MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Department of Economics

14.129 - Contract Theory, Fall 1999

Professor Susan Athey | Phone: 3-6407

Office: E52-252C | Email: athey@mit.edu

Nonlinear Pricing: Solving the Principal's Problem

- Principal solves:

$$\max_{q(\theta)} \left[E(s(q(\theta),\theta) - U(\theta \,|\, \theta)) = E\left[s(q(\theta),\theta) - u_2(q(\theta),\theta)\frac{1-F(\theta)}{f(\theta)}\right]\right.$$

  - Define:

$$\Phi(q,\theta) = s(q,\theta) - u_2(q,\theta)\frac{1-F(\theta)}{f(\theta)}$$

    $\Phi(q,\theta)$ is principal's pointwise objective function
    IC$\Rightarrow$ q nondecreasing

  - Question:
    Does P want q strictly or weakly increasing?

  - If $\Phi(q,\theta)$ satisfies SCP-IR, then $\arg\max_q \Phi(q,\theta)$ is increasing in θ and monotonicity constraint does not bind.

  - Φ differentiable and $\frac{\partial^2\Phi}{\partial q \partial \theta} > 0$, and interior optimum, then separation (i.e. q strictly increasing) is optimal.

- But if $\frac{\partial^2 \Phi}{\partial q \partial \theta} < 0$ on some subinterval of $[\underline{\theta}, \overline{\theta}]$, P does not want to sort and monotonicity constraint binds.

- Sufficient condition for $\Phi_{q\theta} \geq 0$ is

$$\begin{aligned} v_{12} &\geq 0 \\ u_{12} &> 0 \\ u_{122} &\leq 0 \end{aligned}$$

$$\frac{1 - F(\theta)}{f(\theta)} \text{ decreasing in } \theta$$

  - Often $v_{12} = 0$ (i.e. $v(q, \theta) = -c(q)$)
  - If $u = q\theta$, then $u_{122} = 0$ ($u = \theta\ g(q)$ as well)
  - Monotone hazard rate MHR

$$\frac{1 - F(\theta)}{f(\theta)} \searrow \Rightarrow MHR \Rightarrow 1 - F(\theta) \text{ is log concave}$$

  - "Bad" density which leads to "ironing"

– Suppose $\Phi_{q\theta} > 0$ and MHR condition holds $(\frac{1-F(\theta)}{f(\theta)} \searrow)$

$$\max_q \Phi(q, \theta)$$

$$s_q(q(\theta), \theta) = \frac{1 - F(\theta)}{f(\theta)} u_{12}(q(\theta), \theta) \geq 0 \qquad \text{(FOC)}$$

$$F(\overline{\theta}) = 1 \Rightarrow s_q(q(\overline{\theta}), \overline{\theta}) = 0 \Rightarrow \text{ no distortion on the top}$$

By the comparative statics theorems,

$$\theta < \overline{\theta} \Rightarrow q(\theta) < q^{F.B.}$$

Rewriting the F.O.C.:

$$f(\theta)\, s_q(q(\theta), \theta) = (1 - F(\theta))\, u_{12}(q(\theta), \theta)$$

$f(\theta)$ importance of social optimum for this θ

$1 - F(\theta)$ importance of higher types