14.129 Fall 1999, Problem Set 2 Professor Susan Athey, T.A. Svetlana Danilkina

Due Friday, October 8, 1999

1. Consider the normal-exponential multi-task linear contracting model discussed in class, where the principal observes $x_i = e_i + \upsilon_i$, and (υ_1, υ_2) are independent normals with mean zero, and variances σ_1^2 and σ_2^2. The incentive contracts take the form $s(x_1, x_2) = \beta + \alpha_1 x_1 + \alpha_2 x_2$. Suppose that there are two tasks, and that the cost-of-effort function takes the form $c(e_1, e_2) = \gamma e_1 e_2 + \eta_1 e_1 + \eta_2 e_2 + \kappa_1 e_1^2 + \kappa_2 e_2^2$.

   (a) Derive an expression for the "effort supply functions" (that is, the level of effort on each task supplied by the agent) as a function of the linear incentive instruments α_1 and α_2. Show that these functions are linear in α_1 and α_2. How would this change if we relaxed our assumptions on the cost-of-effort function? How does the effort provided at task 1 change with α_1 and α_2? (And how does this depend on γ)?

   (b) Derive an expression for the total certainty equivalent surplus as a function of α_1 and α_2. (Hint: do not substitute in the cost-of-effort functional form until the last moment!) Under what conditions is $\frac{\partial^2}{\partial \alpha_1 \partial \alpha_2} TCE \geq 0$?

   (c) How do the optimal choices of α_1 and α_2 change with σ_1^2?

   (d) Recall a fact from statistics: if $y = f(x_1, x_2)$ and $z = g(x_1, x_2)$, where f and g are non-decreasing, and x_1 and x_2 are independent random variables, then $\text{cov}(y, z) \geq 0$. Then, suppose that there is a population of firms, each identical except in their monitoring technologies, which determine σ_1^2 and σ_2^2. Suppose that in the population, the quality of monitoring in task 1 is statistically independent of the quality of monitoring of task 2. How do you expect the incentives chosen by the firms to covary? Interpret your result.

   (e) What happens if the firm can choose σ_1^2 at a cost $\phi(\sigma_1^2; \lambda)$, where $\phi(\sigma_1^2; \lambda)$ is supermodular. How do the choices about the incentive instruments and the monitoring technology change with λ?

   (f) Would your results change qualitatively if the choices of α_i were restricted to be either 0 or 1?

2. Now suppose that there are two principals, and principal 1 cares only about task 1, while principal 2 cares only about task 2. Further, principal 1 can only observe x_1, while principal 2 can only observe x_2. Each principal offers a linear contract to the agent.

   (a) Write down each principal's objective function, as a function of the sharing rules (where the opposing principal's incentive scheme is taken as given). Be careful when you consider the individual rationality constraint of the agent. In the single-principal case, the principal maximized TCE and extracted utility from the agent using the fixed component of wages, β. Carefully explain what is different about this problem.

   (b) What is the nature of the externality of one principal on the other? Under what conditions is it positive? Negative? Interpret.

(c) Show that in the decentralized game, there is no strategic interaction between α_1 and α_2. Under what additional assumptions (on the cost of effort or the stochastic production technology) would a positive strategic interaction arise? A negative one? (Your answer can be qualitative; you do not need to do a lot of algebra).

(d) Suppose (hypothetically) that each principal "internalized" the externality on the other agent. In other words, suppose that each principal i maximizes total social surplus (including that of the other principal). How does the marginal return to α_i change for principal i when the other principal's utility is taken into account? Discuss the difference between the decentralized game and the case of only one principal, focusing on the extent to which the interaction between the incentive instruments (in terms of social surplus) is accounted for in the solutions.

(e) Use the comparative statics theorems discussed in class to draw conclusions about the conditions under which there is under- and over-provision of incentives, relative to the case of only one principal. Interpret your answer.

(f) Does your analysis have any consequences for how organizations should reward supervisors? (Or how universities should reward professors?)