

axiomTM



The 30 Year Horizon

<i>Manuel Bronstein</i>	<i>William Burge</i>	<i>Timothy Daly</i>
<i>James Davenport</i>	<i>Michael Dewar</i>	<i>Martin Dunstan</i>
<i>Albrecht Fortenbacher</i>	<i>Patrizia Gianni</i>	<i>Johannes Grabmeier</i>
<i>Jocelyn Guidry</i>	<i>Richard Jenks</i>	<i>Larry Lambe</i>
<i>Michael Monagan</i>	<i>Scott Morrison</i>	<i>William Sit</i>
<i>Jonathan Steinbach</i>	<i>Robert Sutor</i>	<i>Barry Trager</i>
<i>Stephen Watt</i>	<i>Jim Wen</i>	<i>Clifton Williamson</i>

Volume 3: Axiom Programmers Guide

Portions Copyright (c) 2005 Timothy Daly

The Blue Bayou image Copyright (c) 2004 Jocelyn Guidry

Portions Copyright (c) 2004 Martin Dunstan

Portions Copyright (c) 2007 Alfredo Portes

Portions Copyright (c) 2007 Arthur Ralfs

Portions Copyright (c) 2005 Timothy Daly

Portions Copyright (c) 1991-2002,
The Numerical ALgorithms Group Ltd.
All rights reserved.

This book and the Axiom software is licensed as follows:

Redistribution and use in source and binary forms, with or without modification, are permitted provided that the following conditions are met:

- Redistributions of source code must retain the above copyright notice, this list of conditions and the following disclaimer.
- Redistributions in binary form must reproduce the above copyright notice, this list of conditions and the following disclaimer in the documentation and/or other materials provided with the distribution.
- Neither the name of The Numerical ALgorithms Group Ltd. nor the names of its contributors may be used to endorse or promote products derived from this software without specific prior written permission.

THIS SOFTWARE IS PROVIDED BY THE COPYRIGHT HOLDERS AND CONTRIBUTORS "AS IS" AND ANY EXPRESS OR IMPLIED WARRANTIES, INCLUDING, BUT NOT LIMITED TO, THE IMPLIED WARRANTIES OF MERCHANTABILITY AND FITNESS FOR A PARTICULAR PURPOSE ARE DISCLAIMED. IN NO EVENT SHALL THE COPYRIGHT OWNER OR CONTRIBUTORS BE LIABLE FOR ANY DIRECT, INDIRECT, INCIDENTAL, SPECIAL, EXEMPLARY, OR CONSEQUENTIAL DAMAGES (INCLUDING, BUT NOT LIMITED TO, PROCUREMENT OF SUBSTITUTE GOODS OR SERVICES; LOSS OF USE, DATA, OR PROFITS; OR BUSINESS INTERRUPTION) HOWEVER CAUSED AND ON ANY THEORY OF LIABILITY, WHETHER IN CONTRACT, STRICT LIABILITY, OR TORT (INCLUDING NEGLIGENCE OR OTHERWISE) ARISING IN ANY WAY OUT OF THE USE OF THIS SOFTWARE, EVEN IF ADVISED OF THE POSSIBILITY OF SUCH DAMAGE.

Inclusion of names in the list of credits is based on historical information and is as accurate as possible. Inclusion of names does not in any way imply an endorsement but represents historical influence on Axiom development.

Michael Albaugh	Cyril Alberga	Roy Adler
Christian Aistleitner	Richard Anderson	George Andrews
S.J. Atkins	Henry Baker	Martin Baker
Stephen Balzac	Yurij Baransky	David R. Barton
Gerald Baumgartner	Gilbert Baumslag	Michael Becker
Nelson H. F. Beebe	Jay Belanger	David Bindel
Fred Blair	Vladimir Bondarenko	Mark Botch
Raoul Bourquin	Alexandre Bouyer	Karen Braman
Peter A. Broadbery	Martin Brock	Manuel Bronstein
Stephen Buchwald	Florian Bundschuh	Luanne Burns
William Burge	Ralph Byers	Quentin Carpent
Robert Caviness	Bruce Char	Ondrej Certik
Tzu-Yi Chen	Cheekai Chin	David V. Chudnovsky
Gregory V. Chudnovsky	Mark Clements	James Cloos
Jia Zhao Cong	Josh Cohen	Christophe Conil
Don Coppersmith	George Corliss	Robert Corless
Gary Cornell	Meino Cramer	Jeremy Du Croz
David Cyganski	Nathaniel Daly	Timothy Daly Sr.
Timothy Daly Jr.	James H. Davenport	David Day
James Demmel	Didier Deshombres	Michael Dewar
Jack Dongarra	Jean Della Dora	Gabriel Dos Reis
Claire DiCrescendo	Sam Dooley	Lionel Ducos
Iain Duff	Lee Duhem	Martin Dunstan
Brian Dupee	Dominique Duval	Robert Edwards
Heow Eide-Goodman	Lars Erickson	Richard Fateman
Bertfried Fauser	Stuart Feldman	John Fletcher
Brian Ford	Albrecht Fortenbacher	George Frances
Constantine Frangos	Timothy Freeman	Korrinn Fu
Marc Gaetano	Rudiger Gebauer	Van de Geijn
Kathy Gerber	Patricia Gianni	Samantha Goldrich
Holger Gollan	Teresa Gomez-Diaz	Laureano Gonzalez-Vega
Stephen Gortler	Johannes Grabmeier	Matt Grayson
Klaus Ebbe Grue	James Griesmer	Vladimir Grinberg
Oswald Gschnitzer	Ming Gu	Jocelyn Guidry
Gaetan Hache	Steve Hague	Satoshi Hamaguchi
Sven Hammarling	Mike Hansen	Richard Hanson
Richard Harke	Bill Hart	Vilya Harvey
Martin Hassner	Arthur S. Hathaway	Dan Hatton
Waldek Hebisch	Karl Hegbloom	Ralf Hemmecke

Henderson	Antoine Hersen	Roger House
Gernot Hueber	Pietro Iglio	Alejandro Jakubi
Richard Jenks	William Kahan	Kai Kaminski
Grant Keady	Wilfrid Kendall	Tony Kennedy
Ted Kosan	Paul Kosinski	Klaus Kusche
Bernhard Kutzler	Tim Lahey	Larry Lambe
Kaj Laurson	George L. Legendre	Franz Lehner
Frederic Lehouby	Michel Levaud	Howard Levy
Ren-Cang Li	Rudiger Loos	Michael Lucks
Richard Luczak	Camm Maguire	Francois Maltey
Alasdair McAndrew	Bob McElrath	Michael McGettrick
Edi Meier	Ian Meikle	David Mentre
Victor S. Miller	Gerard Milmeister	Mohammed Mobarak
H. Michael Moeller	Michael Monagan	Marc Moreno-Maza
Scott Morrison	Joel Moses	Mark Murray
William Naylor	Patrice Naudin	C. Andrew Neff
John Nelder	Godfrey Nolan	Arthur Norman
Jinzhong Niu	Michael O'Connor	Summat Oemrawsingh
Kostas Oikonomou	Humberto Ortiz-Zuazaga	Julian A. Padget
Bill Page	David Parnas	Susan Pelzel
Michel Petitot	Didier Pinchon	Ayal Pinkus
Frederick H. Pitts	Jose Alfredo Portes	Gregorio Quintana-Orti
Claude Quitte	Arthur C. Ralfs	Norman Ramsey
Anatoly Raportirenko	Albert D. Rich	Michael Richardson
Guilherme Reis	Huan Ren	Renaud Rioboo
Jean Rivlin	Nicolas Robidoux	Simon Robinson
Raymond Rogers	Michael Rothstein	Martin Rubey
Philip Santas	Alfred Scheerhorn	William Schelter
Gerhard Schneider	Martin Schoenert	Marshall Schor
Frithjof Schulze	Fritz Schwarz	Steven Segletes
V. Sima	Nick Simicich	William Sit
Elena Smirnova	Jonathan Steinbach	Fabio Stumbo
Christine Sundaresan	Robert Sutor	Moss E. Sweedler
Eugene Surowitz	Max Tegmark	T. Doug Telford
James Thatcher	Balbir Thomas	Mike Thomas
Dylan Thurston	Steve Toleque	Barry Trager
Themos T. Tsikas	Gregory Vanuxem	Bernhard Wall
Stephen Watt	Jaap Weel	Juergen Weiss
M. Weller	Mark Wegman	James Wen
Thorsten Werther	Michael Wester	R. Clint Whaley
John M. Wiley	Berhard Will	Clifton J. Williamson
Stephen Wilson	Shmuel Winograd	Robert Wisbauer
Sandra Wityak	Waldemar Wiwianka	Knut Wolf
Liu Xiaojun	Clifford Yapp	David Yun
Vadim Zhytnikov	Richard Zippel	Evelyn Zoernack
Bruno Zuercher	Dan Zwillinger	

Contents

1	Details for Programmers	1
1.1	Makefile	1

New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

Tim Daly
CAISS, City College of New York
November 10, 2003 ((iHy))

Chapter 1

Details for Programmers

1.1 Makefile

This book is actually a literate program[2] and can contain executable source code. In particular, the Makefile for this book is part of the source of the book and is included below. Axiom uses the “noweb” literate programming system by Norman Ramsey[6].

```
— * —  
  
PROJECT=bookvol3  
TANGLE=/usr/local/bin/NOTANGLE  
WEAVE=/usr/local/bin/NOWEAVE  
LATEX=/usr/bin/latex  
MAKEINDEX=/usr/bin/makeindex  
  
all:  
${WEAVE} -t8 -delay ${PROJECT}.pamphlet >${PROJECT}.tex  
${LATEX} ${PROJECT}.tex 2>/dev/null 1>/dev/null  
${MAKEINDEX} ${PROJECT}.idx  
${LATEX} ${PROJECT}.tex 2>/dev/null 1>/dev/null
```

—————

Bibliography

- [1] Jenks, R.J. and Sutor, R.S. “Axiom – The Scientific Computation System” Springer-Verlag New York (1992) ISBN 0-387-97855-0
- [2] Knuth, Donald E., “Literate Programming” Center for the Study of Language and Information ISBN 0-937073-81-4 Stanford CA (1992)
- [3] Daly, Timothy, “The Axiom Wiki Website”
<http://axiom.axiom-developer.org>
- [4] Watt, Stephen, “Aldor”,
<http://www.aldor.org>
- [5] Lamport, Leslie, “Latex – A Document Preparation System”, Addison-Wesley, New York ISBN 0-201-52983-1
- [6] Ramsey, Norman “Noweb – A Simple, Extensible Tool for Literate Programming”
<http://www.eecs.harvard.edu/~nr/noweb>
- [7] Daly, Timothy, “The Axiom Literate Documentation”
<http://axiom.axiom-developer.org/axiom-website/documentation.html>