14.129 Fall 1999, Problem Set 3 — Professor Susan Athey, T.A. Svetlana Danilkina

Due Monday, October 18, 1999

1. Let $u(x, y, \theta)$ be differentiable and well-behaved, and suppose that $u_2 < 0$.

   (a) Show that if $u_1/|u_2|$ is nondecreasing in θ (the Spence-Mirrlees single crossing property holds), then for any $t : \Re \to \Re$, $u(x, t(x), \theta)$ satisfies single crossing of incremental returns in $(x; \theta)$. Illustrate this with a diagram.

   (b) Suppose $u_1/|u_2|$ is strictly increasing in θ. Show that if t is differentiable and there is an interior optimal choice of x for each θ, the optimal choice of x is strictly increasing. (You may apply the comparative statics theorem to establish weak monotonicity).

2. Consider the n-type screening model described in class.

   (a) Show that all of the IC constraints hold, if and only if ULIC(i) and DLIC(i) hold for all i.

   (b) Show that when the principal chooses the optimal contract, only the DLIC constraints and the lowest type's IR constraint will bind.

   (c) Describe how (b) can be used to show that there is "no distortion at the top."

3. The government wishes to procure inputs. Suppose that a firm has marginal cost of production equal to θ, which is drawn from a distribution $F(\theta)$ with support $[\underline{\theta}, \bar{\theta}]$ and density $f(\theta) > 0$. The firm has quasilinear preferences $t - q\theta$, where t is the payment from the government. The government's payoff is $v(q) - t$, where v is strictly increasing and concave, and it offers an incentive-compatible menu $\{q(\theta), t(\theta)\}$ (not necessarily an optimal contract, just an IC one). The reservation utility of the firm is 0.

   (a) How does q change with θ, qualitatively?

   (b) Derive an expression for the firm's indirect utility as a function of the quantity and transfer offered to the high-cost type $\bar{\theta}$, and the quantity schedule $q(\theta)$; your expression should not depend on any transfer except that offered to the high-cost type.

   (c) Why does it make sense to write this expression as a function of the high-cost type's utility, if we plan to study the principal's optimal contracting problem? In your answer, refer to a 2- or 3-type variant of this model, and discuss which incentive constraints will be binding in the principal's problem. What is different about this model and the model we discussed in class?

   (d) Using your expression and the direct definition of the agent's utility, derive an expression for the transfer $t(\theta)$ as a function of $q(\theta)$. How does t change with θ, qualitatively? Verify that your analysis is consistent with the first-order conditions for truthful reporting, assuming that the relevant functions are differentiable.

(e) Suppose that the principal compares two IC contracts, $q(\theta)$ and $\tilde{q}(\theta)$, where $q(\theta) = \tilde{q}(\theta)$ on $\theta < x$ and $q(\theta) + z = \tilde{q}(\theta)$ on $\theta \geq x$. How do the two schemes compare in terms of the expected value of the social surplus? The expected value of the agent's utility (use your expressions from above to incorporate the consequences of incentive compatibility)? Compare the expressions for social surplus and the agent's utility, and examine the comparison separately for the events $\theta < x$ and $\theta \geq x$.

(f) What is a sufficient condition for $\tilde{q}(\theta)$ to generate more expected social surplus than $q(\theta)$? What is a sufficient condition for $\tilde{q}(\theta)$ to bring the principal more expected utility than $q(\theta)$? (Recall that the principal's expected utility is the difference between the social surplus and the agent's expected utility). Compare the two results, and carefully interpret them. Why does the principal's benefit differ from the social one? Interpret the principal's tradeoff in terms of rents.

4. (Fudenberg-Tirole [1992, Exercise 7.5.]) Consider the following insurance model with adverse selection. The insuree can have a low probability of accident ($\underline{\theta}$) or a high one ($\overline{\theta} > \underline{\theta}$), with probabilities $\underline{p}$ and $\overline{p}$, respectively. The insuree knows his probability of accident, but the insurance company (which is a monopolist and which offers a menu of contracts) does not. The insuree has objective function

$$U(W_1, W_2, \theta) = (1 - \theta)u(W_1) + \theta u(W_2),$$

where W_1 and W_2 are his net incomes in states of nature 1 (no accident) and 2 (accident) and u is his von Neumann-Morgenstern utility function ($u' > 0, u'' < 0$). With W_0 denoting the insuree's initial wealth and D the (monetary) damage in case of accident, the risk-neutral insurer's expected utility is

$$V(W_1, W_2, \theta) = (1 - \theta)(W_0 - W_1) + \theta(W_0 - D - W_2).$$

(a) Give a diagrammatic description of the optimal two-contract menu for the insurance monopolist. In particular, draw the status-quo (no-contract) point in the (W_1, W_2) space, and indifference curves corresponding to the two types for both the insuree and the insurer. Show that the binding IC constraint is that the high-risk insuree would not want to mimic the low-risk insuree, and that the high-risk insuree gets full insurance ($\overline{W}_1 = \overline{W}_2$). Argue informally that the high-risk insuree may or may not get a rent (depending on the probability $\overline{p}$) , and that he gets a rent if some insurance is given to the low-risk insuree.

(b) Perform the same analysis as in question (a), but use algebra. Hints: Use question (a) to guess which constraints are binding (ignore the others and check them later); let Γ denote the inverse function of the insuree's VNM utility function u. Describe a menu as $\{(\underline{U}_1, \underline{U}_2), (\overline{U}_1, \overline{U}_2)\}$,where $\underline{U}_1$ is the low-type's number of utils in state 1, etc. Notice that the monopoly's objective is concave in these utility levels and the constraints are linear. Solve the monopoly's program. (For the answer, see Stiglitz [1977]).