MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Economics

14.129 - Contract Theory, Fall 1999

Professor Susan Athey | Phone: 3-6407

Office: E52-252C | Email: athey@mit.edu

Introduction to Moral Hazard

1. Moral Hazard in Teams

   Setup:

   - efforts are unobserved
   - assume no uncertainty
   - I players, output a product of many players

   $$x = f(e_1, ..., e_I)$$

   - free rider problem (assume risk neutrality, z_i is money)

   $$u_i(z_i, e_i) = z_i - e_i$$

   - $s_i(x)$ sharing rule - how much money goes to agent i
   - budget balancing (BB)

   $$\sum s_i(x) = x$$

   Question: Can we find $s_i(x)$ such that BB $\forall x$ and efficient output?

Answer: No, if s_i is differentiable and $\underbrace{e_i^* > 0}_{\text{optimal effort}}$

PROOF:

Nash Equilibrium: $e_i = \arg\max_{e_i} [s_i(f(e)) - e_i]$

First Order Condition (FOC)

$$\underbrace{s_i'}_{\text{share}} \underbrace{f_i(e)}_{i^{th}\text{ partial der of } f} = 1, \quad \forall i$$

Efficiency: $\max f(e) - \sum e_i, \quad f_i = 1$

Contradiction:

- for every agent $s_i' = 1$
- BB: $\sum s_i'(x) = 1$
  Interpretation: personal benefits go one for one with costs, can't do this if benefits are shared.

- 2 Solutions:
  - Boot Camp, group punishments (violates s_i differentiability assumption)
  - Improve Measurement

2. Single Principal / Single Agent (1 of each)

- Tradeoffs: Insurance vs. Incentives
  - efficient for principal to be insuring the workers
- 3 Formulations:
  (a) State Space - output can take on a limited # of values (linear program)
    - easy to understand

– analytically difficult

(b) Parameterized Distribution - analytically elegant

(c) General Distribution

– * Question: $f(x \mid e)$ - why only a single parameter?
  * Answer: actions can change distribution (in more than one-dimensional way); this expand richness of agent's space
  * conceptually nice
  * analytically difficult

3. Parameterized Distribution Formulation

- $e \rightarrow F(\underbrace{x}_{output} \mid e) \qquad density \ [f(x \mid e)]$
  - $x \sim N(e, 1)\,, \ \ variance = 1$
  - same as $x = e + \varepsilon, \ \ \varepsilon \sim N(0, 1)$

- Principal: $v(w)\,, \ v' > 0$
- Agent: $u(w) - c(e)\,, \ u' > 0, \ c' > 0, \ c'' > 0$
  Principal chooses $s(x)\,, \ v(x - s(x))$

- Behavioral Assumptions: for a given $s(x)$, e is chosen by agent iff

$$\int u(s(x))\, f(x \mid e)\, dx - c(e) \geq \int u(s(x))\, f(x \mid e')\, dx - c(e')\,, \ \forall e' \tag{IC}$$

$$\int u(s(x))\, f(x \mid e)\, dx - c(e) \geq \underline{u} \tag{IR}$$

Critique: agent can be given very low utility for some realizations of output: what about limited liability?

- Program:

$$\max_{s(x)} \int v\left(x - s\left(x\right)\right) f\left(x \mid e\right) dx$$

$$\text{subject to } IR, IC$$

  - maximize Principal's utility, given agent's constraint
  - Can incorporate pointwise constraint
  - trying to push agent to reservation utility - Pareto optimality

- Analyze as constrained optimization
  - principal chooses sharing rule
  - remove agent entirely as a strategic player

- To solve this equation, use a 2 step procedure.
  - For any e, find $s\left(x\right)$ that satisfies IC, IR "implements" e
    $EV\left(e\right) = \int v\left(x - s_e\left(x\right)\right) f\left(x \mid e\right) dx$
  - Choose e to maximize $EV\left(e\right)$