14.129 Problem Set 1 Professor Susan Athey

Due September 27, 1999

1. (Deductibles) Consider the case of automobile insurance where an agent's effort affects the distribution of the accident cost, x. Let $s(x)$ be the consumption of the agent, net of the cost of the accident, so that the agent's payoff is $u(s(x)) - c(e)$. The principal then receives $-x - s(x)$. However, assume that the distribution of x has an atom at $x = 0$. Specifically, $f(0, e) = 1 - p(e)$, and $f(x, e) = p(e)g(x)\ \forall x > 0$. Here, $p(e) > 0$ is the probability of accident when effort is e; p is decreasing and convex. Determine if the appropriate form of MLRP is satisfied and describe the optimal automobile insurance policy.

2. (Limited Liability). Consider a standard moral hazard problem with the following special features:
The Principal and the agent are both risk neutral. Let x be the observed output, e be the agent's unobserved input and $s(x)$ the payment to the agent. Then the two parties evaluate their final utility according to the vNM utility functions:
Principal: $x - s(x)$
Agent: $s(x) - c(e)$
where c is a strictly increasing function of effort, and $s(x) \geq 0$ for all x. We assume that this constraint also guarantees that the agent is willing to work for the principal rather than for someone else (no additional participation constraint necessary).
Output can take three values: $x_1 = 1$, $x_2 = 2$, $x_3 = 3$.
Effort can take two values: $e_L = 0$ or $e_H = 1$. We normalize $c(0) = 0$.
The probability of x given e, $f(x \mid e)$, satisfies the MLRP.

   (a) Assume that a second best solution to this problem induces the agent to choose $e = 1$. Show that in such a solution, $s(x_1) = s(x_2) = 0$ and $s(x_3) > 0$.

   (b) Assume now that the slope of the incentive scheme is restricted to lie between 0 and 1, i.e. that:
   $$\frac{s(x_j) - s(x_i)}{x_j - x_i} \in [0, 1] \quad \forall j > i$$
   (This Assumption can be rationalized by arguing that the agent could otherwise engage in arbitrage by free disposal or by using own funds to boost revenues.) Assume again that a second best solution induces $a = 1$. Show that in such a solution, $s(x_1) = 0$ and that if $s(x_2) > 0$, then $s(x_3) - s(x_2) = 1$.

3. (CARA utility and incentive contracts). Consider the standard moral hazard model discussed in class. Suppose that the agent's effort cost can be measured in monetary units, i.e. $U(w, e) = U(w - c(e))$, and that the agent has constant absolute risk aversion, i.e. $U(w, e) = -\exp(-r(w - c(e)))$. (Note that this is a special case of a muliplicatively separable utility function.) Show the following:

   (a) Suppose that $(s^*(x)\ , e^*)$ solves the principal's program (maximizing principal's expected utility subject to IR and IC) when the agent's reservation utility is $\underline{u} = U(w)$. Show that: (i) The optimal second best action is independent of w, and (ii) The optimal incentive scheme $s(x)$ can be written as $s(x) + k$, where only k depends on w.

(b) Derive the expression for the certainty equivalent wealth when output is distributed normally with mean e and variance σ^2, and the sharing rule is given by $s(x) = \alpha x + \beta$.

4. (Aggregation and Linearity) This problem asks you to fill in the details of a version of the "Aggregation and Linearity" model discussed in class (based on Holmstrom and Milgrom, 1987). The technology in each period is independent and identical. In each period $t = 1, 2$ the agent chooses to work ($e_t = 1$) or to shirk ($e_t = 0$). The outcome x_t can be either 1 or 0 with probability $\text{Prob}(x_t = 1 \mid e_t = 1) = q$ and $\text{Prob}\ (x_t = 1 \mid e_t = 0) = p$, where $p < q$. The private cost of working is $c(e_t)$ in period t,with $c = c(1) > c(0) = 0$. The agent's utility is exponential and takes the form

$$U(w, e_1, e_2) = -e^{-r(w - \sum_t c(e_t))},$$

where w is the payment from the principal. The agent's reservation utitliy has zero certainty equivalent. The agent can observe the outcome x_1 in period 1 before choosing his second period action. The principal's payoff is $\sum x_t - w$. The payment w can depend on both period's outcomes.

(a) Assume it is optimal to have the agent work in both periods. Show that this can be implemented at least cost with an incentive scheme of the form

$$w(x_1, x_2) = \alpha \sum x_t + \beta,$$

i.e., the agent is paid the number of successes times α plus a fixed fee β.

(b) Show that this solution is no longer optimal if the agent cannot observe his first period outcome. (This is the hard part). What is the intuition behind this result?