Numerical Simulation of the Response of Sandy Soils Treated with Prefabricated Vertical Drains

Antonios Vytiniotis

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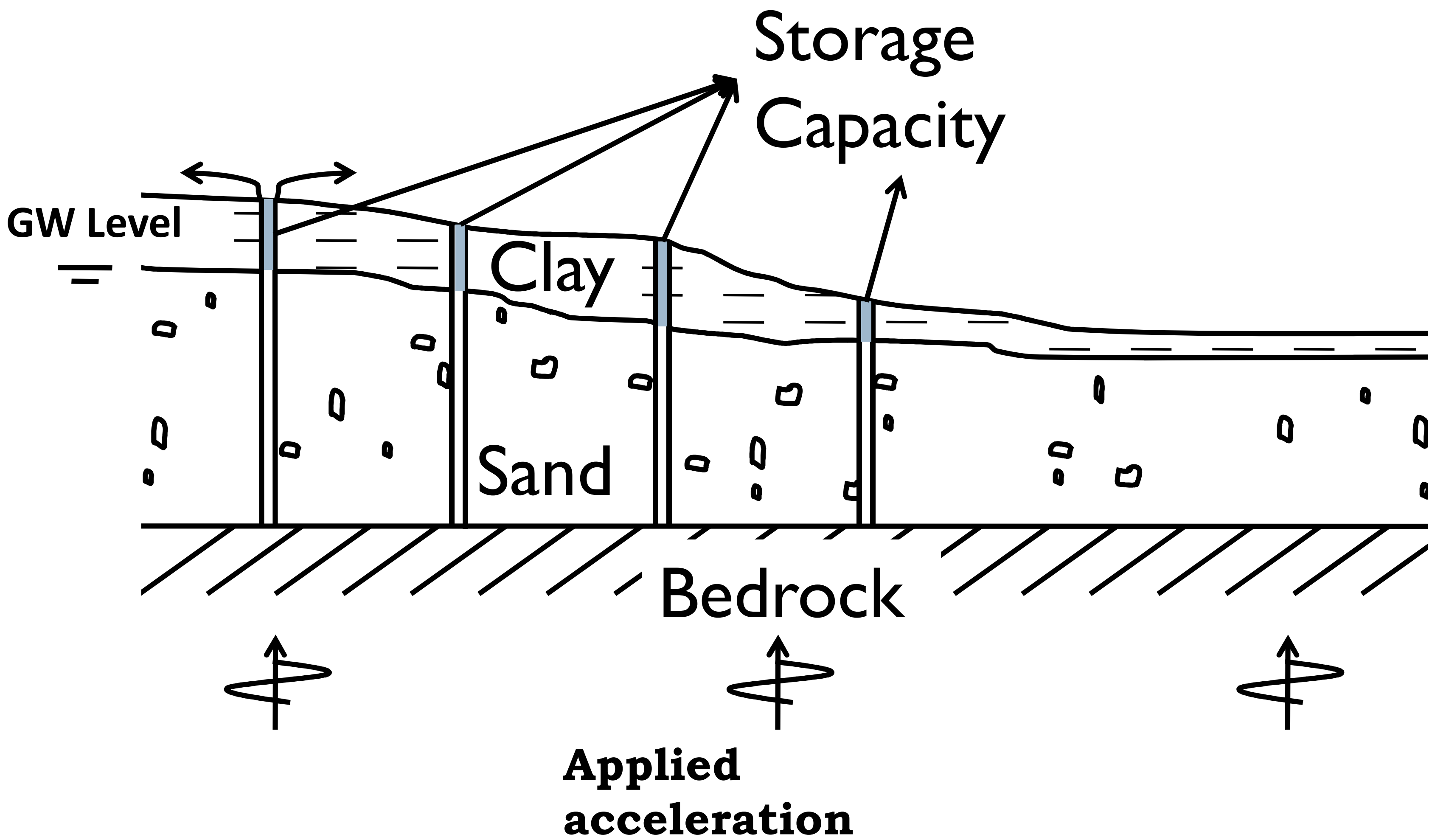
Outline

- Introduce soil improvement with Prefabricated Vertical Drains (PV-Drains)
- Discuss modeling issues of the response of sandy soils treated with PV-Drains
- Validation against Centrifuge Test

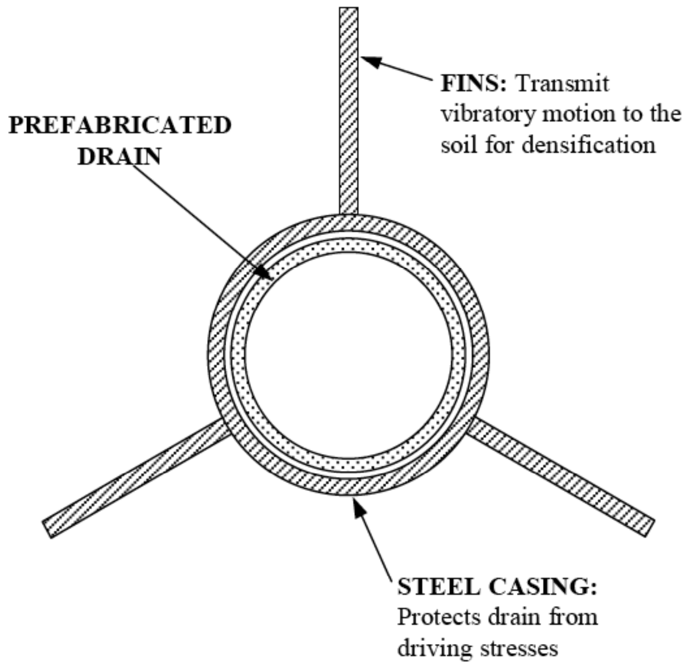
1. PV-Drains

PV-Drains



Storage
Capacity
GW Level
Clay
Sand
Bedrock
Applied
acceleration

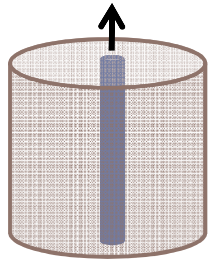
PV-Drains: Cross Section



PV-Drains: Installation

2. Previous research

Analytical Solutions



- Radial Dissipation of Excess Pore Pressure:

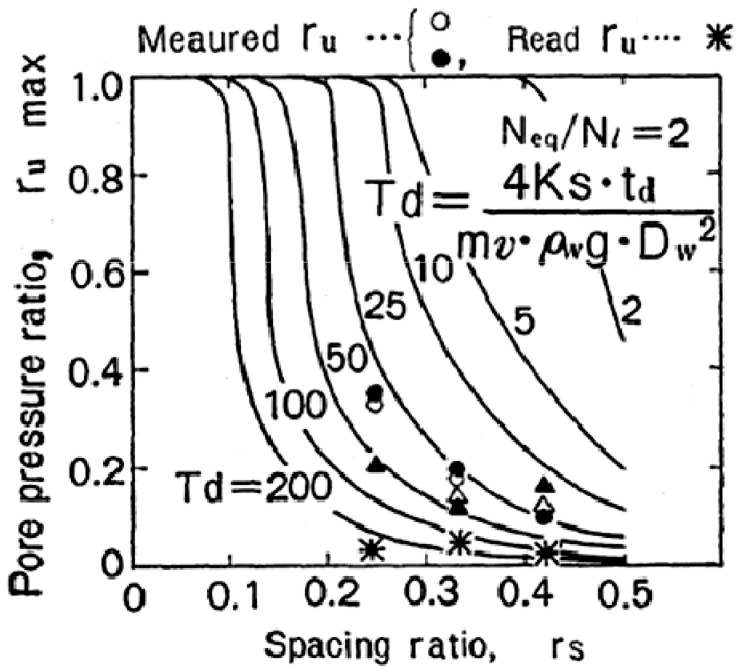
$$\frac{\partial}{\partial r}\left(k\frac{1}{r}\frac{\partial u}{\partial r}\right) = m_v\left(\frac{\partial u}{\partial t} - \frac{\partial u_g}{\partial t}\right)$$

- Excess Pore Pressure Accumulation:

$$r_u = \frac{2}{\pi}\sin^{-1}\left(\frac{N}{N_L}\right)^{\frac{1}{2\theta}}$$

- Key References
  - Seed & Booker: Perfect drains
  - Onoue et al. Effect of well resistance
  - Pestana et al.: FEQDRAIN (includes storage effect)

Validation: Importance of Drain Resistance



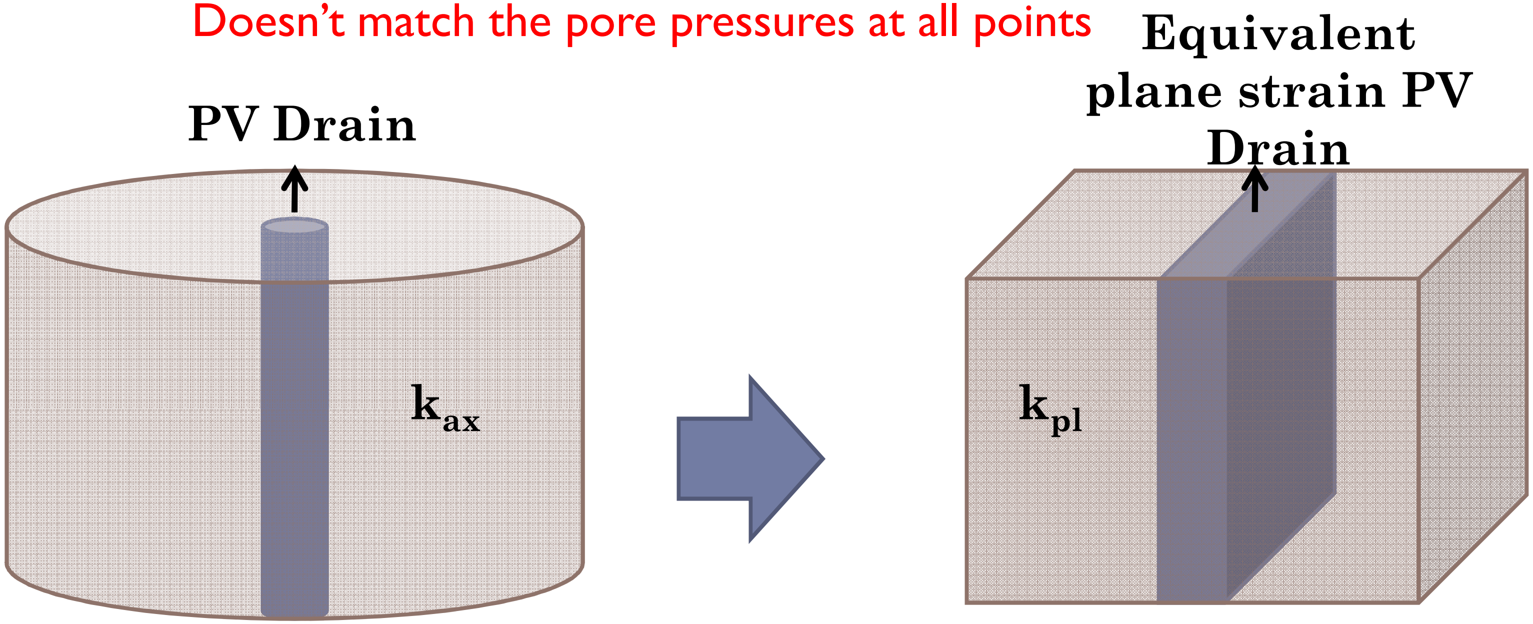
- Predictions of Seed & Booker
  - Read r_u
  - Much lower than measured data
- Measured r_u fit modified theory (well resistance)

Onoue, 1987

2-D (Plane-Strain) Modeling Approximation

Match the degree of consolidation (average diffusion) within soil

Doesn't match the pore pressures at all points



For **infinite permeability** drains inside a **uniform** soil:

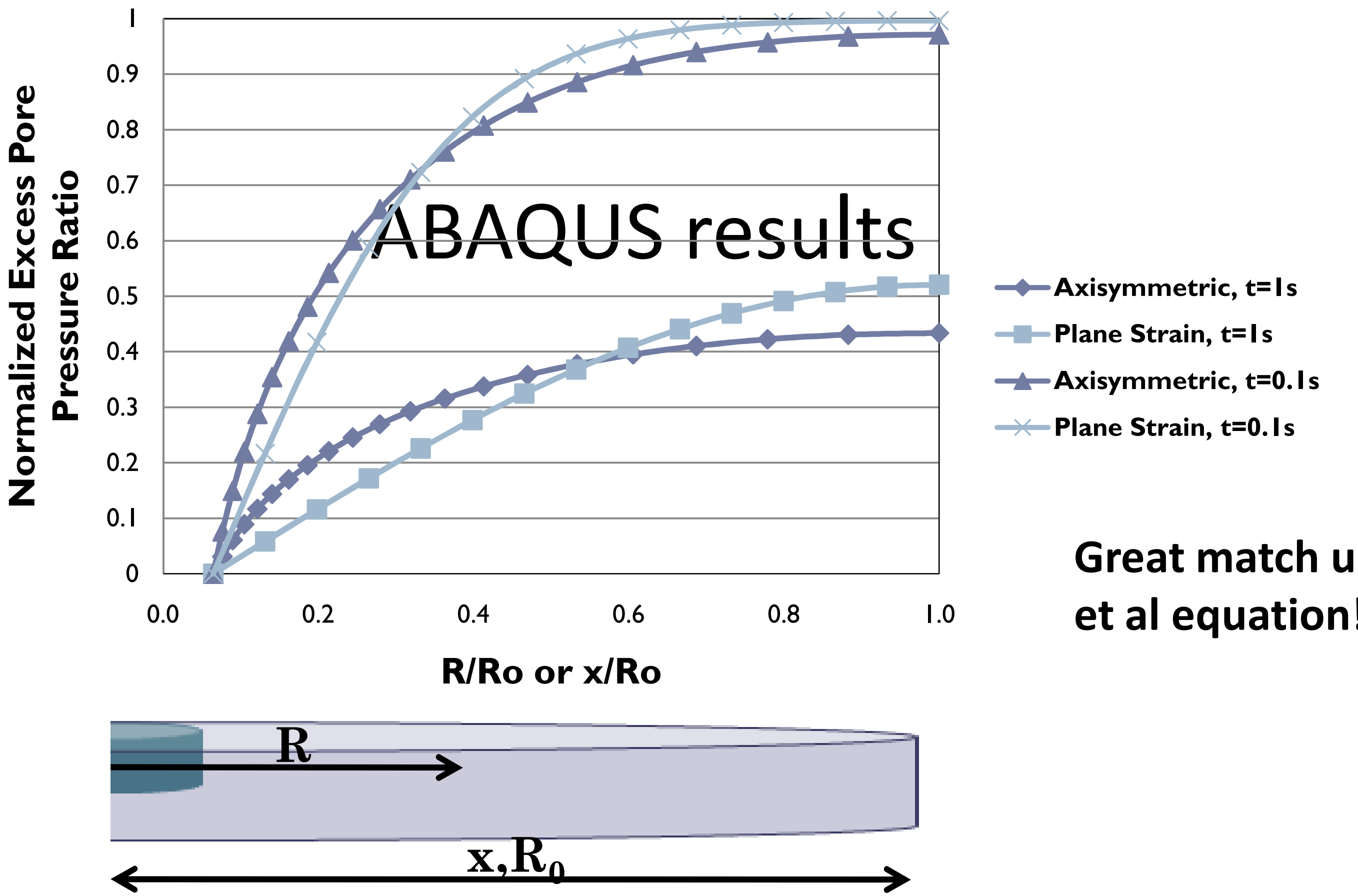
$$k_{pl} = \frac{2k_{ax}}{3\left[\ln(n) - \frac{3}{4}\right]}$$

k_{ax} : true soil permeability,
k_{pl} : equivalent soil permeability in a plane stain analysis
n :drain spacing ratio

Hird et al., 1992

Equivalence between radial and plane strain drainage around a pre-fabricated drain

Normalized Excess Pore Pressure Ratio around a perfect PV drain



Great match using Hird et al equation!!!

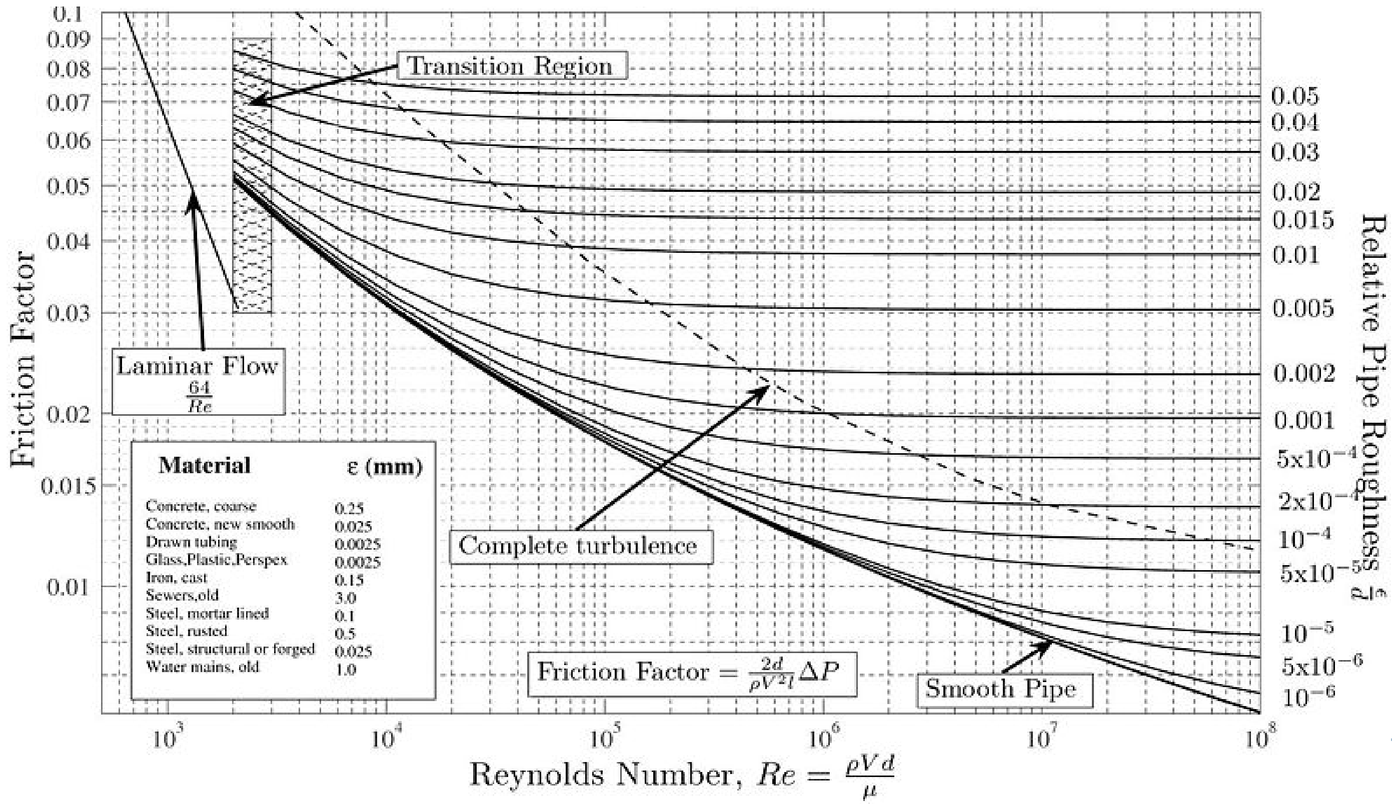
3. Modeling of PV-Drains

PV-Drains: Hydraulics

- Darcy Weisbach Equation

$$\Delta P = \lambda \cdot \frac{L}{D} \cdot \frac{\rho V^2}{2}$$

- ***λ,** dimensionless flow coefficient*
- ***L,** length of pipe*
- ***D,** diameter of pipe*
- ***ρ,** mass density of fluid*
- ***V** , average velocity of the flow*

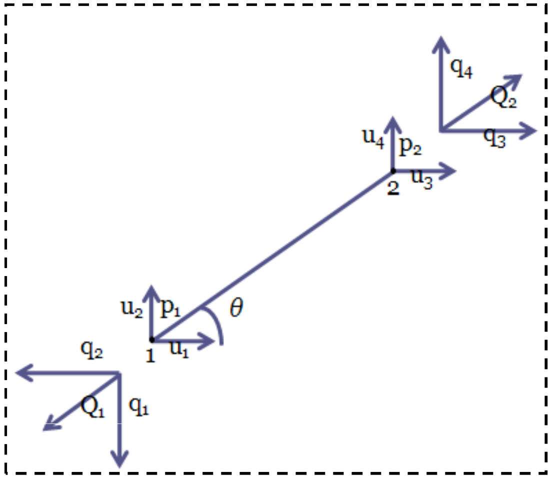


PV-Drain Elements: Laminar flow

$$Q = \frac{A \cdot D^2}{32 \cdot \mu \cdot L} \Delta P$$

$$Q = C_l i$$

C_l $[L^6F^{-1}T^{-1}]$



$$\hat{q} = \begin{bmatrix} q_1 \\ q_2 \\ Q_1 \\ q_3 \\ q_4 \\ Q_2 \end{bmatrix} \quad \hat{u} = \begin{bmatrix} u_1 \\ u_2 \\ p_1 \\ u_3 \\ u_4 \\ p_2 \end{bmatrix} \quad \hat{q} = \hat{k} \cdot \hat{u}$$

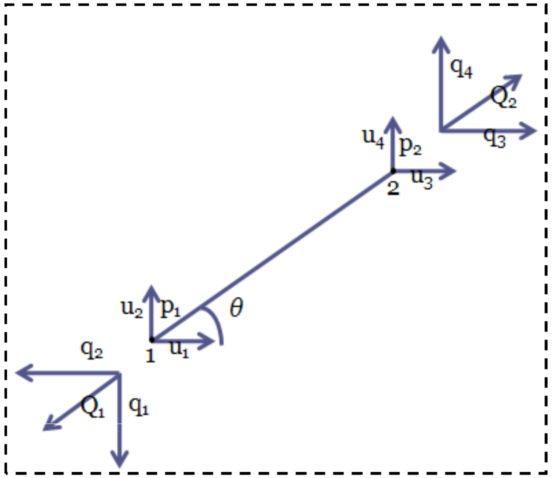
$$\hat{k} = \begin{bmatrix} k_{11} & k_{12} & 0 & -k_{11} & -k_{12} & 0 \\ k_{12} & k_{22} & 0 & -k_{12} & -k_{22} & 0 \\ 0 & 0 & k_{33} & 0 & 0 & -k_{33} \\ -k_{11} & -k_{12} & 0 & k_{11} & k_{12} & 0 \\ -k_{12} & -k_{22} & 0 & k_{12} & k_{22} & 0 \\ 0 & 0 & -k_{33} & 0 & 0 & k_{33} \end{bmatrix}$$

$$k_{11} = \cos^2(\theta)$$
$$k_{22} = \sin^2(\theta)$$
$$k_{12} = \cos(\theta) \cdot \sin(\theta)$$
$$k_{33} = -\frac{A \cdot D^2}{32 \cdot \mu \cdot L}$$

PV-Drain Elements: Fully turbulent flow

$$Q = \sqrt{\frac{2 \cdot D \cdot A^2}{\lambda \cdot L \cdot \rho}} \cdot \sqrt{\Delta P} \qquad Q = C_t \cdot \sqrt{i} \qquad C_t \; [L^{4.5} F^{-0.5} T^{-1}]$$

$$\hat{\dot{q}} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{Q}_1 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{Q}_2 \end{bmatrix} \qquad \hat{\dot{u}} = \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \\ \dot{p}_1 \\ \dot{u}_3 \\ \dot{u}_4 \\ \dot{p}_2 \end{bmatrix} \qquad \hat{\dot{q}} = \hat{k} \cdot \hat{\dot{u}}$$



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$\Delta p_{(n)} > d_c$:

$$k_{33} = -\frac{\Delta Q_n}{\Delta p_{(n)}}$$

$\Delta p_{(n)} < d_c$ and $i > d_c$:

$$k_{33} = -\sqrt{\frac{2 \cdot D \cdot A^2}{\lambda \cdot L \cdot \rho}} \cdot \frac{1}{2\sqrt{\Delta P}}$$

$\Delta p_{(n)} < d_c$ and $i < d_c$:

$$k_{33} = -\frac{C_t}{\sqrt{d_c} \cdot L}$$

$$k_{11} = \cos^2(\theta)$$
$$k_{22} = \sin^2(\theta)$$
$$k_{12} = \cos(\theta) \cdot \sin(\theta)$$

OpenSees Object Oriented Finite Element Framework

- Developed for Research Purposes
- Capability to solve dynamic fully coupled pore pressure displacement analyses
- Modular
- But … with minimal documentation, no pre- and post-processing, and small bugs here and there!

- Most importantly:
  - FUN: interface in tcl/tk and source in C++

PV-Drains: Opensees Implementation

- Laminar flow:
  - `element Pipelin2 eleid node1 node2 Material Area` C_l γ_w

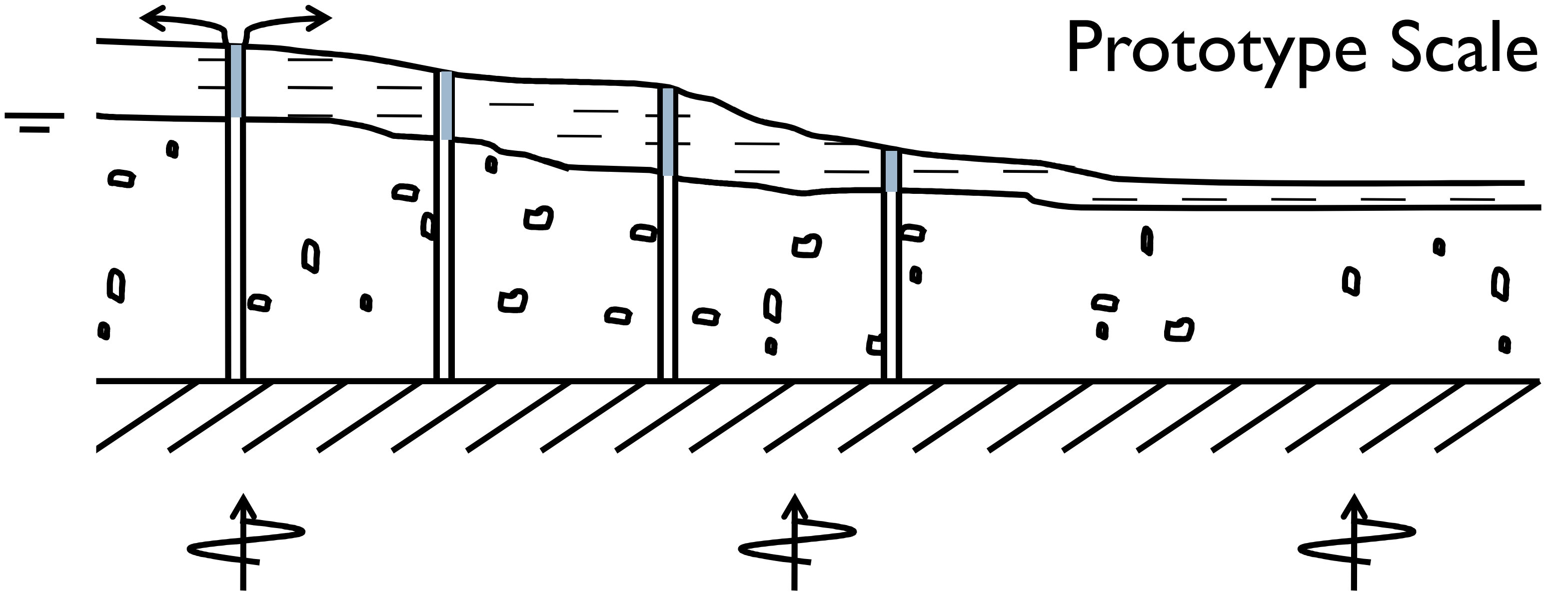
$$C_l = \frac{A \cdot D^2}{32 \cdot \mu}$$

- Turbulent Flow
  - `element Pipe4 eleid node1 node2 Material Area` C_t γ_w d_c

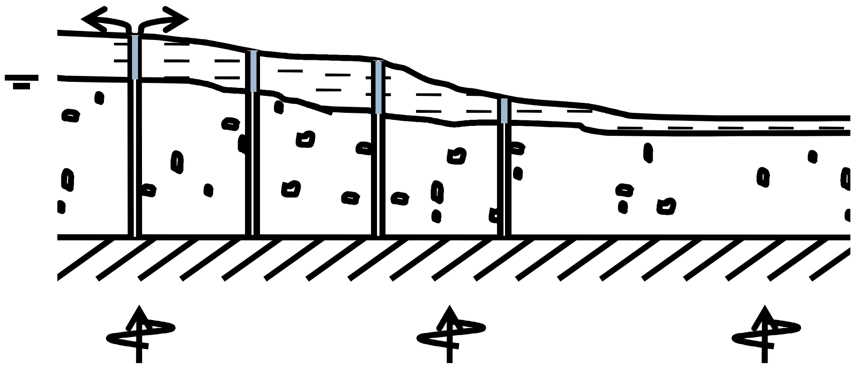
$$C_t = \sqrt{\frac{2 \cdot D \cdot A^2}{\lambda \cdot \rho}}$$

4. Centrifuge Testing

Centrifuge Testing



Prototype Scale



Model Scale

Centrifuge Testing: UC Davis Centrifuge Device

PV-Drains: Scaling Principles

- Scaling of Dimensions:
  - $X^P = X^M \cdot N$
- Scaling of Stresses:
  - $\sigma^P = \sigma^M$
- Scaling of Acceleration:
  - $a^P = a^M / N$
- Scaling of Time:
  - $t^P = t^M \cdot N$
- Scaling of Permeability:
  - $k^P = k^M \cdot N$

PV-Drains: Scale Modeling Issues (I)

- Scaling of flow:
  - $Q^P = Q^M \cdot N^2$
- Scaling of Drain Flow Properties:
  - Laminar flow
    - $C_l^P = C_l^M \cdot N^3$
  - Fully turbulent flow
    - $C_t^P = C_t^M \cdot N^{5/2}$

PV-Drains: Scale Modeling Issues (II)

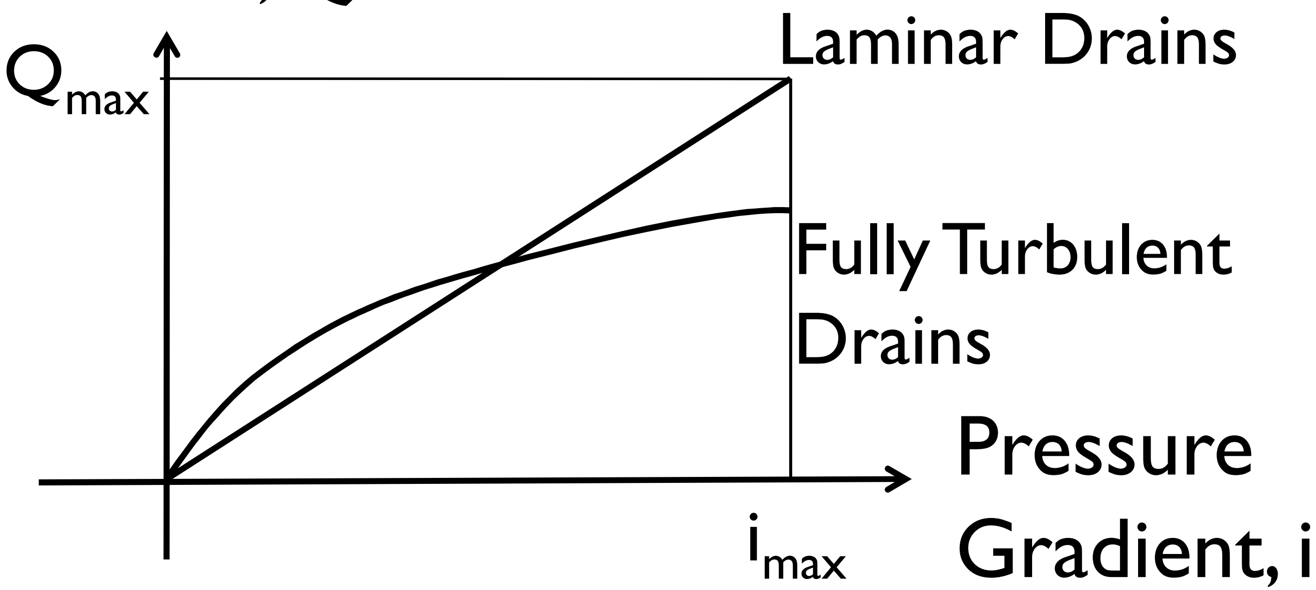
- Scaling of Reynolds number
  - For the same pore fluid:
    - $Re^P = Re^M \cdot N$
  - For different pore fluid (diffusion scaled)
    - $Re^P = Re^M \cdot N^2$

- **Problem Statement:**

  "What is the model scale diameter of a PV-drain (where flow is laminar) that corresponds to a selected prototype scale diameter (where flow is turbulent)?"

PV-Drains: Scale Modeling Issues (III)

Flow Rate, Q

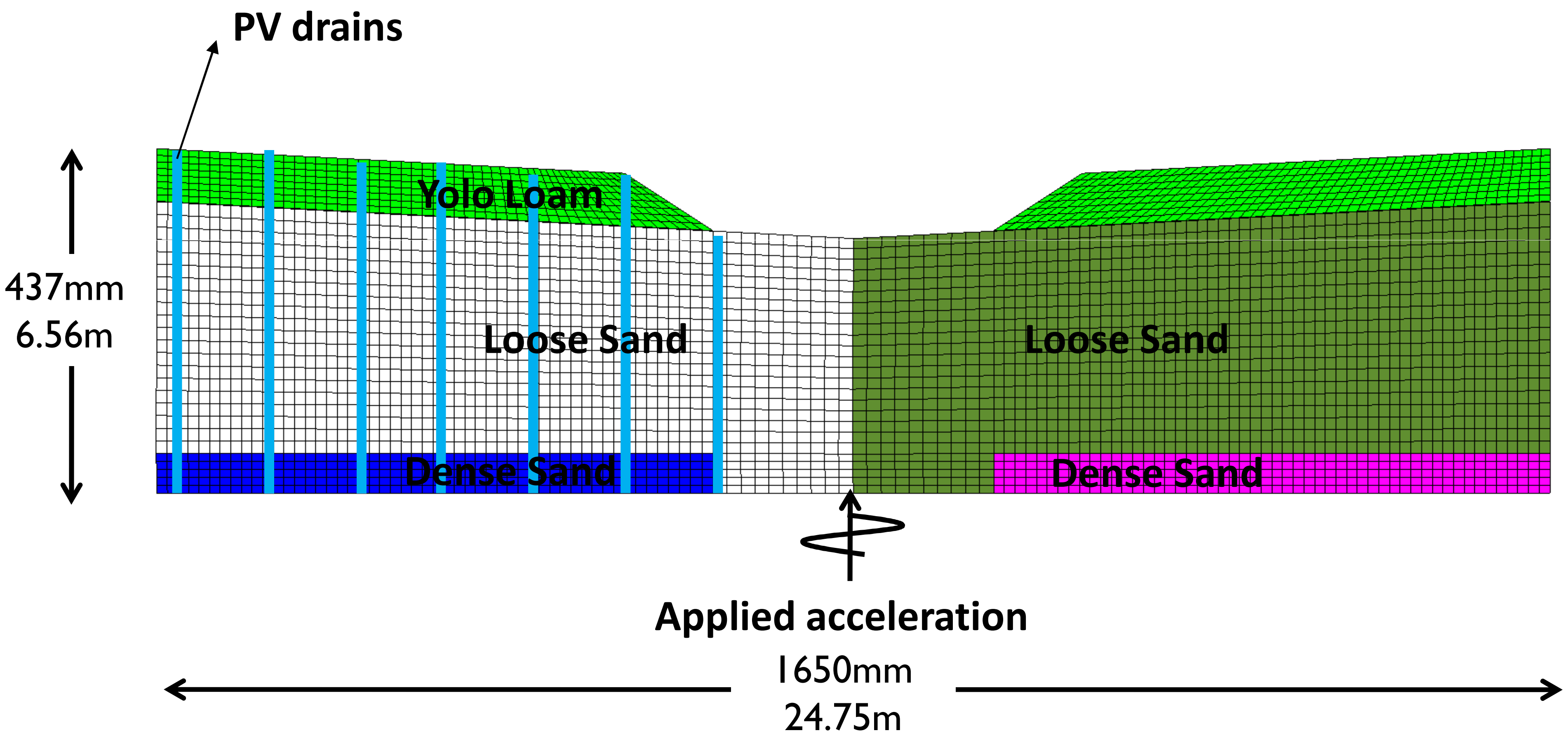


The model scale diameter that minimizes differences between model scale laminar and prototype scale turbulent flow is:

$$D^M = \left(\frac{76.8\,\mu^M}{\sqrt{i_{max}} \cdot N^3} \sqrt{\frac{{D^P}^5}{2 \cdot \lambda \cdot \rho}} \right)^{\frac{1}{4}}$$

5. Validation

PV-Drains Validation: Centrifuge Model



PV-Drains Validation: Centrifuge Model

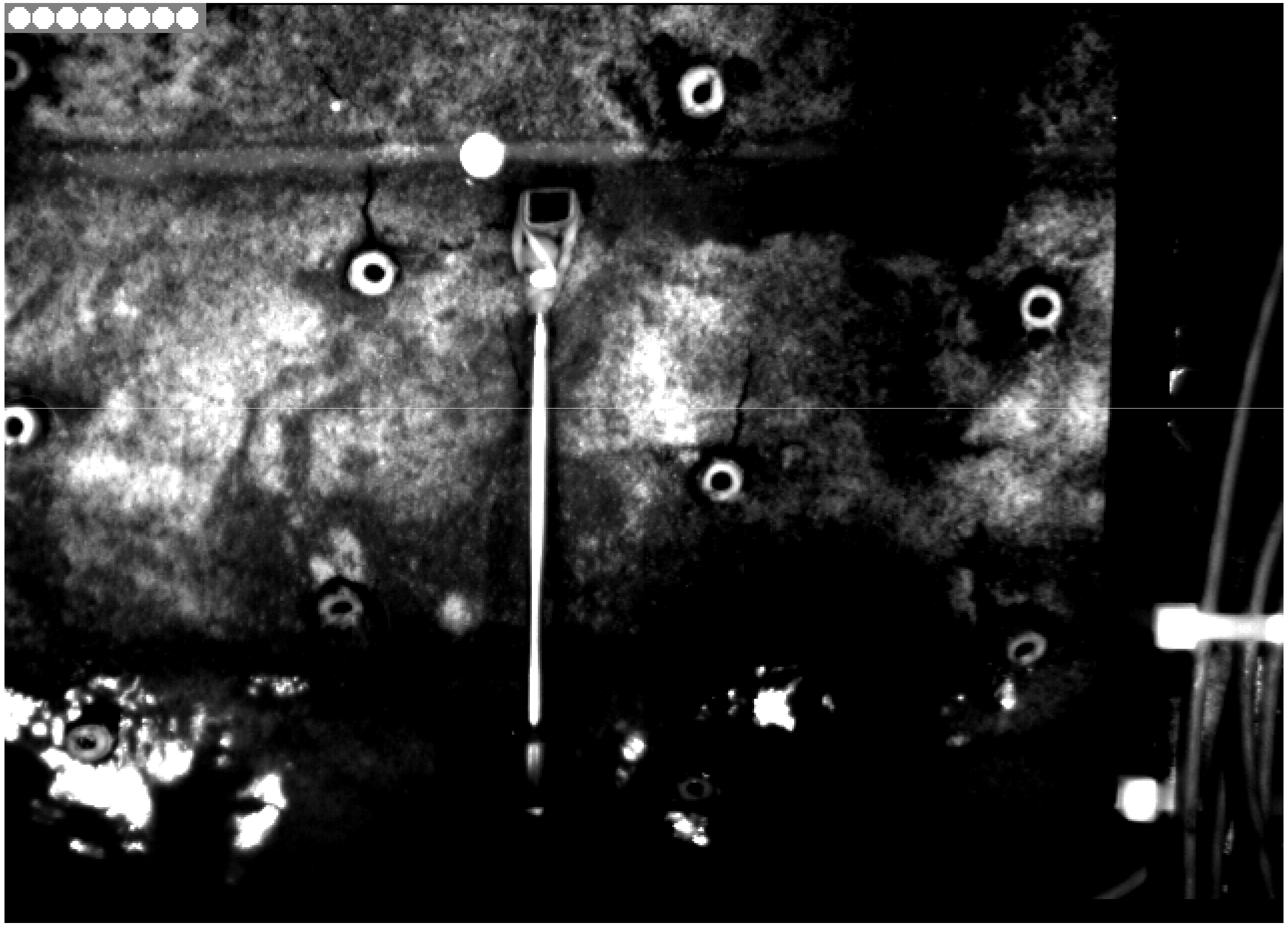
Kamai et al, 2008

PV-Drains Validation: Centrifuge Model

- Shaking sequence

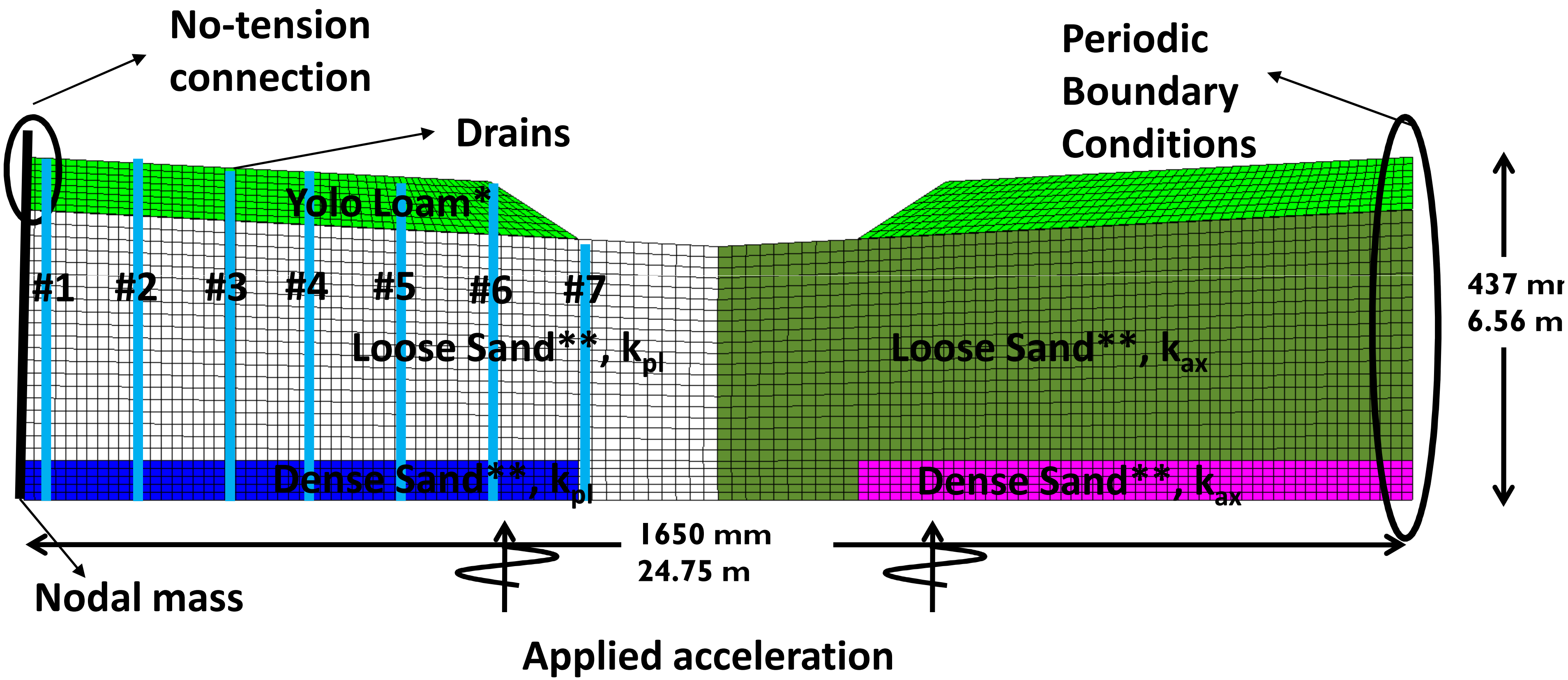
Spin	Event ID	Centrifuge Acceleration (g)	Motion Amplitude[1] (g)	Time	Date
Spin 5	Spin Up	---	---	4:15 PM	3/9/2007
	SSK01_08	15	0.01	4:45 PM	3/9/2007
	SSK01_09	15	0.03	5:06 PM	3/9/2007
	SSK01_10	15	0.07	5:32 PM	3/9/2007
	SSK01_11	15	0.11	6:19 PM	3/9/2007
	SSK01_12	15	0.3	6:45 PM	3/9/2007
	Spin Down	---	---	6:51 PM	3/9/2007

PV-Drains Validation: Centrifuge Model



Kamai et al, 2008

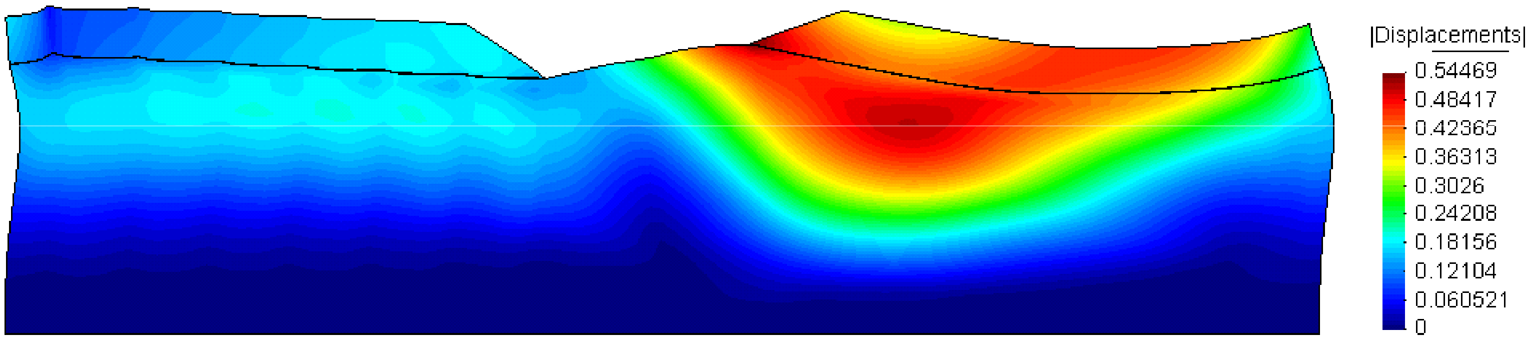
PV-Drains Validation: Base Case Scenario



**pressure independent multiyield model, QUAD Elements*

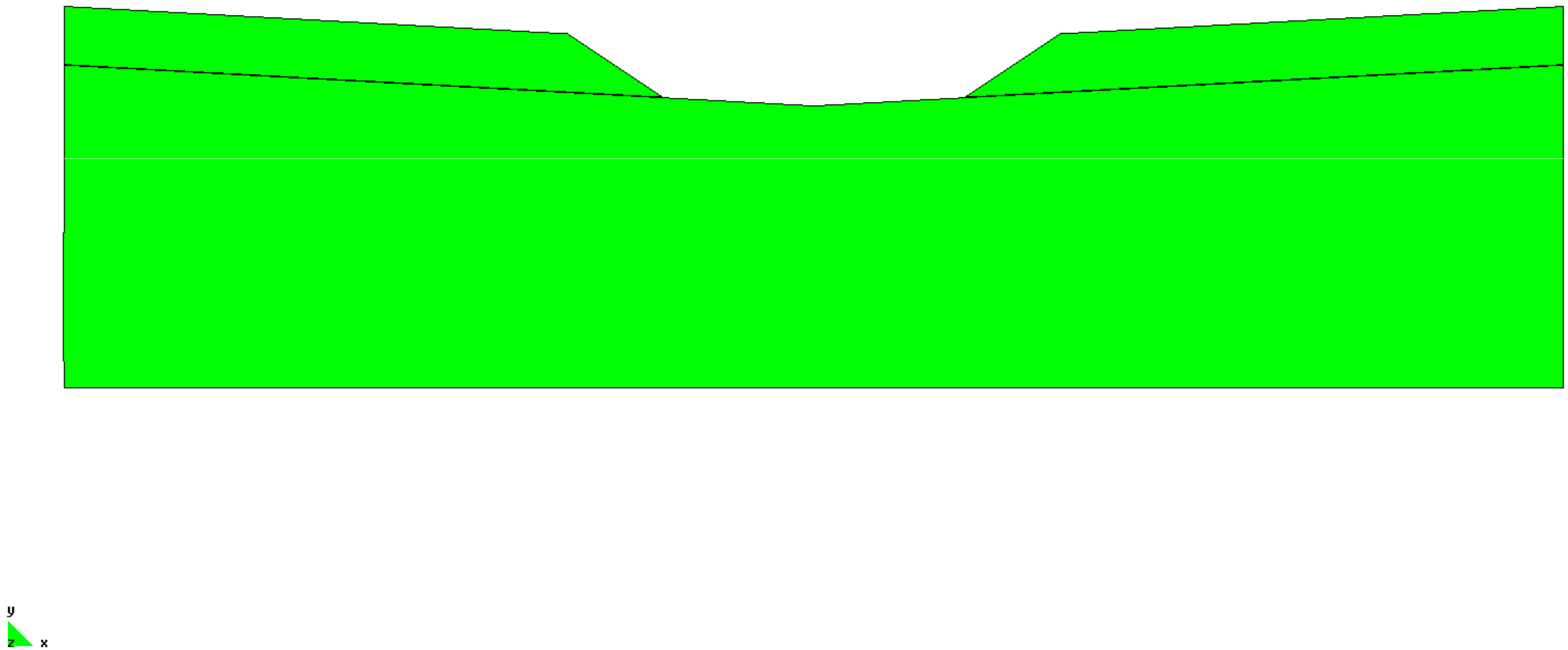
*** pressure dependent multiyield model, QUADUP Elements*

PV-Drains Validation: Final Deformed Shape

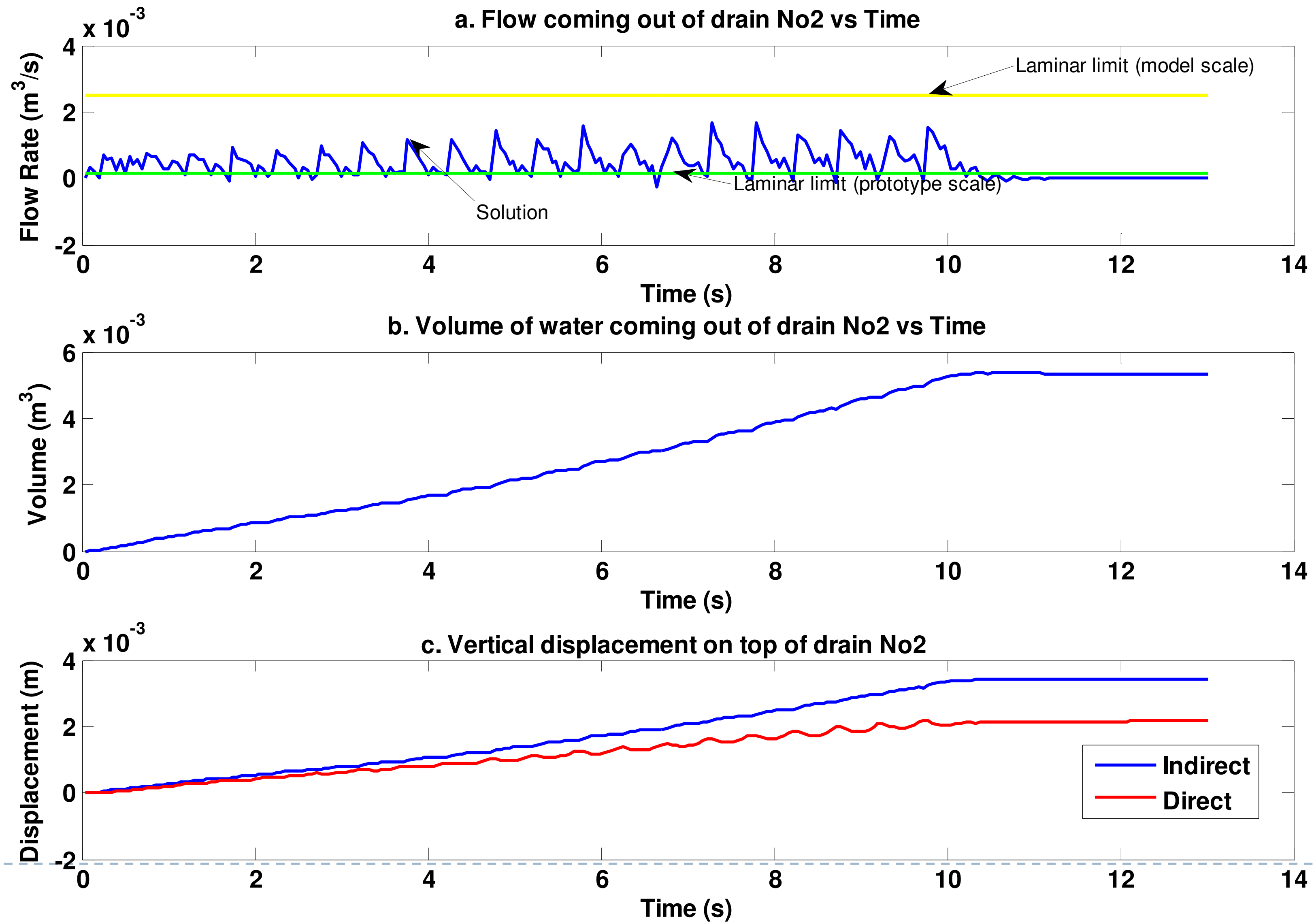


- **Numerical simulations indicate the effectiveness of PV-Drains!!**

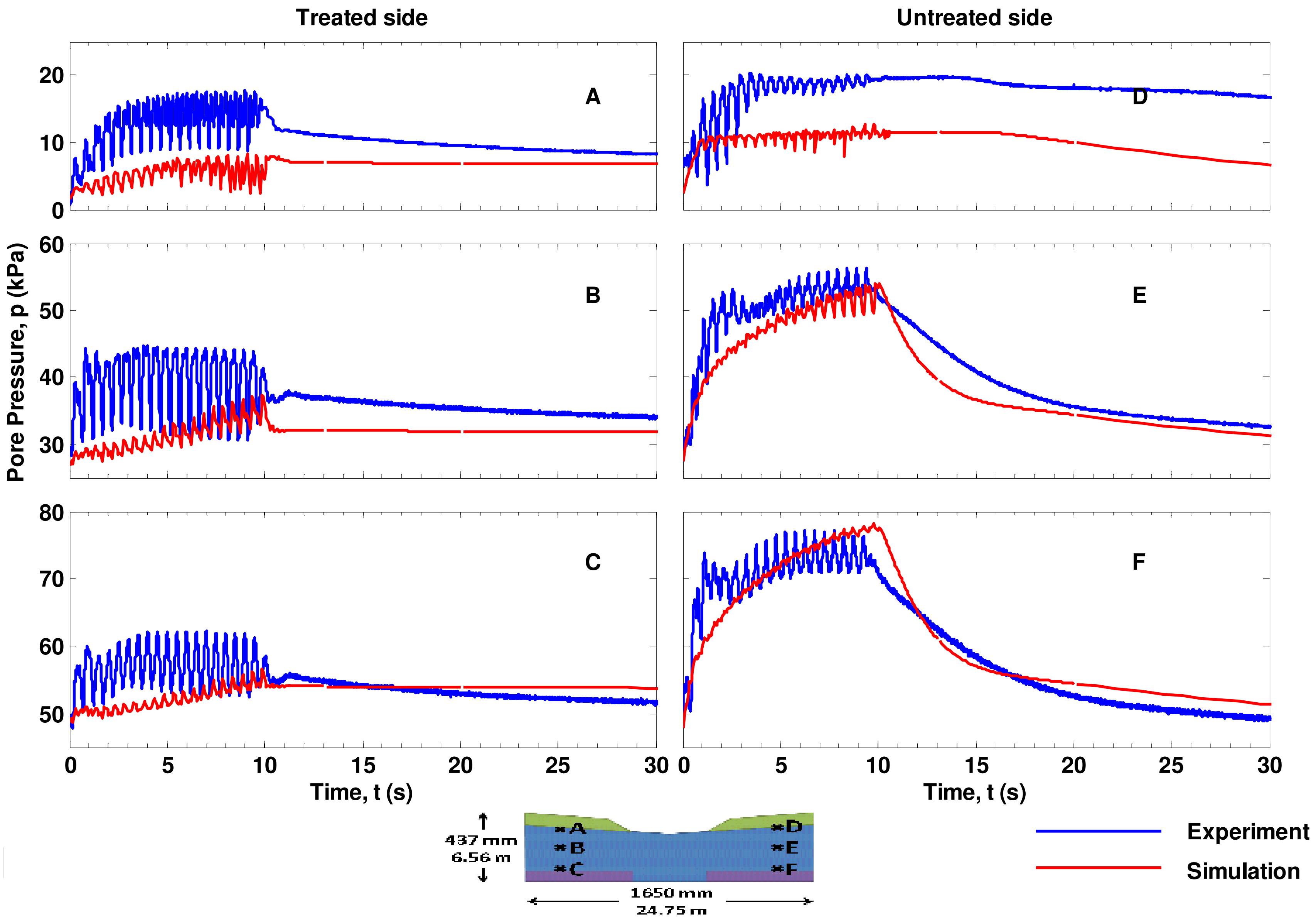
PV-Drains Validation: Pore Pressure



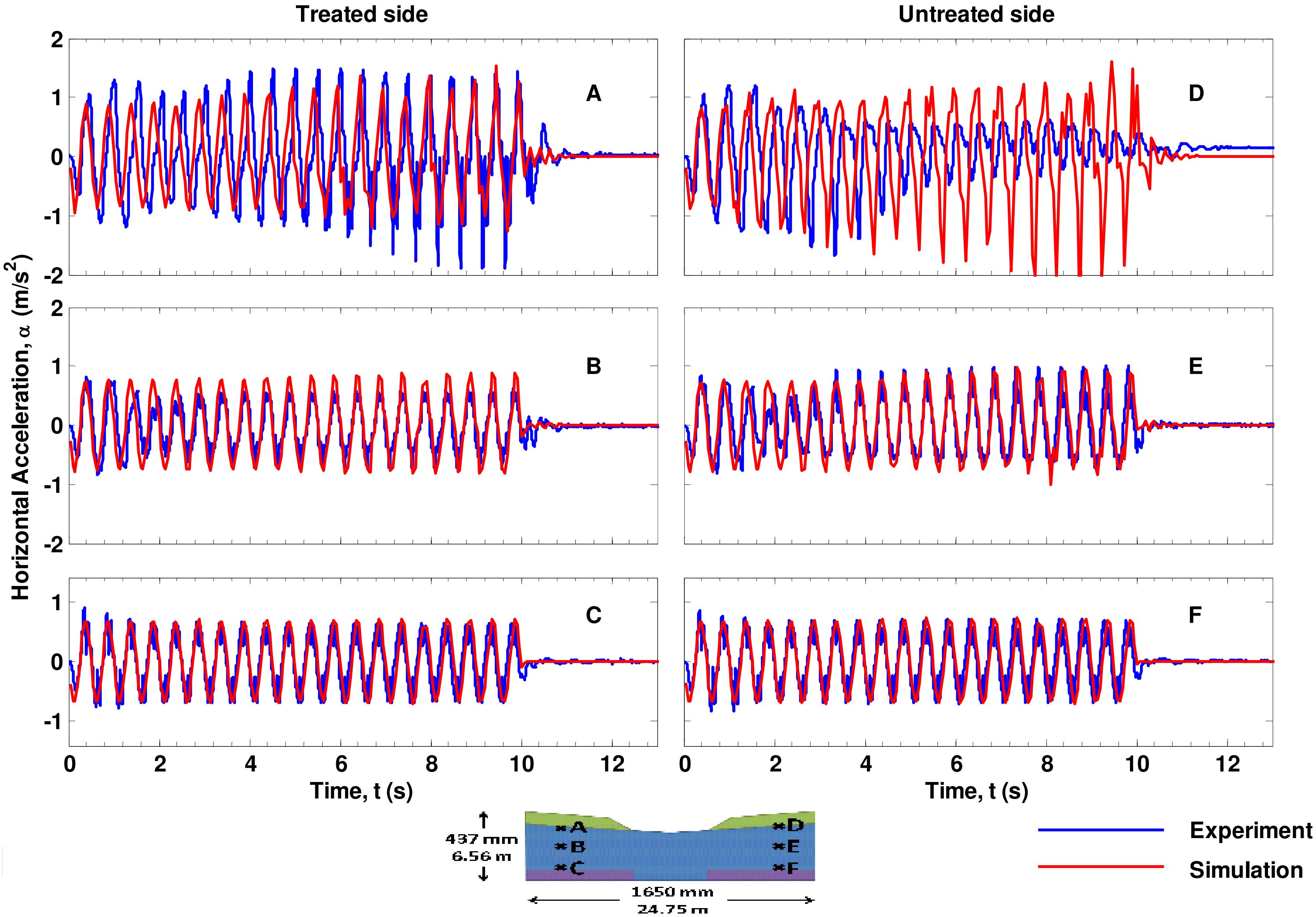
PV-Drains Validation: PV-drain outflow



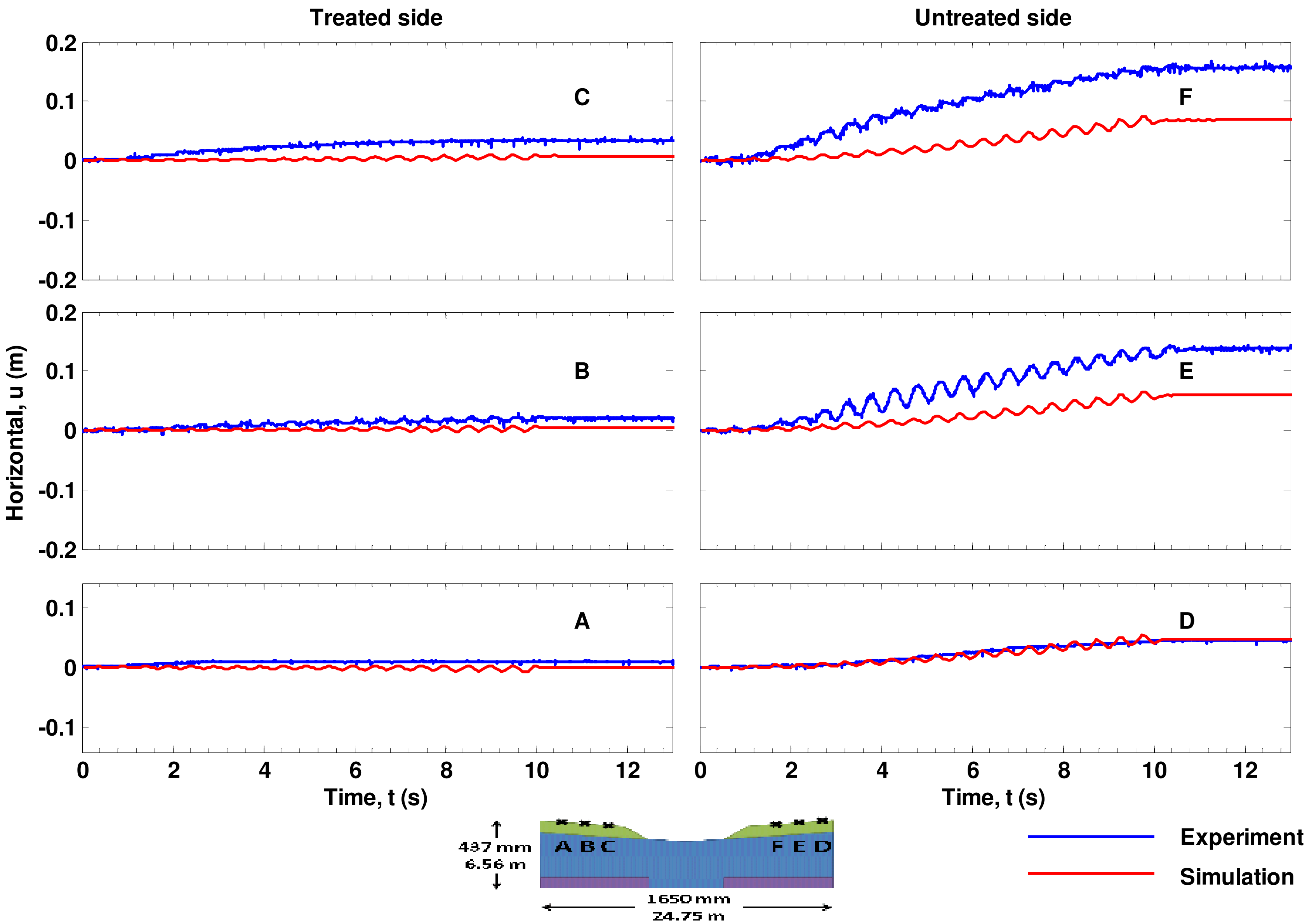
PV-Drains Validation:
Pore Pressures, amax=0.07g



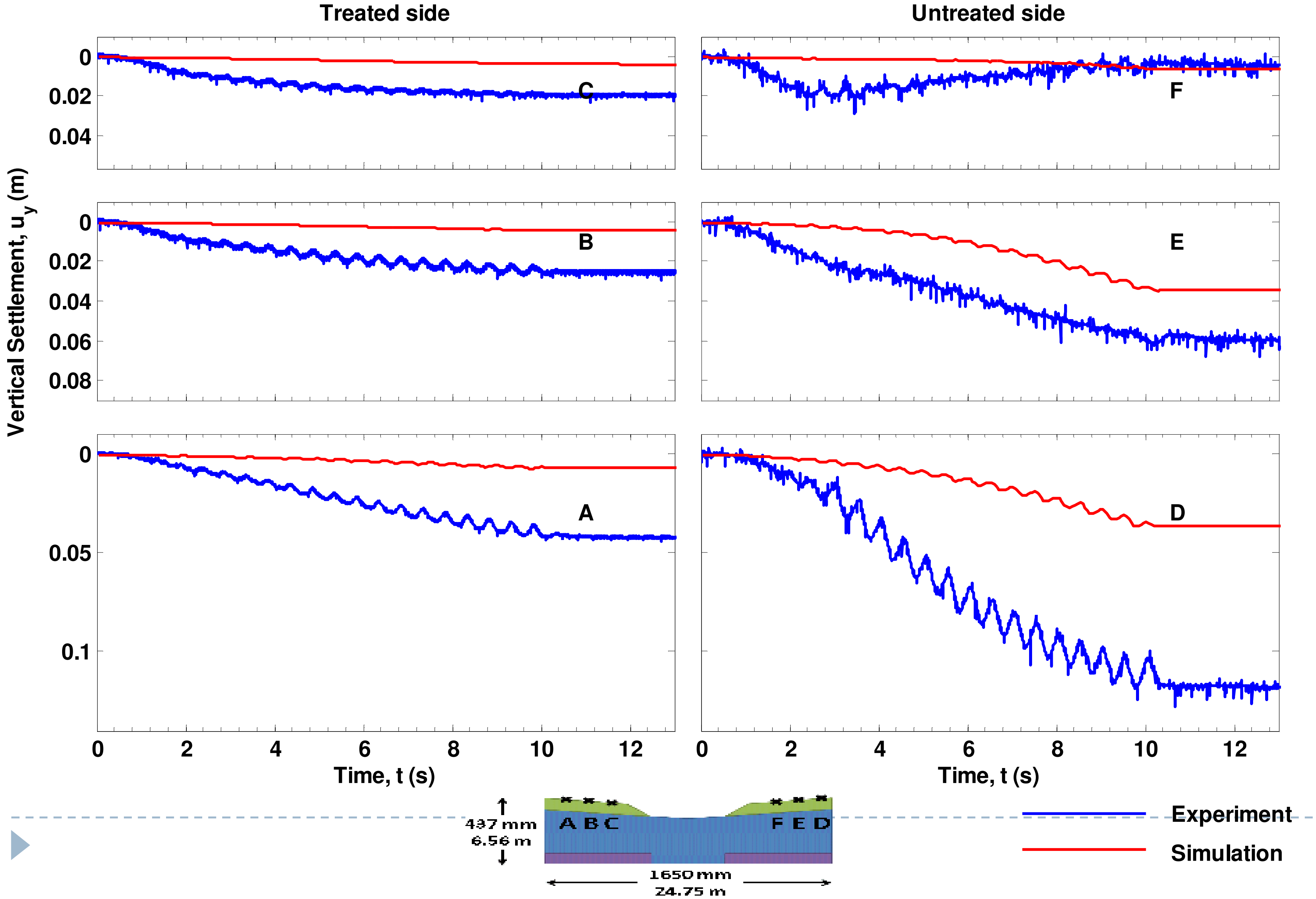
PV-Drains Validation:
Horizontal Accelerations, amax=0.07g



PV-Drains Validation: Surface Horizontal Displacements, amax=0.07g



PV-Drains Validation:
Surface Settlements, amax=0.07g



PV-Drains: Validation Observations

- Excess Pore Pressures
  - Reasonable agreement at mid layer
  - Underestimate at top of sand
- Horizontal Accelerations
  - Good agreement on treated side
  - No de-amplification in untreated sand
    - No ‘prediction’ of liquefaction event
- Horizontal Displacements
  - Reasonable magnitudes
    - No slippage between sand and loam
- Vertical Displacements (surface)
  - Mismatched across model
  - Effect of variable g field (centrifuge) and/or slope failure mechanism?

Coming up …

- Investigation of the non-uniformity of the acceleration field inside the centrifuge device (CMMI 2009)
- Submission of code and manuals to OpenSees (PV-drain elements, element wrappers, and material models)
- Examination of the effect of numerical model by comparing with Dafalias Manzari model (5th IC on RAGEESD)
- Field performance
  - Long term performance?
  - Effect of buckling, clogging, sedimentation & biofouling?
- Free Field Analyses (Incl. Transmitting Boundaries)
- Colloidal Silica Grout Simulation

Thank you!