x^1 was in the period 2 budget set, x^2 must not have been in the period 1 budget set!
  - In other words, rule out: x^1 is available when x^2 is chosen, AND x^2 is available when x^1 is chosen.
  - Graphical approach: illustrate budget sets.

- Formal language: x is revealed preferred to z if z is available and x is chosen.
  - Consider a vector of observe choices and prices, represented by the choice function $c(p)$.
  - x is revealed preferred to z if there exists a price vector q such that $c(q) = x$, and $\sum q_i z_i \leq \sum q_i x_i$.
  - x is strictly revealed preferred to z if there exists a price vector q such that $c(q) = x$, and $\sum q_i z_i < \sum q_i x_i$.
  - Notation: $x \succeq_{RP} z$ iff x is revealed preferred to z; $x \succ_{RP} z$ iff x is strictly revealed preferred to z.
- The Weak Axiom of Revealed Preference (WARP)
  - Let $c(q)$ be single-valued.
  - Definition: The choice function $c(p)$ satisfies WARP if, for all $x, z \in X$, we do not have $x \succeq_{RP} z$ and $z \succeq_{RP} x$.
  - Another way to say it: for all p, q, such that $c(q) = x$ and $c(p) = z$, we cannot have

$$\sum p_i z_i \leq \sum p_i x_i$$

    and

$$\sum q_i z_i \geq \sum q_i x_i.$$

- The Generalized Axiom of Revealed Preference (GARP)

- Definition: The choice function $c(p)$ satisfies GARP if, for all sequences of bundles $x^1, .., x^M \in X$, we do not have

$$x^1 \succeq_{RP} x^2 \succeq_{RP} \cdot\cdot \succeq_{RP} x^M$$

  AND $x^M \succ_{RP} x^1$.

- NOTE: This axiom deals with problem of multiple optima. Non-satiation + strict revealed preference.
- Also stated: rule out $x^1 \succeq_{RP} x^2 \succeq_{RP} \cdot\cdot \succeq_{RP} x^1$ with one strict preference.

• Proposition: $c(\cdot)$ can be rationalized by a preference relation satisfying A1-A6 iff $c(\cdot)$ satisfies GARP.

  - Proof: you can read; not responsible for details.