Implicit Large Eddy Simulation of Transitional Flows Over Airfoils and Wings

A. Uranga(*), P.-O. Persson(**), J. Peraire(*), and M. Drela(*)

(*) Department of Aeronautics & Astronautics
Massachusetts Institute of Technology

(**) Department of Mathematics
University of California, Berkeley

19th AIAA Computational Fluid Dynamics Conference

June 24, 2009

Outline

1. Introduction
2. Methodology
   - Governing Equations and Turbulence Modeling
   - Discontinuous Galerkin Method
   - Computational Grids
3. Results
   - Laminar Regime: $Re = 10{,}000$
   - Transitional Regime: $Re = 60{,}000$
4. Conclusions

Introduction

- Accurate prediction of transition is of crucial importance at low Re
  - transition location has significant impact on aerodynamic performance
- Formation of laminar separation bubble (LSB)
  - Laminar BL separates in adverse pressure gradient
  - Separated flow rapidly transitions to turbulence
  - Subsequent reattachment of turbulent BL
- Flow regime encountered in small aircraft and MAVs

Introduction

- Goal: *predict formation of LSB and subsequent transition*
- Flow around rectangular SD7003 wing at angle of attack of 4°
  - flow exhibits LSB on upper surface
  - extensive experimental data available [1]
  - numerical simulations performed by other groups [2]
- Laminar regime: $Re = 10{,}000$
- Transitional regime: $Re = 60{,}000$
- Implicit Large Eddy Simulations (ILES) using high-order Discontinuous Galerkin (DG) method

[1] Ol, M., McAuliffe, B., Hanff, E., Scholz, U., and Kahler, C., "Comparison of laminar separation bubbles measurements on a low Reynolds number airfoil in three facilities", AIAA-2005-5149, 2005.

[2] Galbraith, M. and Visbal, M., "Implicit Large Eddy Simulaion of low Reynolds number flow past the SD7003 airfoil", AIAA-2008-225 , 2008.

Governing Equations

Compressible Navier-Stokes equations and ideal gas law

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}(\rho u_i) = 0$$

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} \quad \text{for} \quad i \in \{1,2,3\}$$

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x_j}\left[u_j(\rho E + p)\right] - \frac{\partial}{\partial x_j}(u_i \tau_{ji}) + \frac{\partial q_j}{\partial x_j} = 0$$

$$p = (\gamma - 1)\rho\left(E - \frac{1}{2}u_k u_k\right)$$

where
$$\tau_{ij} \equiv \mu\left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}\right]$$

$$q_j = -\frac{\mu}{Pr}\frac{\partial}{\partial x_j}\left(E + \frac{p}{\rho} - \frac{1}{2}u_k u_k\right)$$

Simulation of Turbulence

- Large Eddy Simulation (LES)
  - large-scale motions are resolved
  - small scales modeled through sub-grid-scale model
- Implicit LES (ILES)
  - unresolved scales accounted for by numerical dissipation (no sub-grid-scale model)
  - solving the full compressible Navier-Stokes equations
- ILES approach previously used by Visbal and collaborators [2] with compact difference method for flow around SD7003
- High computational cost ⇒ benefits from high-order methods

[2] Galbraith, M. and Visbal, M., "Implicit Large Eddy Simulaion of low Reynolds number flow past the SD7003 airfoil", AIAA-2008-225 , 2008.

Discontinuous Galerkin Method I

High-order Discontinuous Galerkin (DG) method:

- High-order, low dissipation
- Unstructured meshes
- System of conservation laws

$$\frac{\partial u}{\partial t} + \nabla \cdot F_i(u) - \nabla \cdot F_v(u, q) = 0$$
$$q - \nabla u = 0$$

- Domain Ω triangulated into elements $K \in T_h$

Discontinuous Galerkin Method II

- Seek approximate solutions $u_h \in V_h^p$, $q_h \in \Sigma_h^p$ in spaces of element-wise polynomials of order p

$$\begin{aligned} V_h^p &= \{v \in [L^2(\Omega)]^m \mid v|_K \in [\mathcal{P}_p(K)]^m, \ \ \forall K \in \mathcal{T}_h\} \\ \Sigma_h^p &= \{w \in [L^2(\Omega)]^{dm} \mid r|_K \in [\mathcal{P}_p(K)]^{dm}, \ \ \forall K \in \mathcal{T}_h\} \end{aligned}$$

- Multiply by test functions v, w and integrate over element K

$$\int_K q_h \cdot w \, dx = -\int_K u_h \nabla \cdot w \, dx + \int_{\partial K} \hat{u} w \cdot n \, ds \ , \qquad \forall w \in [\mathcal{P}_p(K)]^{dm},$$

$$\int_K \frac{\partial u_h}{\partial t} v \, dx - \int_K [F_i(u_h) - F_v(u_h, q_h)] \cdot \nabla v \, dx$$

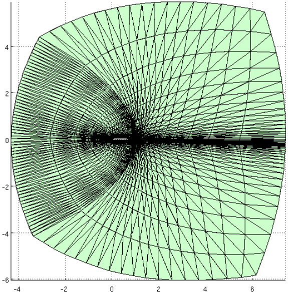
$$= -\int_{\partial K} [\hat{F}_i - \hat{F}_v] \cdot nv \, ds \ , \qquad \forall v \in [\mathcal{P}_p(K)]^m.$$

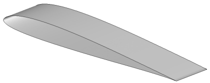
Discontinuous Galerkin Method III

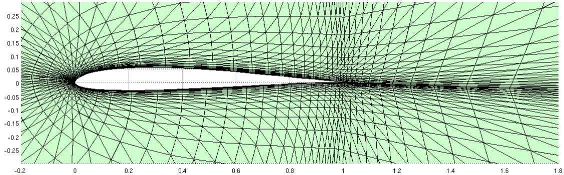
- Numerical fluxes $\hat{F}_i$, $\hat{F}_v$, $\hat{u}$ are approximations to F_i, F_v, u on boundary ∂K of element K
  - Roe scheme for inviscid fluxes F_i
  - CDG method for viscous fluxes F_v
  - flux $\hat{u}$ only function of u_h (*not* of q_h)
- Resulting system of coupled ODEs solved with incomplete factorizations (ILU) and p-multigrid with Newton-GMRES preconditioning
- Code parallelized with block-ILU factorizations
- Error $\mathcal{O}(h^{p+1})$ for smooth problems
- Time stepping with 3^{rd} order diagonal implicit Runge-Kutta (DIRK) method. Time step $\Delta t^* = \Delta t \, U_\infty / c = 0.01$

Computational Grids

- SD7003 at 4° AoA, span $0.2c$
- Domain $[-4.3c\ ,\ 7.4c]\times[-6.0c\ ,\ 5.9c]$
  $\times[0\ ,\ 0.2c]$
- Span-wise periodic BC
- Polynomials of order $p = 3$
  $\rightarrow 4^{\text{th}}$ order method in space
- 52,800 tetrahedral elements
- 1,056,000 DOFs





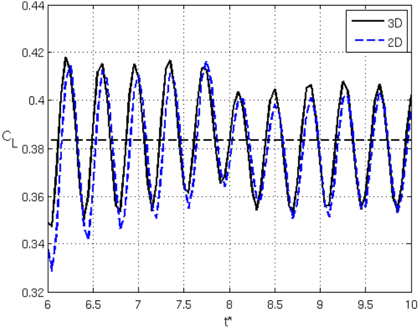


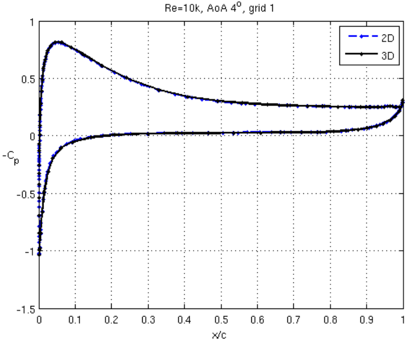
Outline

1. Introduction
2. Methodology
   - Governing Equations and Turbulence Modeling
   - Discontinuous Galerkin Method
   - Computational Grids
3. Results
   - Laminar Regime: $Re = 10{,}000$
   - Transitional Regime: $Re = 60{,}000$
4. Conclusions

Laminar Regime: $Re = 10{,}000$

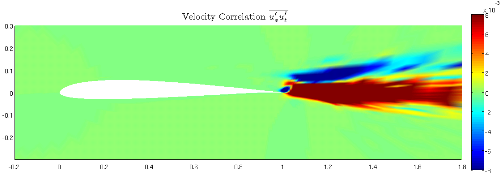
- Flow is essentially 2D
- Close-to-periodic vortex shedding



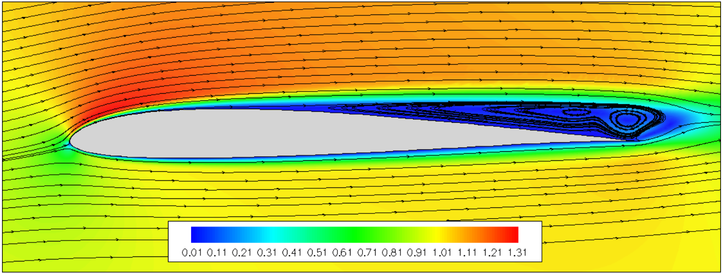


Laminar Regime: Re = 10,000

- Separation at 34%
- No transition along airfoil
- No reattachment



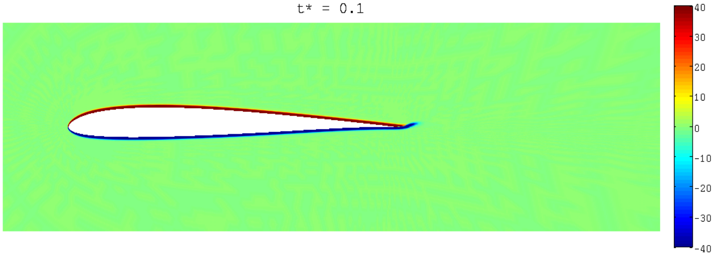
Velocity correlations

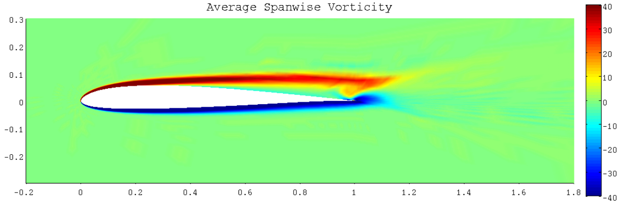


Average streamlines and contours of average velocity magnitude

Laminar Regime: $Re = 10{,}000$

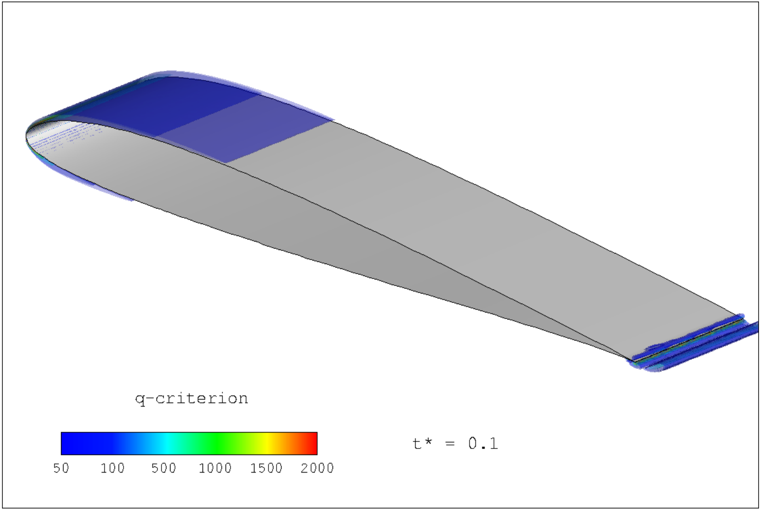
Span-wise vorticity ω_z on wing's middle plane





Laminar Regime: $Re = 10{,}000$

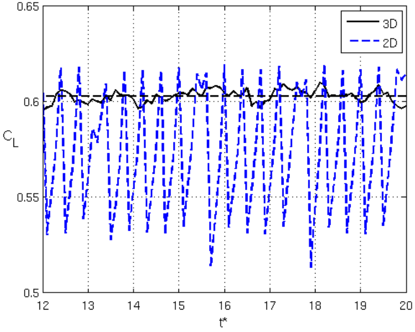
Vortical structures: iso-surfaces of q-criterion ($\nabla^2 p/2\rho$)

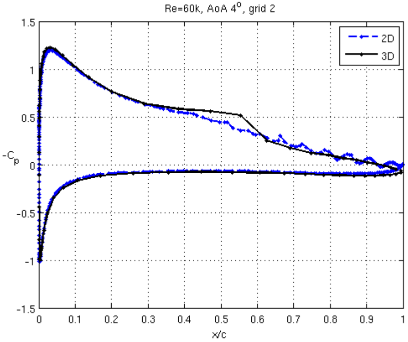


Outline

Transitional Regime: $Re = 60{,}000$

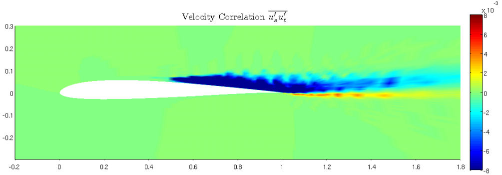
- 3D solution very different from 2D solution
- Significant 3D vortical structures present
- Non-periodic vortex shedding



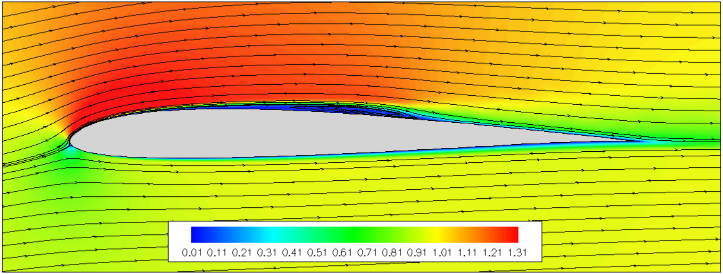


Transitional Regime: $Re = 60{,}000$

- Separation at 24%
- Transition at 51%
- Reattachment at 60%



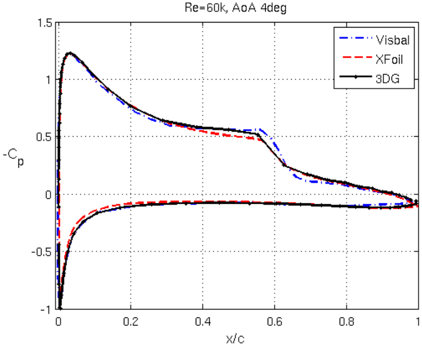
Velocity correlations

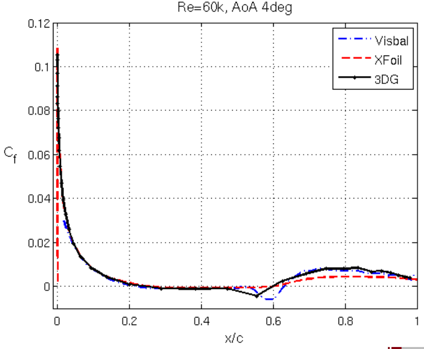


Average streamlines and contours of average velocity magnitude

Transitional Regime: $Re = 60{,}000$

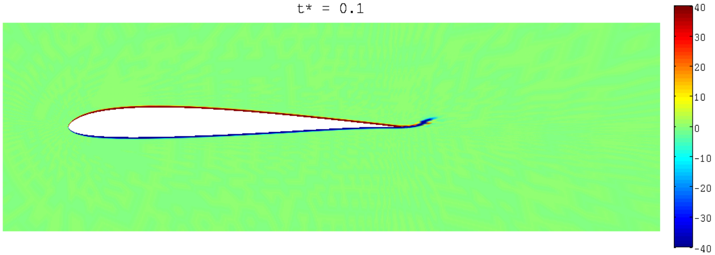
- Average pressure and skin friction coefficients
- Separation and transition well captured
- Good agreement with XFoil and previously published ILES [Galbraith & Visbal 2008]

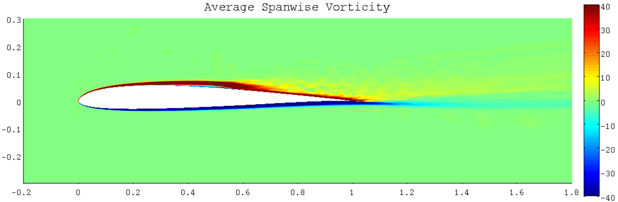




Transitional Regime: $Re = 60{,}000$

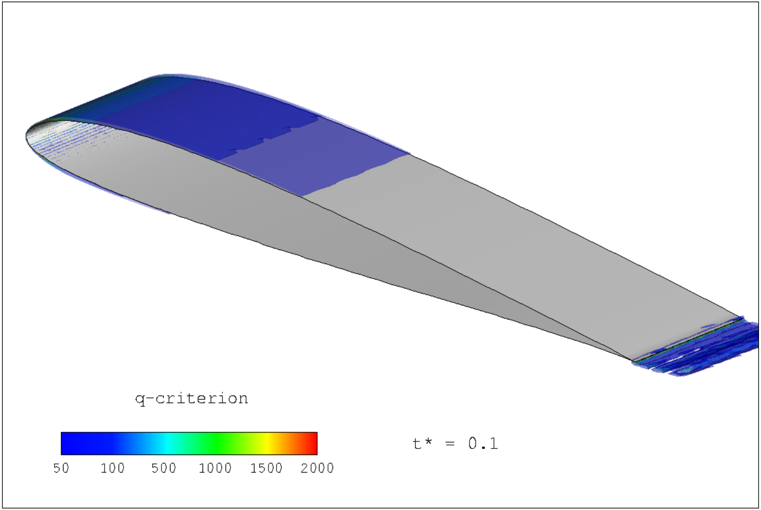
Span-wise vorticity ω_z on wing's middle plane





Transitional Regime: $Re = 60{,}000$

Vortical structures: iso-surfaces of q-criterion ($\nabla^2 p/2\rho$)

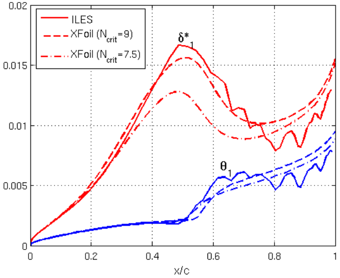


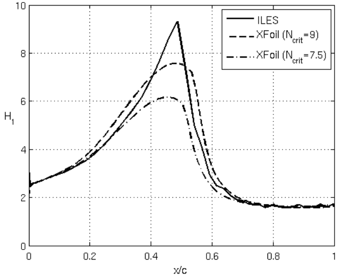
Transitional Regime: $Re = 60{,}000$

Boundary layer integral parameters of time-average flow

- Pseudo-velocity profile $\vec{u}^*(n) = \int_0^n \vec{\omega} \times \hat{n}\, dn$
  (asymptotes outside BL even with strong curvature)
- BL edge n_e defined where $|\vec{\omega}|n < \epsilon_0|\vec{u}^*|$ and $|d\vec{\omega}/dn|n^2 < \epsilon_1|\vec{u}^*|$
- Streamwise pseudo profile $u_1(n) = \vec{u}^*(n) \cdot \vec{u}_e^*/u_e^*$
  - displacement thickness, momentum thickness, shape factor

$$\delta_1^* = \int_0^{n_e} \left(1 - \frac{u_1}{u_e}\right) dn \qquad \theta_{11} = \int_0^{n_e} \left(1 - \frac{u_1}{u_e}\right) \frac{u_1}{u_e} dn \qquad H_{11} = \frac{\delta_1^*}{\theta_{11}}$$



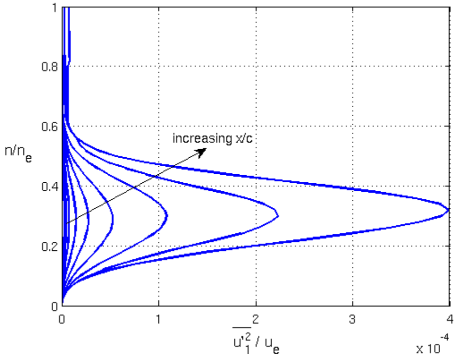


Transitional Regime: $Re = 60{,}000$

Transition Mechanism

- Fluctuating stream-wise pseudo-velocity

$$u_1'(\vec{x}, t) = u_1(\vec{x}, t) - \overline{u_1}(x)$$



- Consistent with Tollmien-Schlichting (TS) modes
- Perturbation amplitude increases downstream

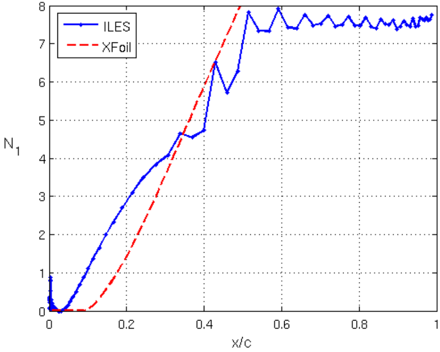
Transitional Regime: $Re = 60{,}000$

Transition Mechanism

- Amplification of stream-wise TS waves at any chord-wise location x:

$$A_1(x) = \frac{1}{n_e u_e(x)} \sqrt{\int_0^{n_e} \overline{{u'_1}^2}\, dn}$$

- Stream-wise amplification factor: $N_1(x) = \ln\left(\frac{A_1(x)}{A_{1_0}}\right)$ (all waves)



- XFoil line: N-factor of the single most-amplified wave at given location
- Similar slopes
- $\Rightarrow$ TS transition

Outline

Conclusions

- Use of DG method for ILES of low Reynolds number flows
- At $Re = 10{,}000$, flow laminar and essentially 2D over wing surface
- At $Re = 60{,}000$, transition associated with LSB observed
- Transition found to be the result of unstable TS waves
- Remarkable agreement with XFoil predicitons, finer LES results [Visbal and collaborators], experimental data [Ol and collaborators]
  $\longrightarrow$ *in spite of use of relatively coarse meshes*
- Results suggest that DG is particularly suited to simulate these flows

Acknowledgments

- Air Force Office of Scientific Research (AFOSR) support under MURI project *Biologically Inspired Flight*
- Scholarship from Mexican's National Council for Science and Technology (CONACYT)
- NSF through TeraGrid resources provided by TACC

Questions