The 30 Year Horizon

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Volume 8.1: Axiom Gallery

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Liu Xiaojun
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Bruno Zuercher
Dan Zwillinger

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New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

Tim Daly
CAISS, City College of New York
November 10, 2003 ((iHy))

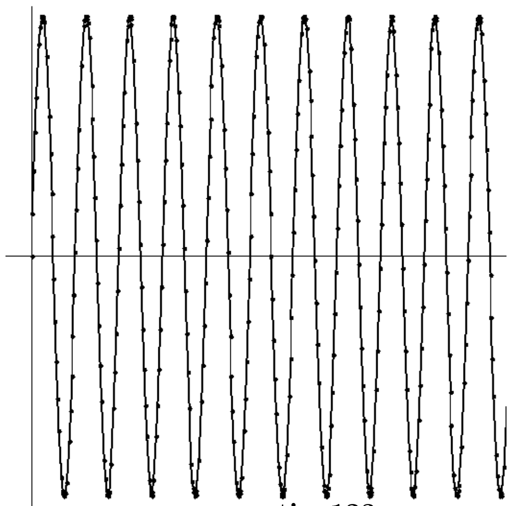
Chapter 1

General examples

These examples come from code that ships with Axiom in various input files.

1.1 Two dimensional functions

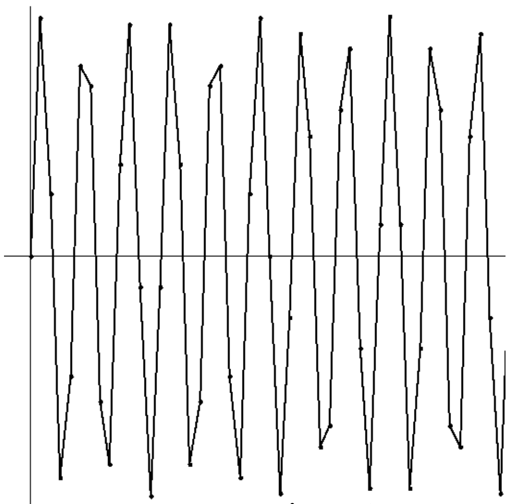
A Simple Sine Function



— **equation123** —

```
draw(sin(11*x),x = 0..2*%pi)
```

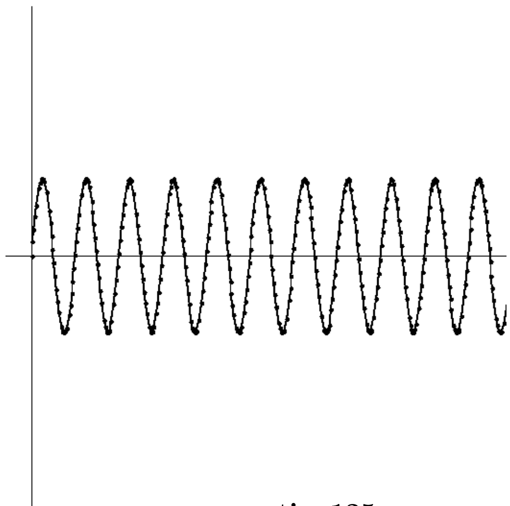
A Simple Sine Function, Non-adaptive plot



— **equation124** —

```
draw(sin(11*x),x = 0..2*%pi,adaptive == false,title == "Non-adaptive plot")
```

A Simple Sine Function, Drawn to Scale

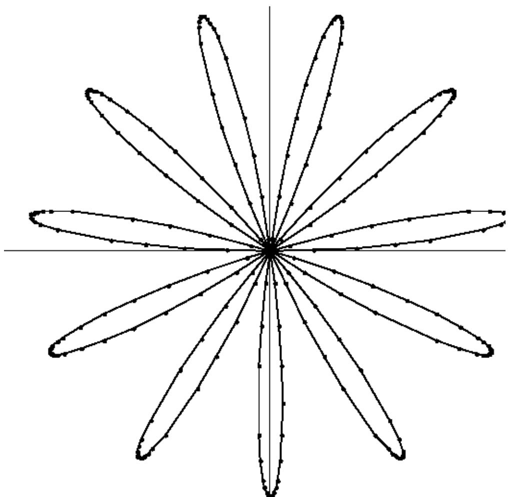


— **equation125** —

```
draw(sin(11*x),x = 0..2*%pi,toScale == true,title == "Drawn to scale")
```

————

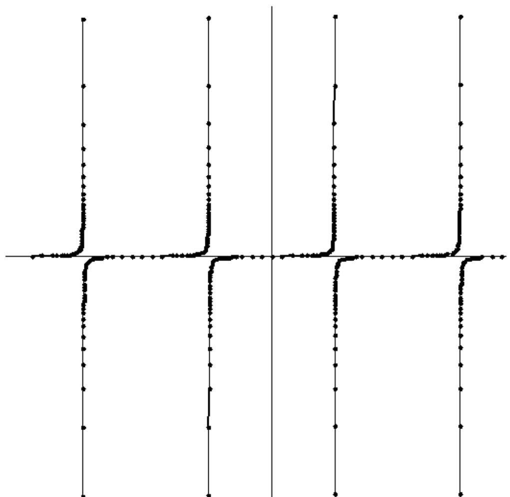
A Simple Sine Function, Polar Plot



— **equation126** —

```
draw(sin(11*x),x = 0..2*%pi,coordinates == polar,title == "Polar plot")
```

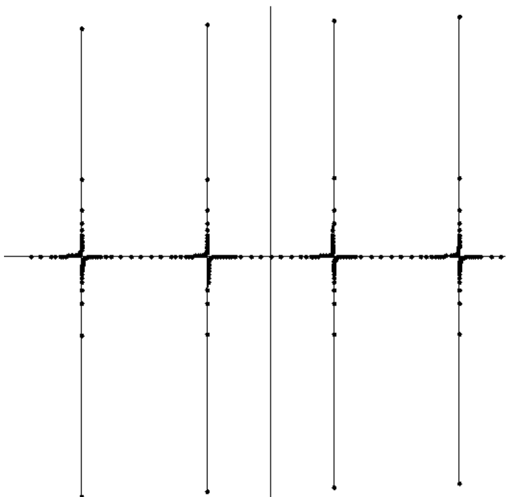
A Simple Tangent Function, Clipping On



— **equation127** —

```
draw(tan x,x = -6..6,title == "Clipping on")
```

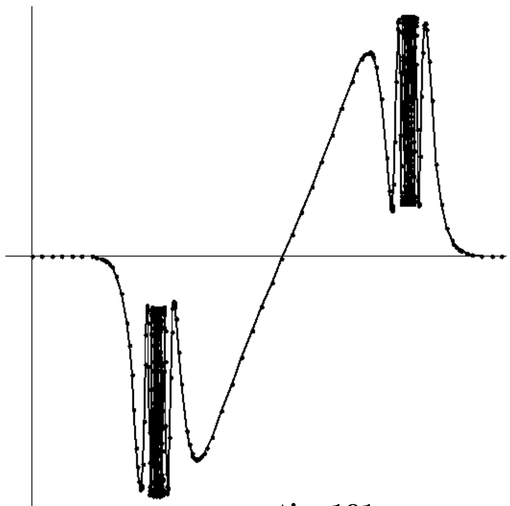
A Simple Tangent Function, Clipping On



— **equation128** —

```
draw(tan x,x = -6..6,clip == false,title == "Clipping off")
```

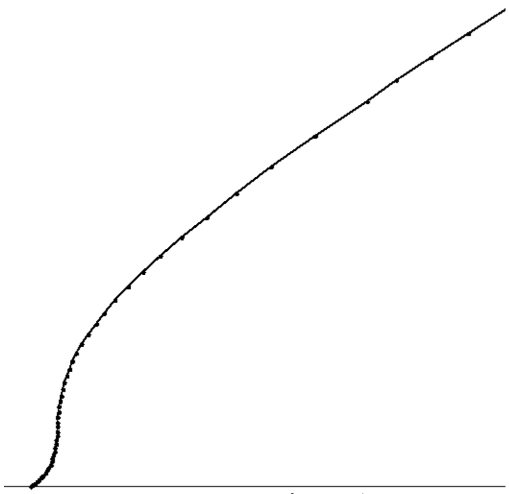
Tangent and Sine



— **equation101** —

```
f(x:DFLOAT):DFLOAT == sin(tan(x))-tan(sin(x))
draw(f,0..6)
```

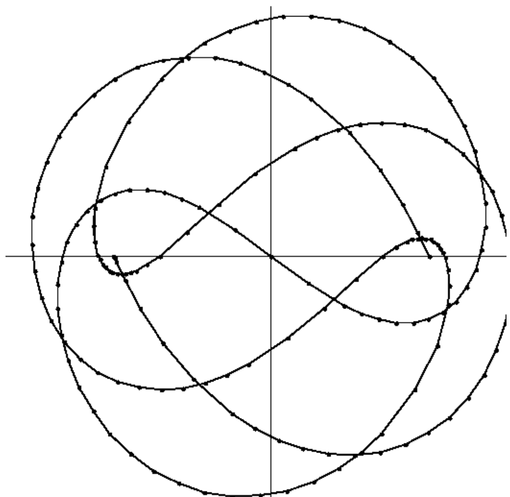
A 2D Sine Function in BiPolar Coordinates



— **equation107** —

```
draw(sin(x),x=0.5..%pi,coordinates == bipolar(1$DFLOAT))
```

A 2D Sine Function in Elliptic Coordinates

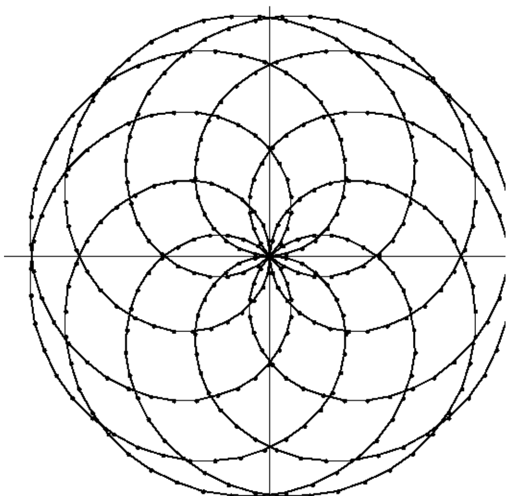


— **equation110** —

```
draw(sin(4*t/7),t=0..14*%pi,coordinates == elliptic(1$DFLOAT))
```

——————

A 2D Sine Wave in Polar Coordinates

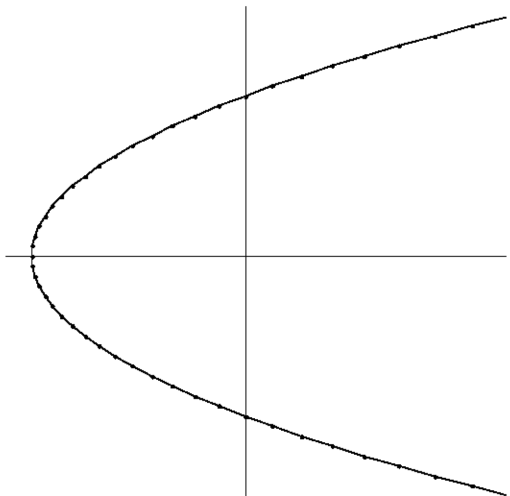


— equation118 —

```
draw(sin(4*t/7),t=0..14*%pi,coordinates == polar)
```

1.2 Two dimensional curves

A Line in Parabolic Coordinates

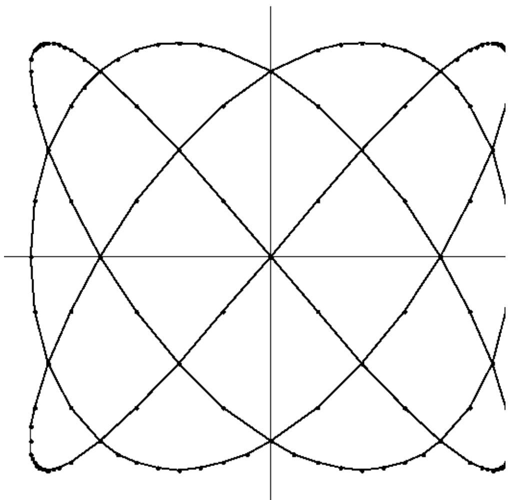


— **equation114** —

```
h1(t:DFLOAT):DFLOAT == t
h2(t:DFLOAT):DFLOAT == 2
draw(curve(h1,h2),-3..3,coordinates == parabolic)
```

————

Lissajous Curve

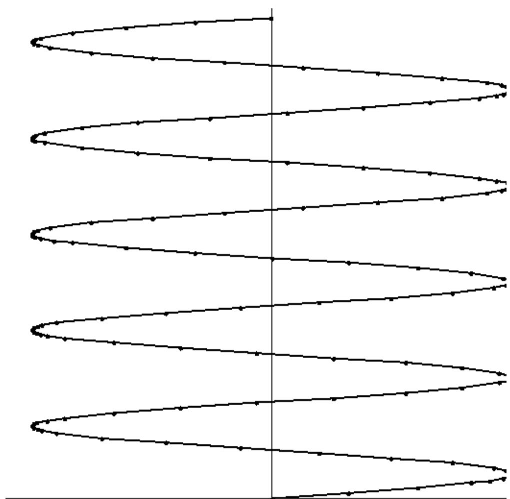


— **equation102** —

```
i1(t:DFLOAT):DFLOAT == 9*sin(3*t/4)
i2(t:DFLOAT):DFLOAT == 8*sin(t)
draw(curve(i1,i2),-4*%pi..4*%pi,toScale == true, title == "Lissajous Curve")
```

————-

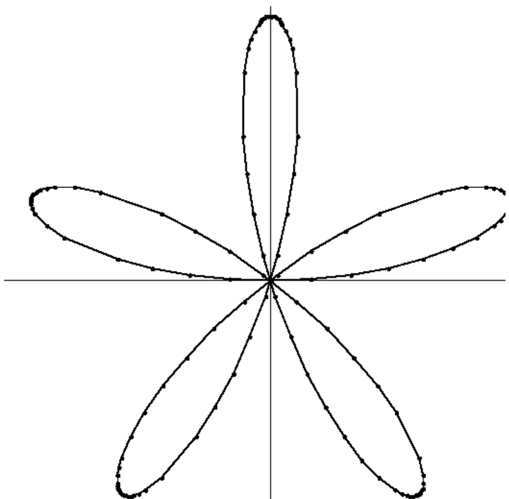
A Parametric Curve



— **equation129** —

```
draw(curve(sin(5*t),t),t = 0..2*%pi,title == "Parametric curve")
```

A Parametric Curve in Polar Coordinates

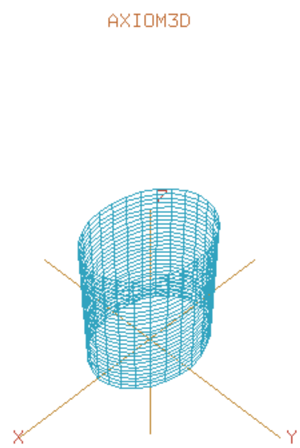


— **equation130** —

```
draw(curve(sin(5*t),t),t = 0..2*%pi,_
          coordinates == polar,title == "Parametric polar curve")
```

1.3 Three dimensional functions

A 3D Constant Function in Elliptic Coordinates

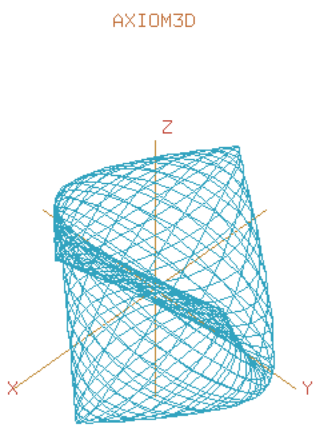


— **equation111** —

```
m(u:DFLOAT,v:DFLOAT):DFLOAT == 1
draw(m,0..2*%pi,0..%pi,coordinates == elliptic(1$DFLOAT))
```

———

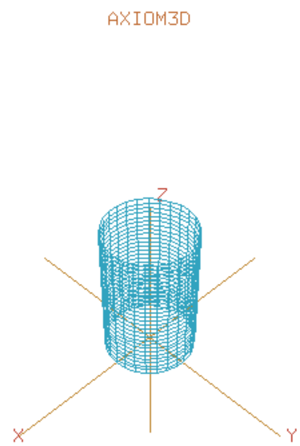
A 3D Constant Function in Oblate Spheroidal



— **equation113** —

```
m(u:DFLOAT,v:DFLOAT):DFLOAT == 1
draw(m,-%pi/2..%pi/2,0..2*%pi,coordinates == oblateSpheroidal(1$DFLOAT))
```

A 3D Constant in Polar Coordinates

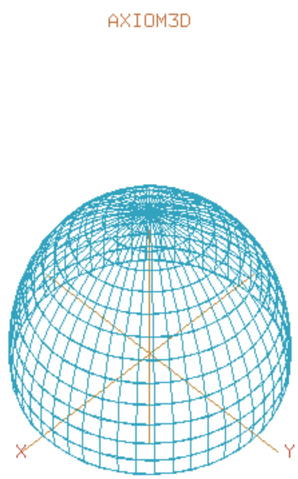


— **equation119** —

```
m(u:DFLOAT,v:DFLOAT):DFLOAT == 1
draw(m,0..2*%pi, 0..%pi,coordinates == polar)
```

———

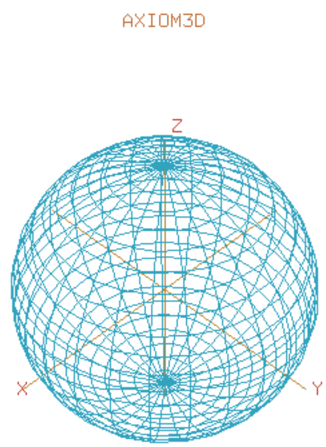
A 3D Constant in Prolate Spheroidal Coordinates



— **equation120** —

```
m(u:DFLOAT,v:DFLOAT):DFLOAT == 1
draw(m,-%pi/2..%pi/2,0..2*%pi,coordinates == prolateSpheroidal(1$DFLOAT))
```

A 3D Constant in Spherical Coordinates

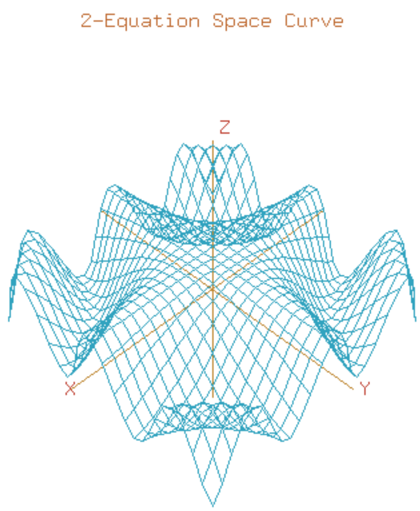


— **equation121** —

```
m(u:DFLOAT,v:DFLOAT):DFLOAT == 1
draw(m,0..2*%pi,0..%pi,coordinates == spherical)
```

———

A 2-Equation Space Function



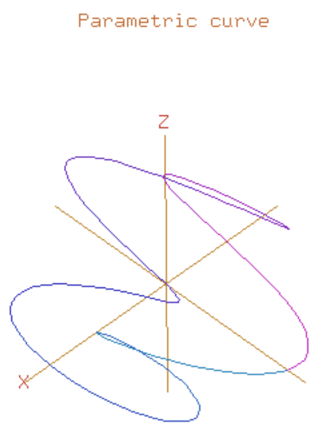
— **equation104** —

```
f(x:DFLOAT,y:DFLOAT):DFLOAT == cos(x*y)
colorFxn(x:DFLOAT,y:DFLOAT):DFLOAT == 1/(x**2 + y**2 + 1)
draw(f,-3..3,-3..3, colorFunction == colorFxn,title=="2-Equation Space Curve")
```

————-

1.4 Three dimensional curves

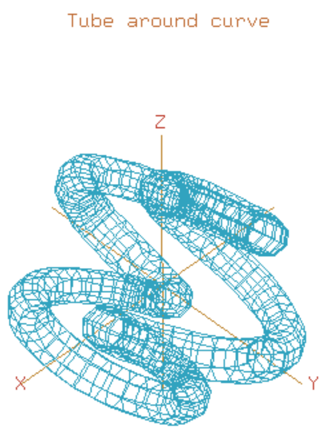
A Parametric Space Curve



— **equation131** —

```
draw(curve(sin(t)*cos(3*t/5),cos(t)*cos(3*t/5),cos(t)*sin(3*t/5)),_
          t = 0..15*%pi,title == "Parametric curve")
```

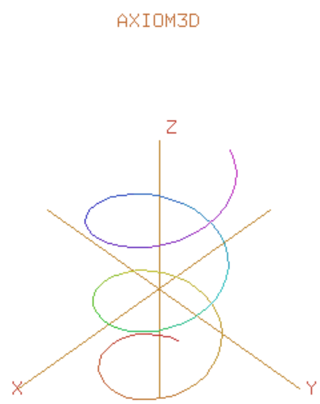
A Tube around a Parametric Space Curve



— **equation132** —

```
draw(curve(sin(t)*cos(3*t/5),cos(t)*cos(3*t/5),cos(t)*sin(3*t/5)),_
           t = 0..15*%pi,tubeRadius == .15,title == "Tube around curve")
```

A 2-Equation Cylindrical Curve

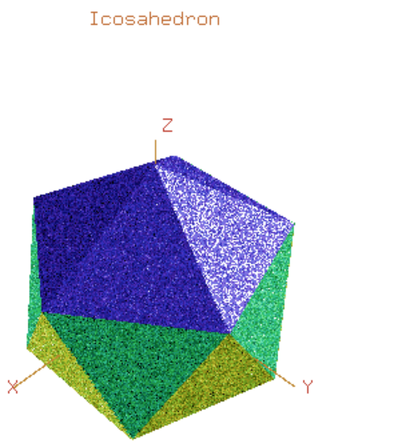


— **equation109** —

```
j1(t:DFLOAT):DFLOAT == 4
j2(t:DFLOAT):DFLOAT == t
draw(curve(j1,j2,j2),-9..9,coordinates == cylindrical)
```

1.5 Three dimensional surfaces

A Icosahedron

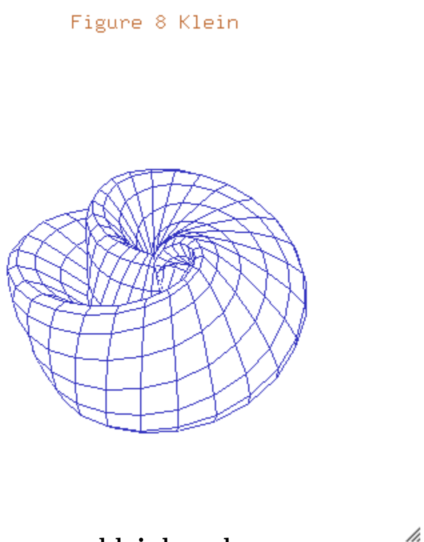


— **Icosahedron** —

```
)se exp add con InnerTrigonometricManipulations
exp(%i*2*%pi/5)
FG2F %
% -1
complexForm %
norm %
simplify %
s:=sqrt %
ph:=exp(%i*2*%pi/5)
A1:=complex(1,0)
A2:=A1*ph
A3:=A2*ph
A4:=A3*ph
A5:=A4*ph
ca1:=map(numeric , complexForm FG2F simplify A1)
ca2:=map(numeric , complexForm FG2F simplify A2)
ca3:=map(numeric ,complexForm FG2F simplify A3)
ca4:=map(numeric ,complexForm FG2F simplify A4)
ca5:=map(numeric ,complexForm FG2F simplify A5)
B1:=A1*exp(2*%i*%pi/10)
B2:=B1*ph
B3:=B2*ph
B4:=B3*ph
B5:=B4*ph
cb1:=map (numeric ,complexForm FG2F simplify B1)
cb2:=map (numeric ,complexForm FG2F simplify B2)
cb3:=map (numeric ,complexForm FG2F simplify B3)
```

```
cb4:=map (numeric ,complexForm FG2F simplify B4)
cb5:=map (numeric ,complexForm FG2F simplify B5)
u:=numeric sqrt(s*s-1)
p0:=point([0,0,u+1/2])@Point(SF)
p1:=point([real ca1,imag ca1,0.5])@Point(SF)
p2:=point([real ca2,imag ca2,0.5])@Point(SF)
p3:=point([real ca3,imag ca3,0.5])@Point(SF)
p4:=point([real ca4,imag ca4,0.5])@Point(SF)
p5:=point([real ca5,imag ca5,0.5])@Point(SF)
p6:=point([real cb1,imag cb1,-0.5])@Point(SF)
p7:=point([real cb2,imag cb2,-0.5])@Point(SF)
p8:=point([real cb3,imag cb3,-0.5])@Point(SF)
p9:=point([real cb4,imag cb4,-0.5])@Point(SF)
p10:=point([real cb5,imag cb5,-0.5])@Point(SF)
p11:=point([0,0,-u-1/2])@Point(SF)
space:=create3Space()$ThreeSpace DFLOAT
polygon(space,[p0,p1,p2])
polygon(space,[p0,p2,p3])
polygon(space,[p0,p3,p4])
polygon(space,[p0,p4,p5])
polygon(space,[p0,p5,p1])
polygon(space,[p1,p6,p2])
polygon(space,[p2,p7,p3])
polygon(space,[p3,p8,p4])
polygon(space,[p4,p9,p5])
polygon(space,[p5,p10,p1])
polygon(space,[p2,p6,p7])
polygon(space,[p3,p7,p8])
polygon(space,[p4,p8,p9])
polygon(space,[p5,p9,p10])
polygon(space,[p1,p10,p6])
polygon(space,[p6,p11,p7])
polygon(space,[p7,p11,p8])
polygon(space,[p8,p11,p9])
polygon(space,[p9,p11,p10])
polygon(space,[p10,p11,p6])
makeViewport3D(space,title=="Icosahedron",style=="smooth")
```

A 3D figure 8 immersion (Klein bagel)

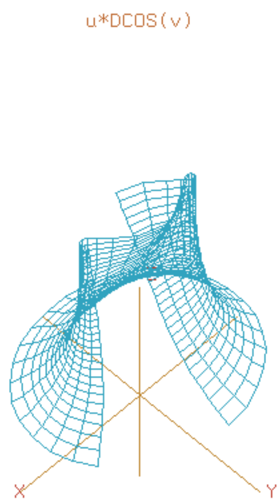


— **kleinbagel** —

```
r := 1
X(u,v) == (r+cos(u/2)*sin(v)-sin(u/2)*sin(2*v))*cos(u)
Y(u,v) == (r+cos(u/2)*sin(v)-sin(u/2)*sin(2*v))*sin(u)
Z(u,v) == sin(u/2)*sin(v)+cos(u/2)*sin(2*v)
v3d:=draw(surface(X(u,v),Y(u,v),Z(u,v)),u=0..2*%pi,v=0..2*%pi,_
          style=="solid",title=="Figure 8 Klein")
colorDef(v3d,blue(),blue())
axes(v3d,"off")
```

From `en.wikipedia.org/wiki/Klein_bottle`. The "figure 8" immersion (Klein bagel) of the Klein bottle has a particularly simple parameterization. It is that of a "figure 8" torus with a 180 degree "Möbius" twist inserted. In this immersion, the self-intersection circle is a geometric circle in the x-y plane. The positive constant r is the radius of this circle. The parameter u gives the angle in the x-y plane, and v specifies the position around the 8-shaped cross section. With the above parameterization the cross section is a 2:1 Lissajous curve.

A 2-Equation bipolarCylindrical Surface

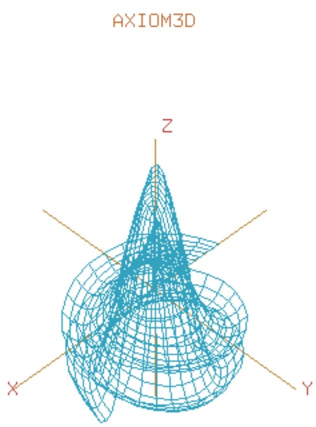


— **equation108** —

```
draw(surface(u*cos(v),u*sin(v),u),u=1..4,v=1..2*%pi,_
     coordinates == bipolarCylindrical(1$DFLOAT))
```

———-

A 3-Equation Parametric Space Surface

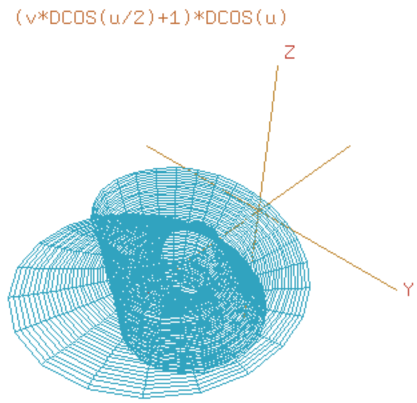


— **equation105** —

```
n1(u:DFLOAT,v:DFLOAT):DFLOAT == u*cos(v)
n2(u:DFLOAT,v:DFLOAT):DFLOAT == u*sin(v)
n3(u:DFLOAT,v:DFLOAT):DFLOAT == v*cos(u)
colorFxn(x:DFLOAT,y:DFLOAT):DFLOAT == 1/(x**2 + y**2 + 1)
draw(surface(n1,n2,n3),-4..4,0..2*%pi, colorFunction == colorFxn)
```

———-

A 3D Vector of Points in Elliptic Cylindrical

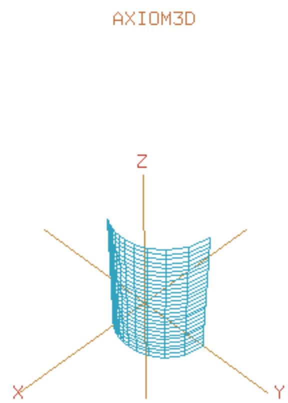


— **equation112** —

```
U2:Vector Expression Integer := vector [0,0,1]
x(u,v) == beta(u) + v*delta(u)
beta u == vector [cos u, sin u, 0]
delta u == (cos(u/2)) * beta(u) + sin(u/2) * U2
vec := x(u,v)
draw(surface(vec.1,vec.2,vec.3),v=-0.5..0.5,u=0..2*%pi,_
     coordinates == ellipticCylindrical(1$DFLOAT),_
     var1Steps == 50,var2Steps == 50)
```

———-

A 3D Constant Function in BiPolar Coordinates

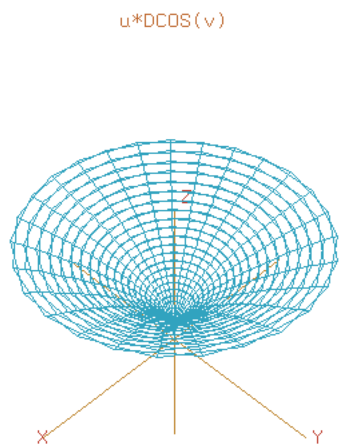


— **equation106** —

```
m(u:DFLOAT,v:DFLOAT):DFLOAT == 1
draw(m,0..2*%pi, 0..%pi,coordinates == bipolar(1$DFLOAT))
```

———-

A Swept in Parabolic Coordinates

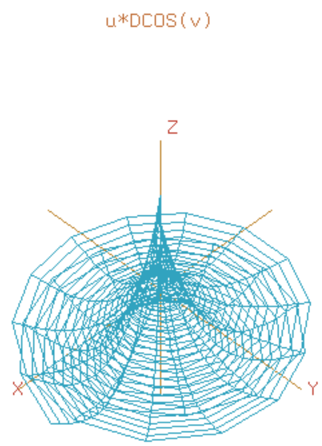


— **equation115** —

```
draw(surface(u*cos(v),u*sin(v),2*u),u=0..4,v=0..2*%pi,coordinates==parabolic)
```

————

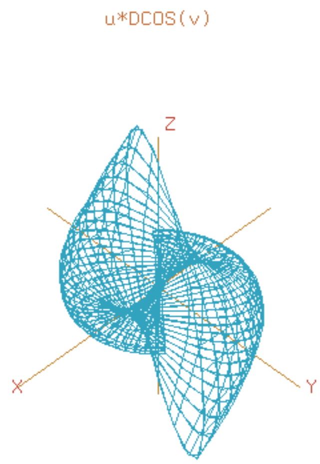
A Swept Cone in Parabolic Cylindrical Coordinates



— **equation116** —

```
draw(surface(u*cos(v),u*sin(v),v*cos(u)),u=0..4,v=0..2*%pi,_
     coordinates == parabolicCylindrical)
```

A Truncated Cone in Toroidal Coordinates

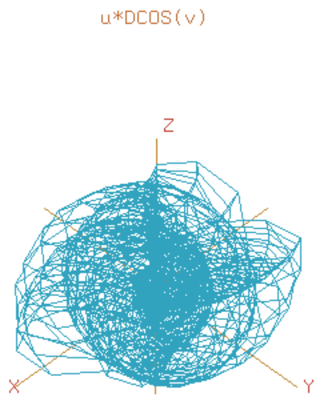


— **equation122** —

```
draw(surface(u*cos(v),u*sin(v),u),u=1..4,v=1..4*%pi,_
     coordinates == toroidal(1$DFLOAT))
```

————

A Swept Surface in Paraboloidal Coordinates



— **equation117** —

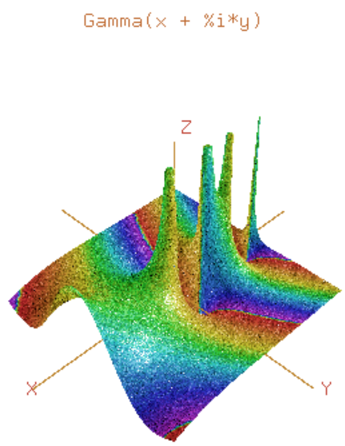
```
draw(surface(u*cos(v),u*sin(v),u*v),u=0..4,v=0..2*%pi,_
     coordinates==paraboloidal,var1Steps == 50, var2Steps == 50)
```

———

Chapter 2

Jenks Book images

The Complex Gamma Function

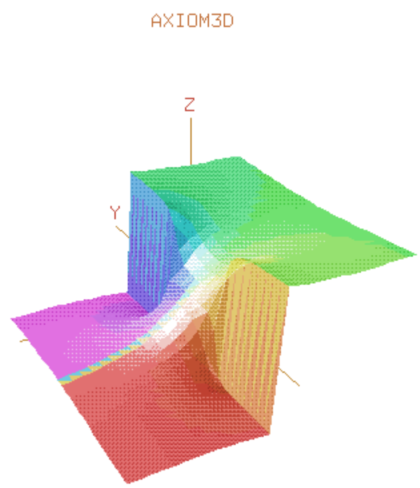


— **complexgamma** —

```
gam(x:DoubleFloat,y:DoubleFloat):Point(DoubleFloat) == _
  ( g:Complex(DoubleFloat):= Gamma complex(x,y) ; _
    point [x,y,max(min(real g, 4), -4), argument g] )
v3d:=draw(gam, -%pi..%pi, -%pi..%pi, title == "Gamma(x + %i*y)", _
          var1Steps == 100, var2Steps == 100, style=="smooth")
```

A 3-d surface whose height is the real part of the Gamma function, and whose color is the argument of the Gamma function.

The Complex Arctangent Function



— **complexarctangent** —

```
atf(x:DoubleFloat,y:DoubleFloat):Point(DoubleFloat) == _
 ( a := atan complex(x,y) ; _
   point [x,y,real a, argument a] )
v3d:=draw(atf, -3.0..%pi, -3.0..%pi, style=="shade")
rotate(v3d,210,-60)
```

The complex arctangent function. The height is the real part and the color is the argument.

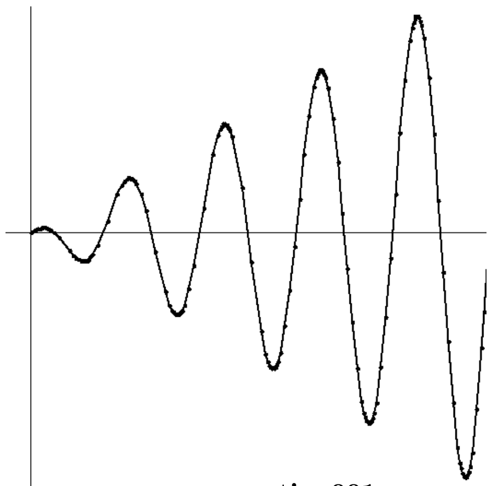
Chapter 3

Hyperdoc examples

Examples in this section come from the Hyperdoc documentation tool. These examples are accessed from the Basic Examples Draw section.

3.1 Two dimensional examples

A function of one variable



— equation001 —

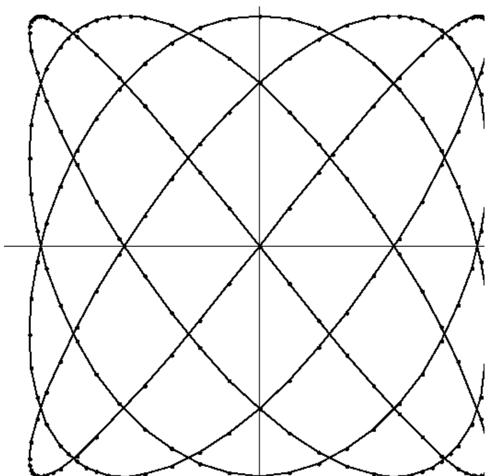
```
draw(x*cos(x),x=0..30,title=="y = x*cos(x)")
```

This is one of the demonstration equations used in hypertex. It demonstrates a function of one variable. It draws

$$y = f(x)$$

where y is the dependent variable and x is the independent variable.

A Parametric function



— **equation002** —

```
draw(curve(-9*sin(4*t/5),8*sin(t)),t=-5*%pi..5*%pi,title=="Lissajous")
```

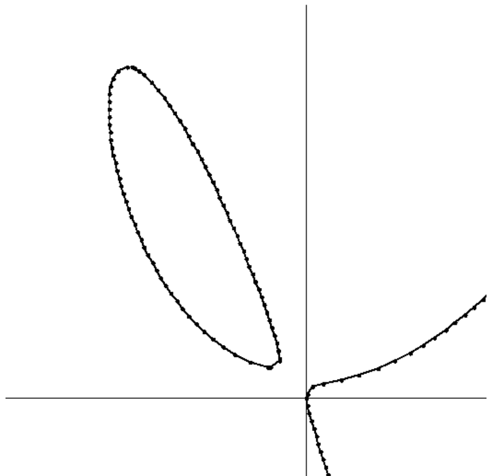
————-

This is one of the demonstration equations used in hypertex. It draw a parametrically defined curve

$$f1(t), f2(t)$$

in terms of two functions $f1$ and $f2$ and an independent variable t.

A Polynomial in 2 variables



— equation003 —

```
draw(y**2+7*x*y-(x**3+16*x) = 0,x,y,range==[-15..10,-10..50])
```

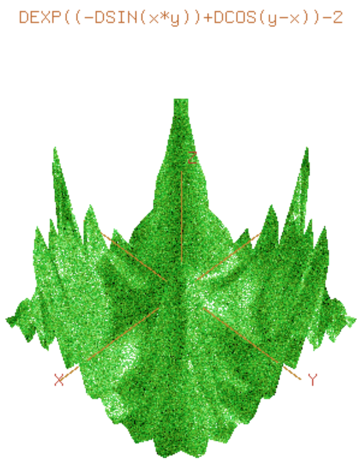
This is one of the demonstration equations used in hypertex. Plotting the solution to

$$p(x,y) = 0$$

where p is a polynomial in two variables x and y.

3.2 Three dimensional examples

A function of two variables



— equation004 —

```
cf(x,y) == 0.5
draw(exp(cos(x-y)-sin(x*y))-2,x=-5..5,y=-5..5,_
     colorFunction==cf,style=="smooth")
```

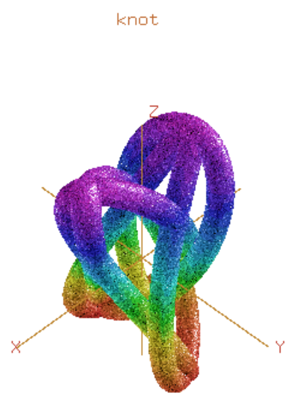
————-

This is one of the demonstration equations used in hypertex. A function of two variables

$$z = f(x, y)$$

where z is the dependent variable and where x and y are the dependent variables.

A parametrically defined curve



— equation005 —

```
f1(t) == 1.3*cos(2*t)*cos(4*t)+sin(4*t)*cos(t)
f2(t) == 1.3*sin(2*t)*cos(4*t)-sin(4*t)*sin(t)
f3(t) == 2.5*cos(4*t)
cf(x,y) == 0.5
draw(curve(f1(t),f2(t),f3(t)),t=0..4*%pi,tubeRadius==.25,tubePoints==16,_
     title=="knot",colorFunction==cf,style=="smooth")
```

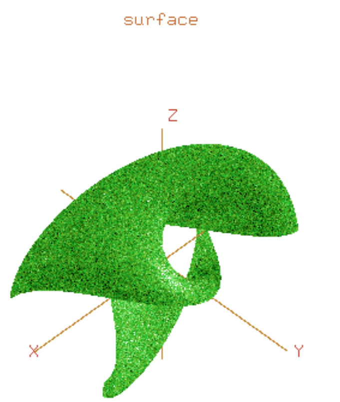
———-

This is one of the demonstration equations used in hypertex. This ia parametrically defined curve

$$f1(t), f2(t), f3(t)$$

in terms of three functions $f1$, $f2$, and $f3$ and an independent variable t.

A parametrically defined surface



— equation006 —

```
f1(u,v) == u*sin(v)
f2(u,v) == v*cos(u)
f3(u,v) == u*cos(v)
cf(x,y) == 0.5
draw(surface(f1(u,v),f2(u,v),f3(u,v)),u=-%pi..%pi,v=-%pi/2..%pi/2,_
     title=="surface",colorFunction==cf,style=="smooth")
```

———-

This is one of the demonstration equations used in hypertex. This ia parametrically defined curve

$$f1(t), f2(t), f3(t)$$

in terms of three functions $f1$, $f2$, and $f3$ and an independent variable t.

Chapter 4

CRC Standard Curves and Surfaces [7]

4.1 Standard Curves and Surfaces

In order to have an organized and thorough evaluation of the Axiom graphics code we turn to the CRC Standard Curves and Surfaces (SCC). This volume was written years after the Axiom graphics code was written so there was no attempt to match the two until now. However, the SCC volume will give us a solid foundation to both evaluate the features of the current code and suggest future directions.

According to the SCC we can organize the various curves by the taxonomy:

1 **random**

  1.1 **fractal**

  1.2 **gaussian**

  1.3 **non-gaussian**

2 **determinate**

  2.1 **algebraic** – A polynomial is defined as a summation of terms composed of integral powers of x and y. An algebraic curve is one whose implicit function

$$f(x, y) = 0$$

  is a polynomial in x and y (after rationalization, if necessary). Because a curve is often defined in the explicit form

$$y = f(x)$$

  there is a need to distinguish rational and irrational functions of x.

2.1.1 **irrational** – An irrational function of x is a quotient of two polynomials, one or both of which has a term (or terms) with power p/q, where p and q are integers.

2.1.2 **rational** – A rational function of x is a quotient of two polynomials in x, both having only integer powers.

2.1.2.1 **polynomial**

2.1.2.2 **non-polynomial**

2.2 **integral** – Certain continuous functions are not expressible in algebraic or transcendental forms but are familiar mathematical tools. These curves are equal to the integrals of algebraic or transcendental curves by definition; examples include Bessel functions, Airy integrals, Fresnel integrals, and the error function.

2.3 **transcendental** – The transcendental curves cannot be expressed as polynomials in x and y. These are curves containing one or more of the following forms: exponential (e^x), logarithmic ($\log(x)$), or trigonometric ($\sin(x)$, $\cos(x)$).

2.3.1 **exponential**

2.3.2 **logarithmic**

2.3.3 **trigonometric**

2.4 **piecewise continuous** – Other curves, except at a few singular points, are smooth and differentiable. The class of nondifferentiable curves have discontinuity of the first derivative as a basic attribute. They are often composed of straight-line segments. Simple polygonal forms, regular fractal curves, and turtle tracks are examples.

2.4.1 **periodic**

2.4.2 **non-periodic**

2.4.3 **polygonal**

2.4.3.1 **regular**

2.4.3.2 **irregular**

2.4.3.3 **fractal**

4.2 CRC graphs

Functions with $x^{n/m}$

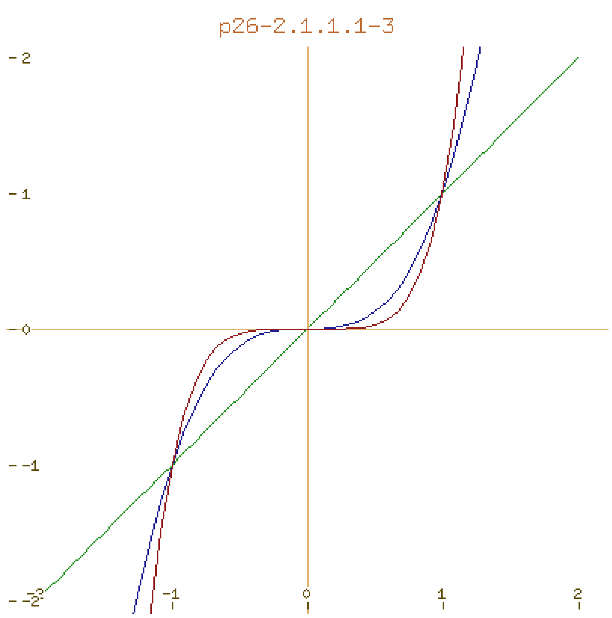
Page 26 2.1.1

$$y = cx^n$$

$$y - cx^n = 0$$

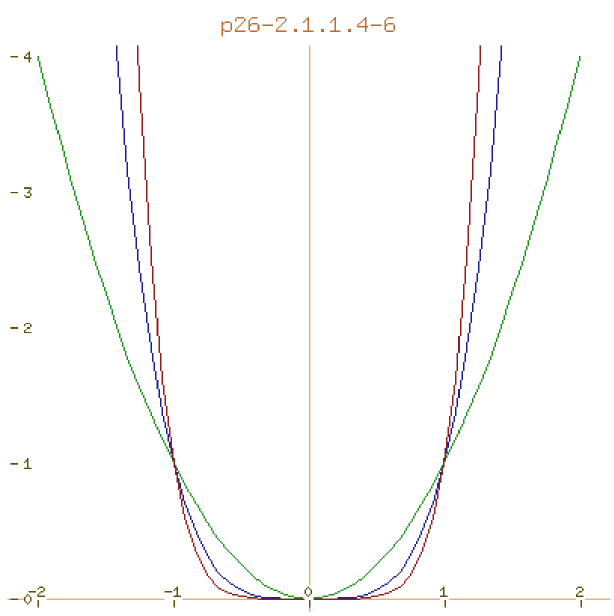
— **p26-2.1.1.1-3** —

```
)clear all
)set mes auto off
f(c,x,n) == c*x^n
lineColorDefault(green())
viewport1:=draw(f(1,x,1), x=-2..2, adaptive==true, unit==[1.0,1.0],_
                title=="p26-2.1.1.1-3")
graph2111:=getGraph(viewport1,1)

lineColorDefault(blue())
viewport2:=draw(f(1,x,3), x=-2..2, adaptive==true, unit==[1.0,1.0])
graph2112:=getGraph(viewport2,1)

lineColorDefault(red())
viewport3:=draw(f(1,x,5), x=-2..2, adaptive==true, unit==[1.0,1.0])
graph2113:=getGraph(viewport3,1)

putGraph(viewport1,graph2112,2)
putGraph(viewport1,graph2113,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



— **p26-2.1.1.4-6** —

```
)clear all
)set mes auto off
f(c,x,n) == c*x^n
lineColorDefault(green())
viewport1:=draw(f(1,x,2), x=-2..2, adaptive==true, unit==[1.0,1.0],_
                title=="p26-2.1.1.4-6")
graph2111:=getGraph(viewport1,1)

lineColorDefault(blue())
viewport2:=draw(f(1,x,4), x=-2..2, adaptive==true, unit==[1.0,1.0])
graph2112:=getGraph(viewport2,1)

lineColorDefault(red())
viewport3:=draw(f(1,x,6), x=-2..2, adaptive==true, unit==[1.0,1.0])
graph2113:=getGraph(viewport3,1)

putGraph(viewport1,graph2112,2)
putGraph(viewport1,graph2113,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")

makeViewport2D(viewport1)
```



Page 26 2.1.2

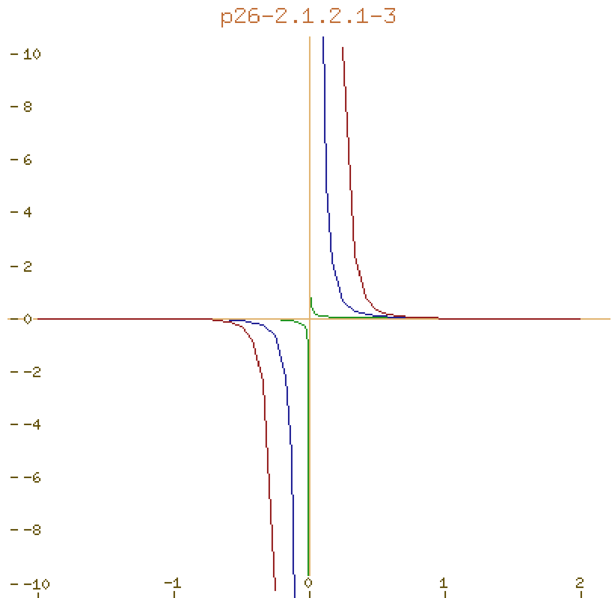
$$y = \frac{c}{x^n}$$

$$yx^n - c = 0$$

— p26-2.1.2.1-3 —

```
)clear all
f(c,x,n) == c/x^n
lineColorDefault(green())
viewport1:=draw(f(0.01,x,1),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p26-2.1.2.1-3")
graph2111:=getGraph(viewport1,1)
lineColorDefault(blue())
viewport2:=draw(f(0.01,x,3),x=-4..4,adaptive==true,unit==[1.0,1.0])
graph2122:=getGraph(viewport2,1)
lineColorDefault(red())
viewport3:=draw(f(0.01,x,5),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2123:=getGraph(viewport3,1)
putGraph(viewport1,graph2122,2)
putGraph(viewport1,graph2123,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

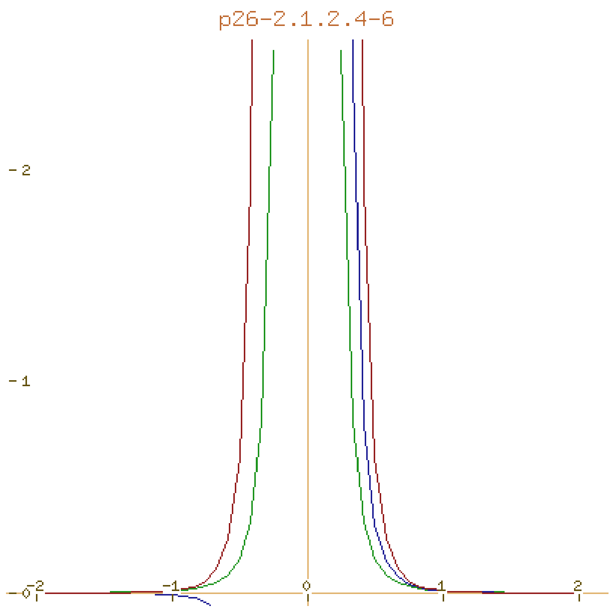
———-



Note that Axiom's plot of viewport2 disagrees with CRC

— **p26-2.1.2.4-6** —

```
)clear all
f(c,x,n) == c/x^n
lineColorDefault(green())
viewport1:=draw(f(0.01,x,4),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p26-2.1.2.4-6")
graph2124:=getGraph(viewport1,1)
lineColorDefault(blue())
viewport2:=draw(f(0.01,x,5),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2125:=getGraph(viewport2,1)
lineColorDefault(red())
viewport3:=draw(f(0.01,x,6),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2126:=getGraph(viewport3,1)
putGraph(viewport1,graph2125,2)
putGraph(viewport1,graph2126,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 28 2.1.3

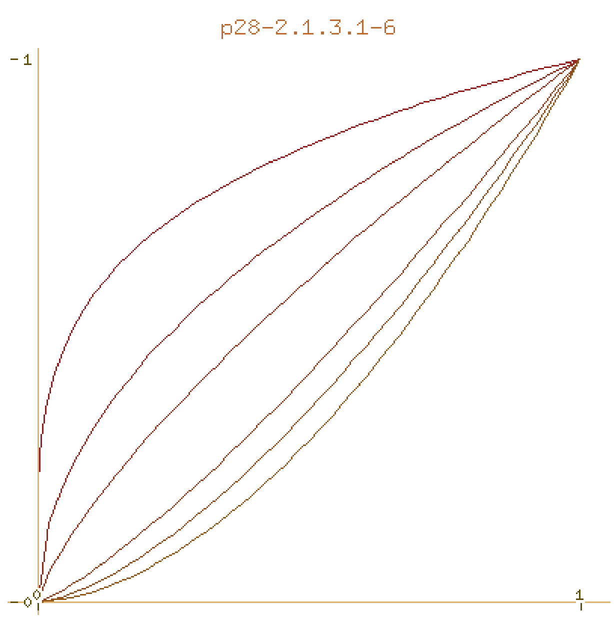
$$y = cx^{n/m}$$

$$y - cx^{n/m} = 0$$

— p28-2.1.3.1-6 —

```
)clear all
f(c,x,n,m) == c*x^(n/m)

lineColorDefault(color(1))
viewport1:=draw(f(1,x,1,4),x=0..1,adaptive==true,unit==[1.0,1.0],_
               title=="p28-2.1.3.1-6")
graph2131:=getGraph(viewport1,1)

lineColorDefault(color(2))
viewport2:=draw(f(1,x,1,2),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2132:=getGraph(viewport2,1)

lineColorDefault(color(3))
viewport3:=draw(f(1,x,3,4),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2133:=getGraph(viewport3,1)

lineColorDefault(color(4))
viewport4:=draw(f(1,x,5,4),x=0..1,adaptive==true,unit==[1.0,1.0])
```

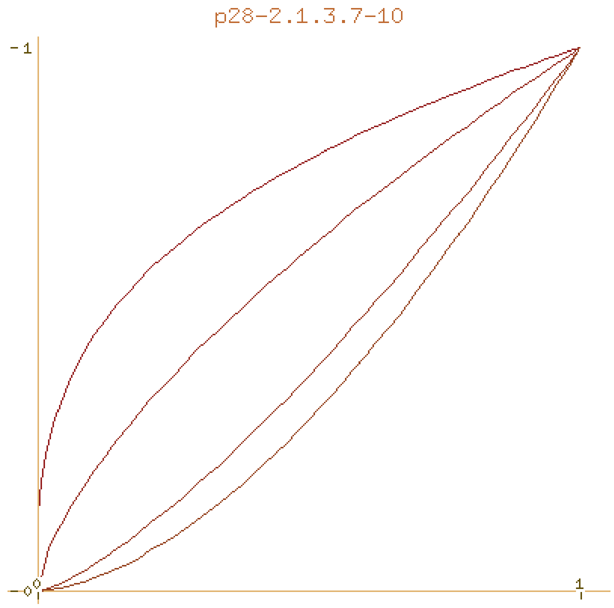
```
graph2134:=getGraph(viewport4,1)

lineColorDefault(color(5))
viewport5:=draw(f(1,x,3,2),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2135:=getGraph(viewport5,1)

lineColorDefault(color(6))
viewport6:=draw(f(1,x,7,4),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2136:=getGraph(viewport6,1)

putGraph(viewport1,graph2132,2)
putGraph(viewport1,graph2133,3)
putGraph(viewport1,graph2134,4)
putGraph(viewport1,graph2135,5)
putGraph(viewport1,graph2136,6)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



— **p28-2.1.3.7-10** —

```
)clear all
```

```
f(c,x,n,m) == c*x^(n/m)

lineColorDefault(color(1))
viewport1:=draw(f(1,x,1,3),x=0..1,adaptive==true,unit==[1.0,1.0],_
                title=="p28-2.1.3.7-10")
graph2137:=getGraph(viewport1,1)

lineColorDefault(color(2))
viewport2:=draw(f(1,x,2,3),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2138:=getGraph(viewport2,1)

lineColorDefault(color(3))
viewport3:=draw(f(1,x,4,3),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2139:=getGraph(viewport3,1)

lineColorDefault(color(4))
viewport4:=draw(f(1,x,5,3),x=0..1,adaptive==true,unit==[1.0,1.0])
graph21310:=getGraph(viewport4,1)

putGraph(viewport1,graph2138,2)
putGraph(viewport1,graph2139,3)
putGraph(viewport1,graph21310,4)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 28 2.1.4

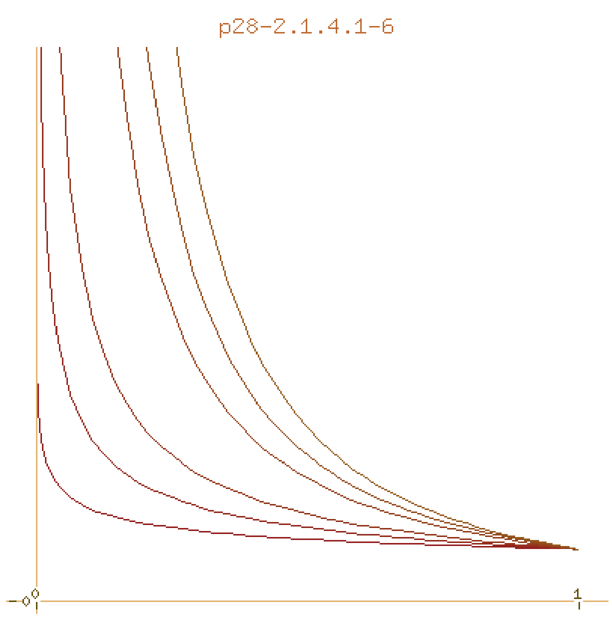
$$y = \frac{c}{x^{n/m}}$$

$$yx^{n/m} - c = 0$$

— **p28-2.1.4.1-6** —

```
)clear all
f(c,x,n,m) == c/x^(n/m)

lineColorDefault(color(1))
viewport1:=draw(f(0.01,x,1,4),x=0..1,adaptive==true,unit==[1.0,1.0],_
                title=="p28-2.1.4.1-6")
graph2141:=getGraph(viewport1,1)

lineColorDefault(color(2))
viewport2:=draw(f(0.01,x,1,2),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2142:=getGraph(viewport2,1)

lineColorDefault(color(3))
viewport3:=draw(f(0.01,x,3,4),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2143:=getGraph(viewport3,1)

lineColorDefault(color(4))
```

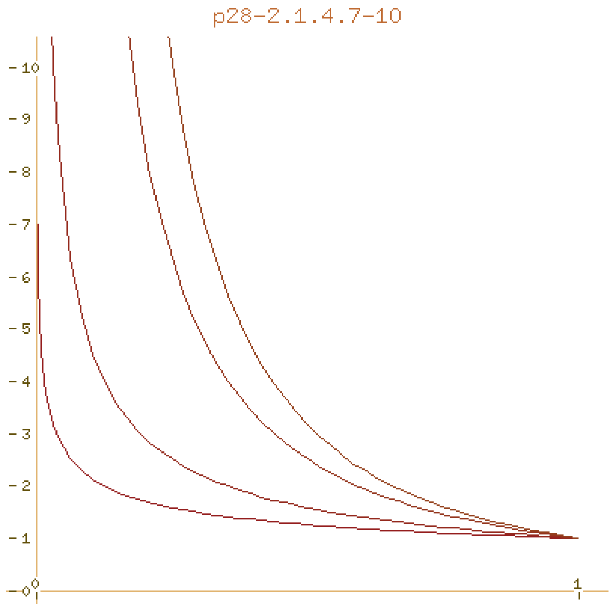
```
viewport4:=draw(f(0.01,x,5,4),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2144:=getGraph(viewport4,1)

lineColorDefault(color(5))
viewport5:=draw(f(0.01,x,3,2),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2145:=getGraph(viewport5,1)

lineColorDefault(color(6))
viewport6:=draw(f(0.01,x,7,4),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2146:=getGraph(viewport6,1)

putGraph(viewport1,graph2142,2)
putGraph(viewport1,graph2143,3)
putGraph(viewport1,graph2144,4)
putGraph(viewport1,graph2145,5)
putGraph(viewport1,graph2146,6)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



— p28-2.1.4.7-10 —

```
)clear all
```

```
f(c,x,n,m) == c/x^(n/m)

lineColorDefault(color(1))
viewport1:=draw(f(1,x,1,3),x=0..1,adaptive==true,unit==[1.0,1.0],_
                title=="p28-2.1.4.7-10")
graph2147:=getGraph(viewport1,1)

lineColorDefault(color(2))
viewport2:=draw(f(1,x,2,3),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2148:=getGraph(viewport2,1)

lineColorDefault(color(3))
viewport3:=draw(f(1,x,4,3),x=0..1,adaptive==true,unit==[1.0,1.0])
graph2149:=getGraph(viewport3,1)

lineColorDefault(color(4))
viewport4:=draw(f(1,x,5,3),x=0..1,adaptive==true,unit==[1.0,1.0])
graph21410:=getGraph(viewport4,1)

putGraph(viewport1,graph2148,2)
putGraph(viewport1,graph2149,3)
putGraph(viewport1,graph21410,4)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Functions with x^n and $(a + bx)^m$

Page 30 2.2.1

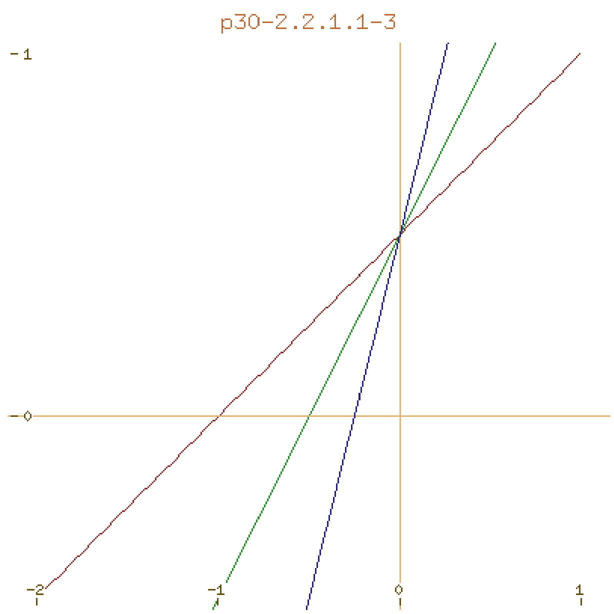
$$y = c(a + bx)$$

$$y - bcx - ac = 0$$

— p30-2.2.1.1-3 —

```
)clear all
f(x,a,b,c) == c*(a+b*x)

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0],_
                title=="p30-2.2.1.1-3")
graph2211:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2212:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2213:=getGraph(viewport3,1)
```

```
putGraph(viewport1,graph2212,2)
putGraph(viewport1,graph2213,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 30 2.2.2

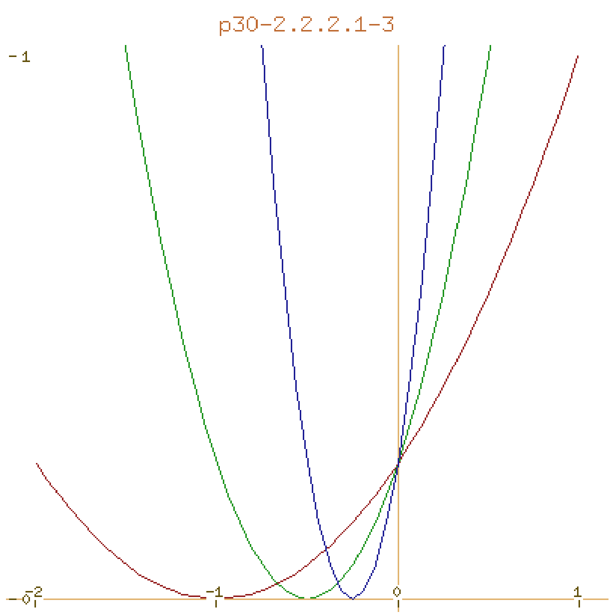
$$y = c(a + bx)^2$$

$$y - cb^2x^2 - 2abcx - a^2c = 0$$

— p30-2.2.2.1-3 —

```
)clear all
f(x,a,b,c) == c*(a+b*x)^2

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0],_
```

```
                title=="p30-2.2.2.1-3")
graph2221:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2222:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2223:=getGraph(viewport3,1)

putGraph(viewport1,graph2222,2)
putGraph(viewport1,graph2223,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 30 2.2.3

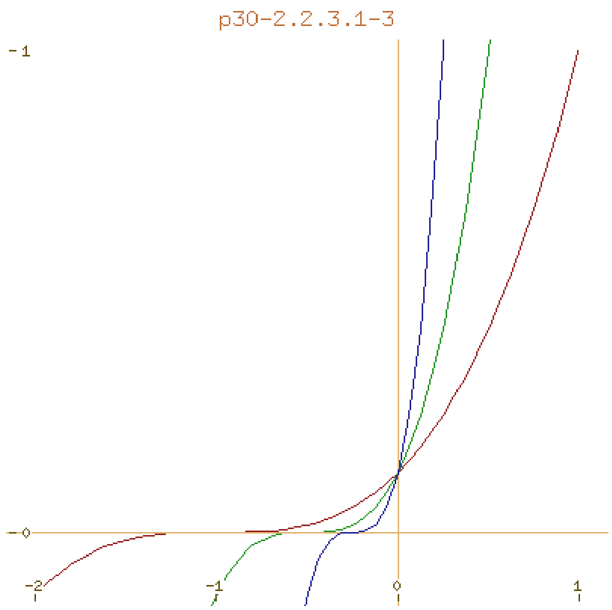
$$y = c(a + bx)^3$$

$$y - b^3cx^3 - 3ab^2cx^2 - 3a^2bcx - a^3c = 0$$

— **p30-2.2.3.1-3** —

```
)clear all
f(x,a,b,c) == c*(a+b*x)^3

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0],_
                title=="p30-2.2.3.1-3")
graph2231:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2232:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2233:=getGraph(viewport3,1)

putGraph(viewport1,graph2232,2)
putGraph(viewport1,graph2233,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 30 2.2.4

$$y = cx(a + bx)$$

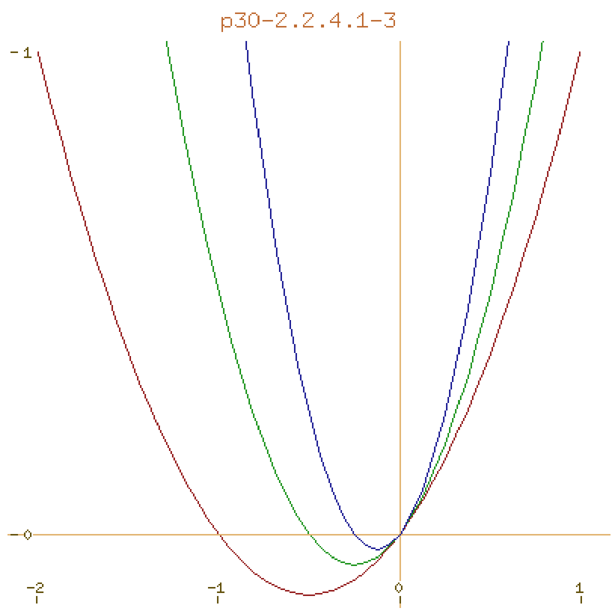
$$y - bcx^2 - acx = 0$$

— **p30-2.2.4.1-3** —

```
)clear all
f(x,a,b,c) == c*x*(a+b*x)

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0],_
                title=="p30-2.2.4.1-3")
graph2241:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2242:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2243:=getGraph(viewport3,1)

putGraph(viewport1,graph2242,2)
putGraph(viewport1,graph2243,3)
```

```
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 30 2.2.5

$$y = cx(a + bx)^2$$

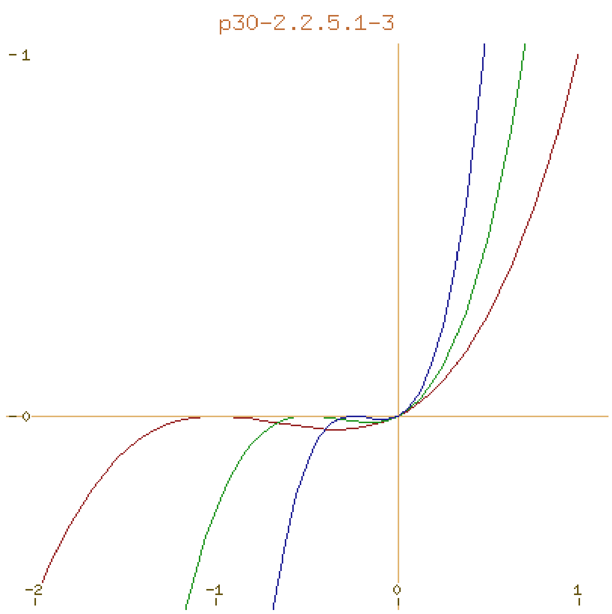
$$y - b^2cx^3 - 2abcx^2 - a^2cx = 0$$

— p30-2.2.5.1-3 —

```
)clear all
f(x,a,b,c) == c*x*(a+b*x)^2

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0],_
                title=="p30-2.2.5.1-3")
graph2251:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2252:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2253:=getGraph(viewport3,1)

putGraph(viewport1,graph2252,2)
putGraph(viewport1,graph2253,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

——————



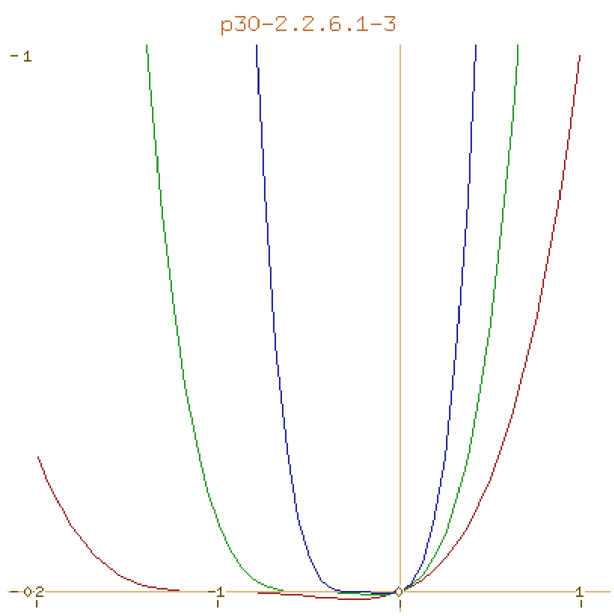
Page 30 2.2.6

$$y = cx(a + bx)^3$$

$$y - b^3cx^4 - 3ab^2cx^3 - 3a^2bcx^2 - a^3cx = 0$$

— **p30-2.2.6.1-3** —

```
)clear all
f(x,a,b,c) == c*x*(a+b*x)^3

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0],_
                title=="p30-2.2.6.1-3")
graph2261:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2262:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2263:=getGraph(viewport3,1)

putGraph(viewport1,graph2262,2)
putGraph(viewport1,graph2263,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 32 2.2.7

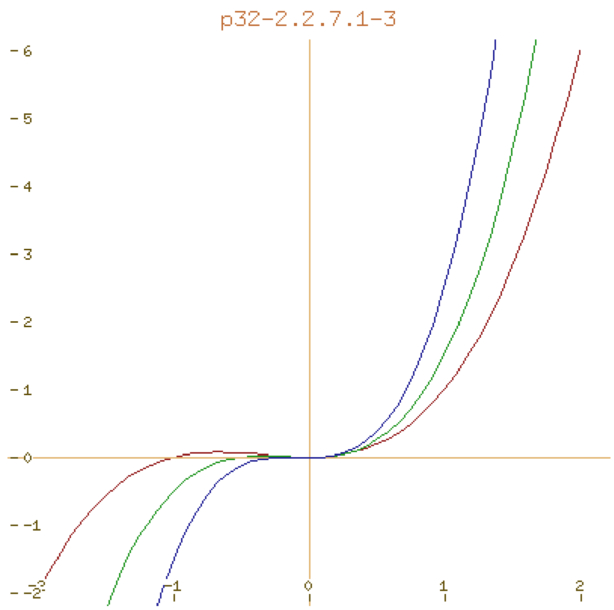
$$y = cx^2(a + bx)$$

$$y - bcx^3 - acx^2 = 0$$

— **p30-2.2.7.1-3** —

```
)clear all
f(x,a,b,c) == c*x^2*(a+b*x)

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0],_
                title=="p30-2.2.7.1-3")
graph2271:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2272:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2273:=getGraph(viewport3,1)

putGraph(viewport1,graph2272,2)
putGraph(viewport1,graph2273,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 32 2.2.8

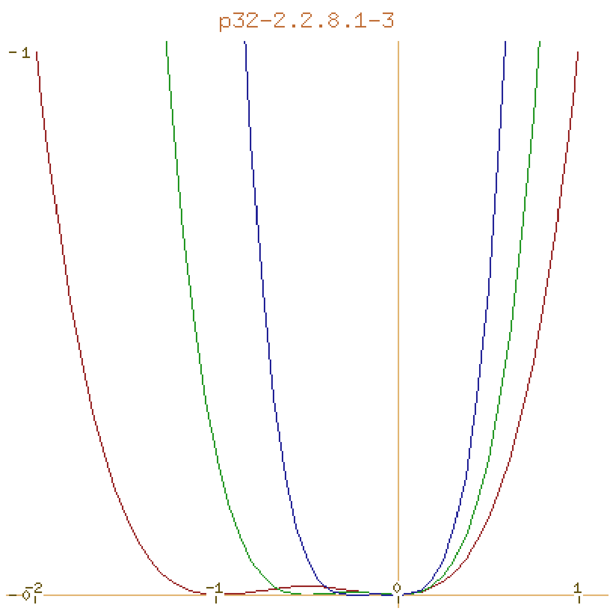
$$y = cx^2(a + bx)^2$$

$$y - b^2cx^4 - 2abcx^3 - a^2cx^2 = 0$$

— **p32-2.2.8.1-3** —

```
)clear all
f(x,a,b,c) == c*x^2*(a+b*x)^2

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0],_
                title=="p32-2.2.8.1-3")
graph2281:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2282:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2283:=getGraph(viewport3,1)

putGraph(viewport1,graph2282,2)
putGraph(viewport1,graph2283,3)
```

```
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 32 2.2.9

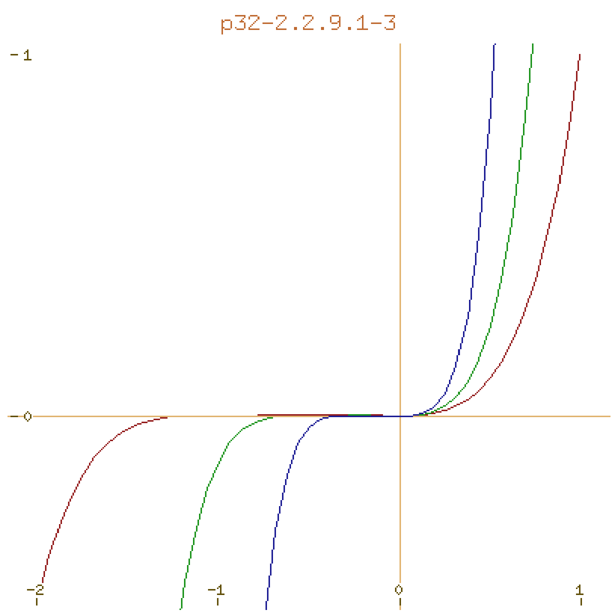
$$y = cx^2(a + bx)^3$$

$$y - b^3cx^5 - 3ab^2cx^4 - 3a^2bcx^3 - a^3cx^2 = 0$$

— p32-2.2.9.1-3 —

```
)clear all
f(x,a,b,c) == c*x^2*(a+b*x)^3

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0],_
                title=="p32-2.2.9.1-3")
graph2291:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2292:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..1,adaptive==true,unit==[1.0,1.0])
graph2293:=getGraph(viewport3,1)

putGraph(viewport1,graph2292,2)
putGraph(viewport1,graph2293,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



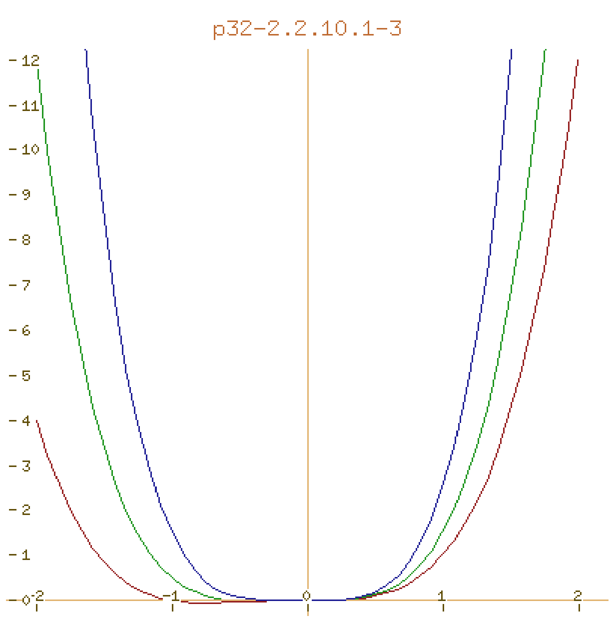
Page 32 2.2.10

$$y = cx^3(a + bx)$$

$$y - bcx^4 - acx^3 = 0$$

— **p32-2.2.10.1-3** —

```
)clear all
f(x,a,b,c) == c*x^3*(a+b*x)

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p32-2.2.10.1-3")
graph22101:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22102:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22103:=getGraph(viewport3,1)

putGraph(viewport1,graph22102,2)
putGraph(viewport1,graph22103,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 32 2.2.11

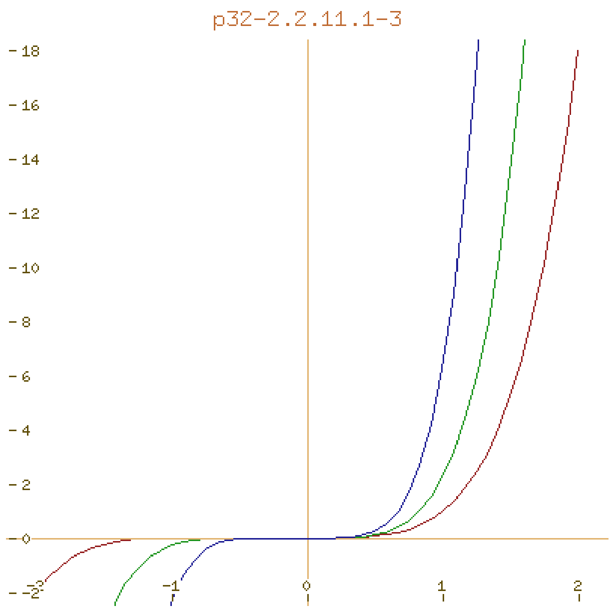
$$y = cx^3(a + bx)^2$$

$$y - b^2cx^5 - 2abcx^4 - a^2cx^3 = 0$$

— **p32-2.2.11.1-3** —

```
)clear all
f(x,a,b,c) == c*x^3*(a+b*x)^2

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p32-2.2.11.1-3")
graph22111:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22112:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22113:=getGraph(viewport3,1)

putGraph(viewport1,graph22112,2)
putGraph(viewport1,graph22113,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 32 2.2.12

$$y = cx^3(a+bx)^3$$

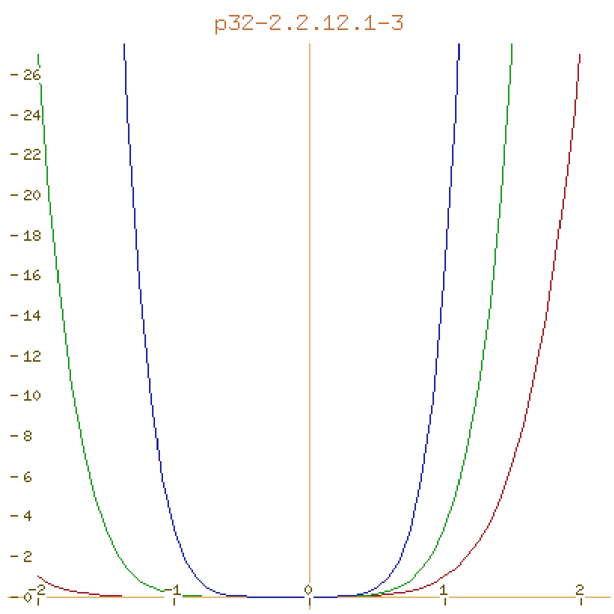
$$y - b^3cx^6 - 3ab^2cx^5 - 3a^2bcx^4 - a^3cx^3 = 0$$

— **p32-2.2.12.1-3** —

```
)clear all
f(x,a,b,c) == c*x^3*(a+b*x)^3

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p32-2.2.12.1-3")
graph22121:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22122:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,2.0,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22123:=getGraph(viewport3,1)

putGraph(viewport1,graph22122,2)
putGraph(viewport1,graph22123,3)
```

```
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 34 2.2.13

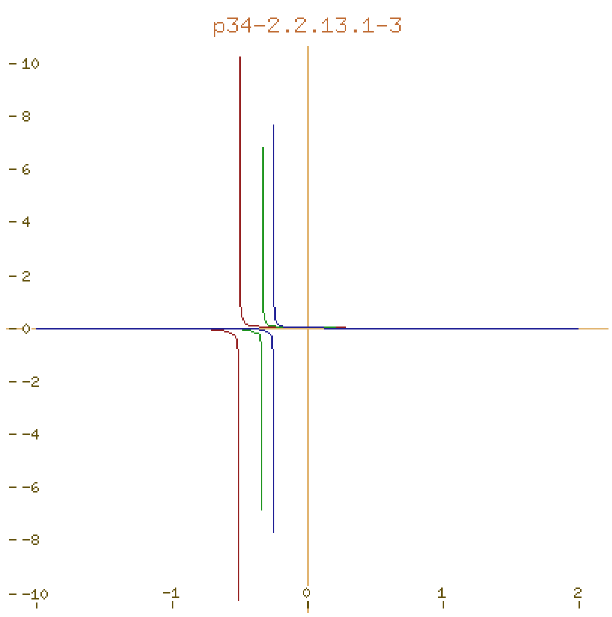
$$y = \frac{c}{(a + bx)}$$

$$ay + bxy - c = 0$$

— **p34-2.2.13.1-3** —

```
)clear all
f(x,a,b,c) == c/(a+b*x)

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p34-2.2.13.1-3")
graph22131:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22132:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22133:=getGraph(viewport3,1)

putGraph(viewport1,graph22132,2)
putGraph(viewport1,graph22133,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



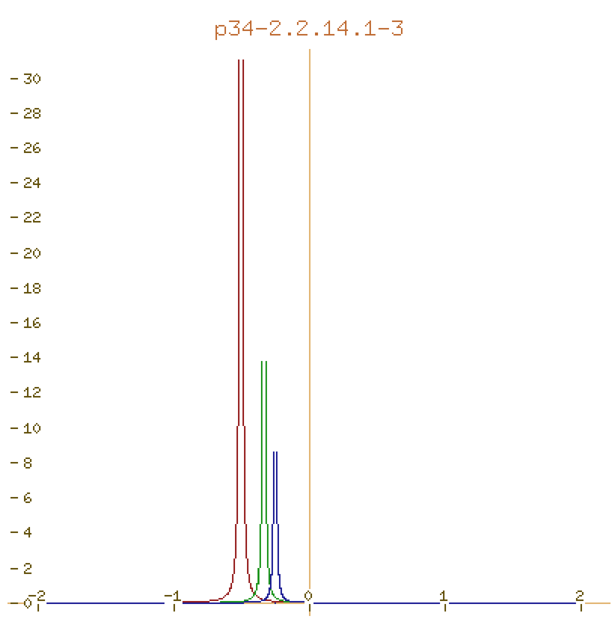
Page 34 2.2.14

$$y = \frac{c}{(a+bx)^2}$$

$$a^2y + 2abxy + b^2x^2y - c = 0$$

— p34-2.2.14.1-3 —

```
)clear all
f(x,a,b,c) == c/(a+b*x)^2

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p34-2.2.14.1-3")
graph22141:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22142:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22143:=getGraph(viewport3,1)

putGraph(viewport1,graph22142,2)
putGraph(viewport1,graph22143,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 34 2.2.15

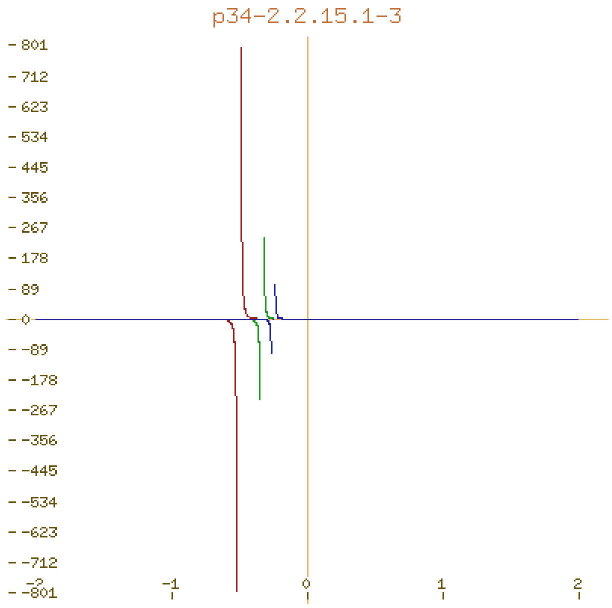
$$y = \frac{c}{(a+bx)^3}$$

$$a^3y + 2a^2bxy + 2ab^2x^2y + b^3x^3y - c = 0$$

— **p34-2.2.15.1-3** —

```
)clear all
f(x,a,b,c) == c/(a+b*x)^3

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p34-2.2.15.1-3")
graph22151:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22152:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22153:=getGraph(viewport3,1)

putGraph(viewport1,graph22152,2)
putGraph(viewport1,graph22153,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 34 2.2.16

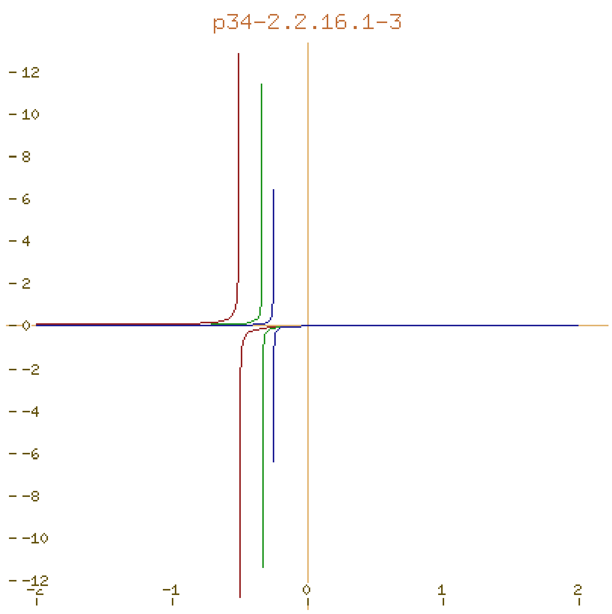
$$y = \frac{cx}{(a + bx)}$$

$$ay + bxy - cx = 0$$

— **p34-2.2.16.1-3** —

```
)clear all
f(x,a,b,c) == c*x/(a+b*x)

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p34-2.2.16.1-3")
graph22161:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22162:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22163:=getGraph(viewport3,1)

putGraph(viewport1,graph22162,2)
```

```
putGraph(viewport1,graph22163,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 34 2.2.17

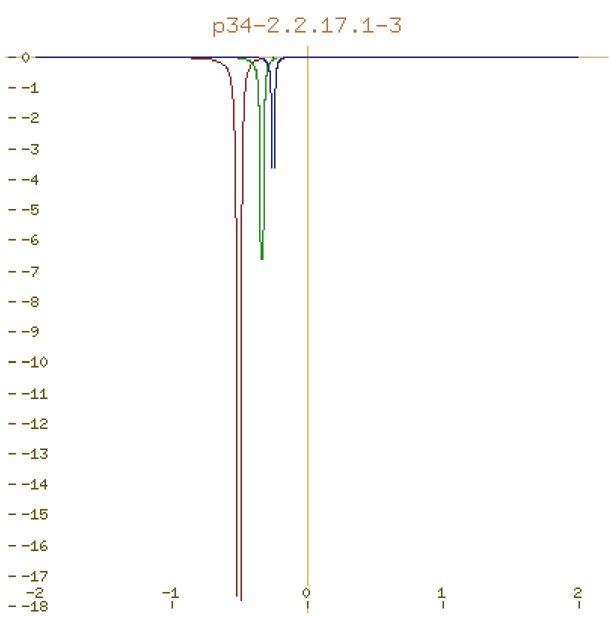
$$y = \frac{cx}{(a+bx)^2}$$

$$a^2y + 2abxy + b^2x^2y - cx = 0$$

— **p34-2.2.17.1-3** —

```
)clear all
f(x,a,b,c) == c*x/(a+b*x)^2

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p34-2.2.17.1-3")
graph22171:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22172:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22173:=getGraph(viewport3,1)

putGraph(viewport1,graph22172,2)
putGraph(viewport1,graph22173,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 34 2.2.18

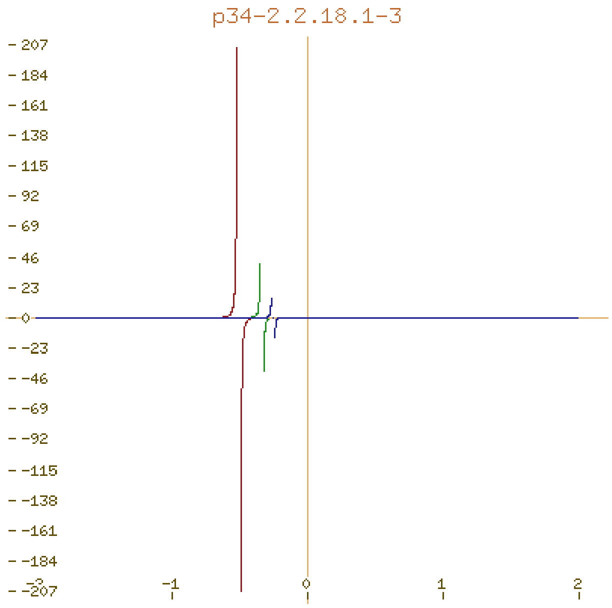
$$y = \frac{cx}{(a+bx)^3}$$

$$a^3y + 3a^2bxy + 3ab^2x^2y + b^3x^3y - cx = 0$$

— p34-2.2.18.1-3 —

```
)clear all
f(x,a,b,c) == c*x/(a+b*x)^3

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p34-2.2.18.1-3")
graph22181:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22182:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22183:=getGraph(viewport3,1)

putGraph(viewport1,graph22182,2)
putGraph(viewport1,graph22183,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 36 2.2.19

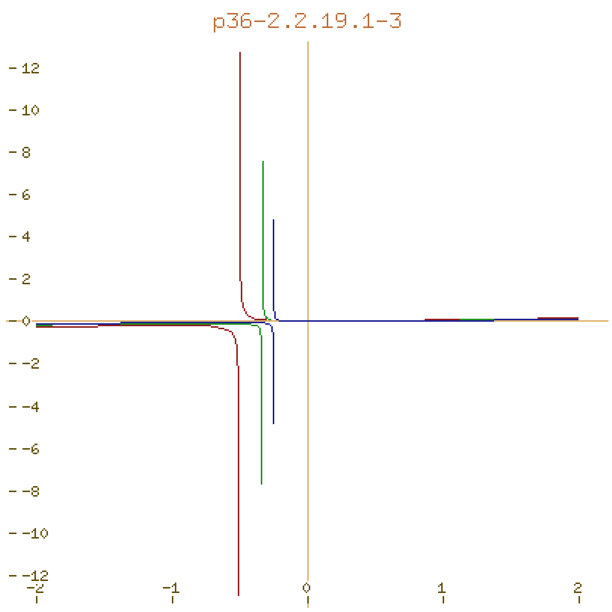
$$y = \frac{cx^2}{(a + bx)}$$

$$ay + bxy - cx^2 = 0$$

— p36-2.2.19.1-3 —

```
)clear all
f(x,a,b,c) == c*x^2/(a+b*x)

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p36-2.2.19.1-3")
graph22191:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22192:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22193:=getGraph(viewport3,1)

putGraph(viewport1,graph22192,2)
```

```
putGraph(viewport1,graph22193,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 36 2.2.20

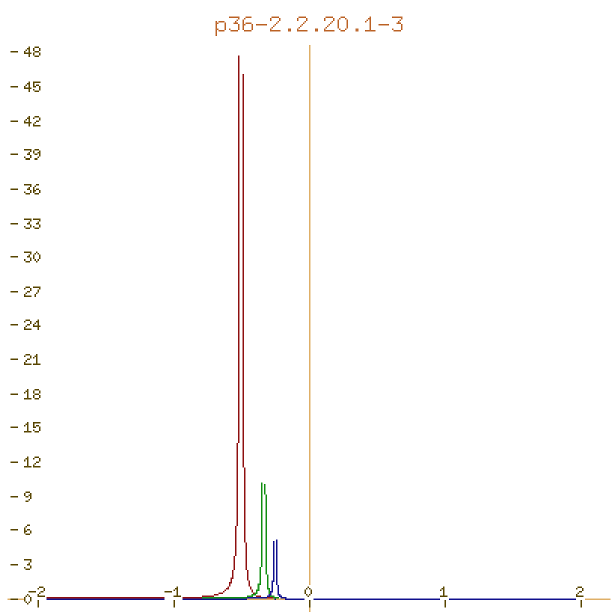
$$y = \frac{cx^2}{(a+bx)^2}$$

$$a^2y + 2abxy + b^2x^2y - cx^2 = 0$$

— **p36-2.2.20.1-3** —

```
)clear all
f(x,a,b,c) == c*x^2/(a+b*x)^2

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0],_
               title=="p36-2.2.20.1-3")
```

```
graph22201:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22202:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22203:=getGraph(viewport3,1)

putGraph(viewport1,graph22202,2)
putGraph(viewport1,graph22203,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 36 2.2.21

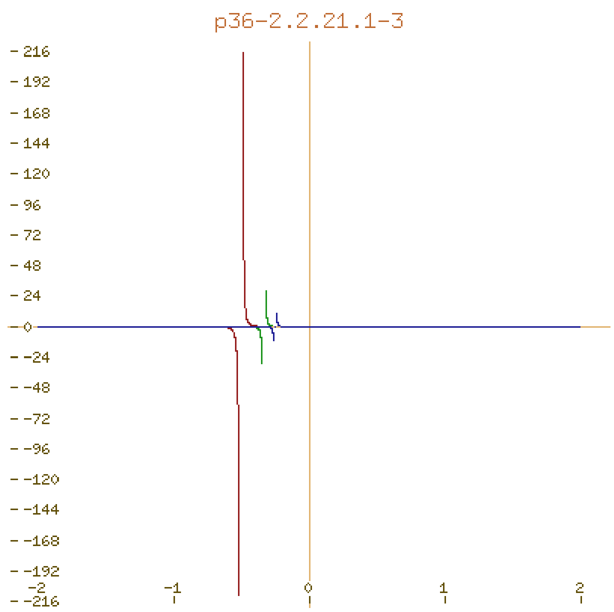
$$y = \frac{cx^2}{(a+bx)^3}$$

$$a^3y + 3a^2bxy + 3ab^2x^2y + b^3x^3y - cx^2 = 0$$

— **p36-2.2.21.1-3** —

```
)clear all
f(x,a,b,c) == c*x^2/(a+b*x)^3

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p36-2.2.21.1-3")
graph22211:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22212:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22213:=getGraph(viewport3,1)

putGraph(viewport1,graph22212,2)
putGraph(viewport1,graph22213,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 36 2.2.22

$$y = \frac{cx^3}{(a + bx)}$$

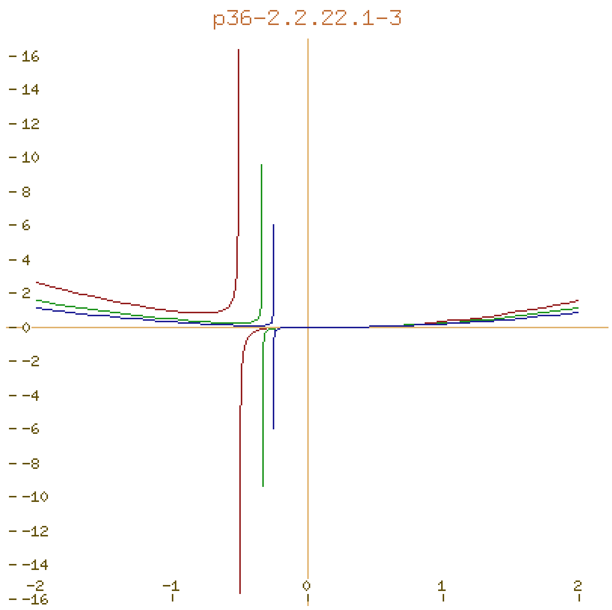
$$ay + bxy - cx^3 = 0$$

— p36-2.2.22.1-3 —

```
)clear all
f(x,a,b,c) == c*x^3/(a+b*x)

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p36-2.2.22.1-3")
graph22221:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22222:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22223:=getGraph(viewport3,1)

putGraph(viewport1,graph22222,2)
```

```
putGraph(viewport1,graph22223,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

——–



Page 36 2.2.23

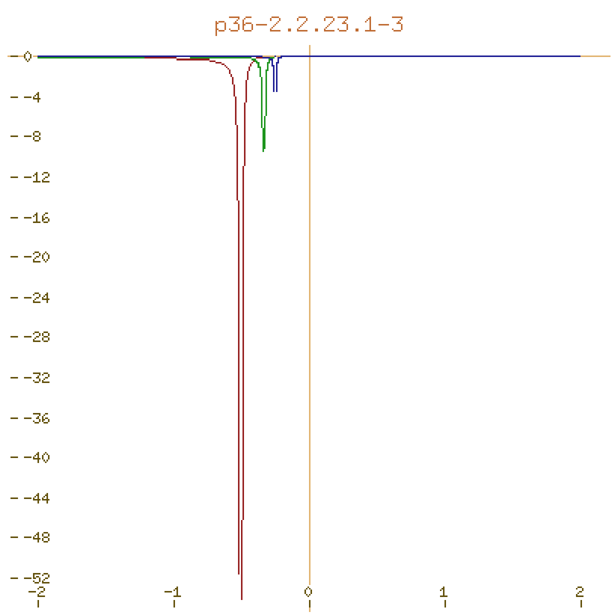
$$y = \frac{cx^3}{(a+bx)^2}$$

$$a^2y + 2abxy + b^2x^2y - cx^3 = 0$$

— **p36-2.2.23.1-3** —

```
)clear all
f(x,a,b,c) == c*x^3/(a+b*x)^2

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p36-2.2.23.1-3")
```

```
graph22231:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22232:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22233:=getGraph(viewport3,1)

putGraph(viewport1,graph22232,2)
putGraph(viewport1,graph22233,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 36 2.2.24

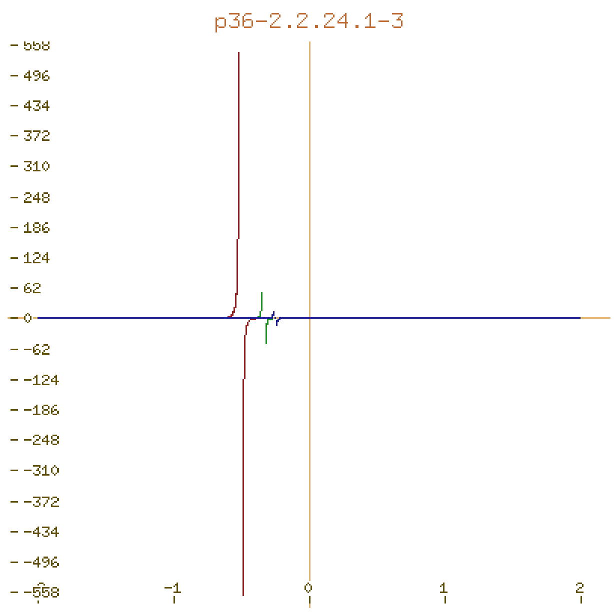
$$y = \frac{cx^3}{(a+bx)^3}$$

$$a^3y + 3a^2bxy + 3ab^2x^2y + b^3x^3y - cx^3 = 0$$

— p36-2.2.24.1-3 —

```
)clear all
f(x,a,b,c) == c*x^3/(a+b*x)^3

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p36-2.2.24.1-3")
graph22241:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,3.0,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22242:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,4.0,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22243:=getGraph(viewport3,1)

putGraph(viewport1,graph22242,2)
putGraph(viewport1,graph22243,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 38 2.2.25

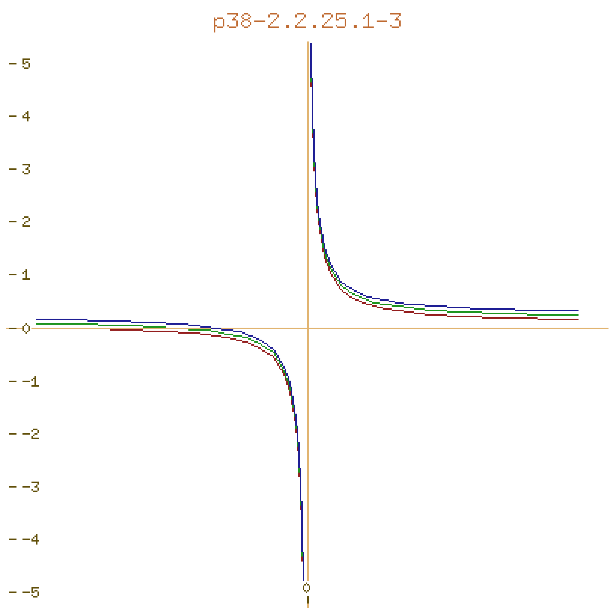
$$y = \frac{c(a + bx)}{x}$$

$$xy - bcx - ca = 0$$

— **p38-2.2.25.1-3** —

```
)clear all
f(x,a,b,c) == c*(a+b*x)/x

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.04),x=-0.5..0.5,adaptive==true,unit==[1.0,1.0],_
                title=="p38-2.2.25.1-3")
graph22251:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,4.0,0.04),x=-0.5..0.5,adaptive==true,unit==[1.0,1.0])
graph22252:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,6.0,0.04),x=-0.5..0.5,adaptive==true,unit==[1.0,1.0])
graph22253:=getGraph(viewport3,1)

putGraph(viewport1,graph22252,2)
```

```
putGraph(viewport1,graph22253,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 38 2.2.26

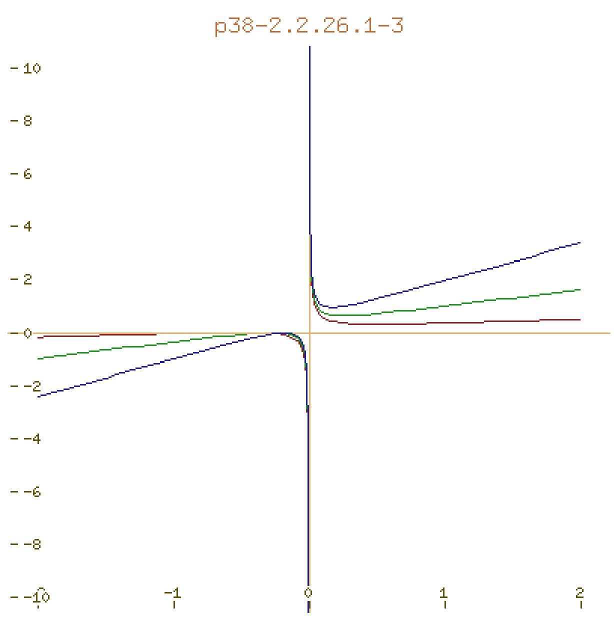
$$y = \frac{c(a+bx)^2}{x}$$

$$xy - b^2cx^2 - 2abcx - a^2c = 0$$

— **p38-2.2.26.1-3** —

```
)clear all
f(x,a,b,c) == c*(a+b*x)^2/x

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.04),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p38-2.2.26.1-3")
graph22261:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,1.0,4.0,0.04),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22262:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,6.0,0.04),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22263:=getGraph(viewport3,1)

putGraph(viewport1,graph22262,2)
putGraph(viewport1,graph22263,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 38 2.2.27

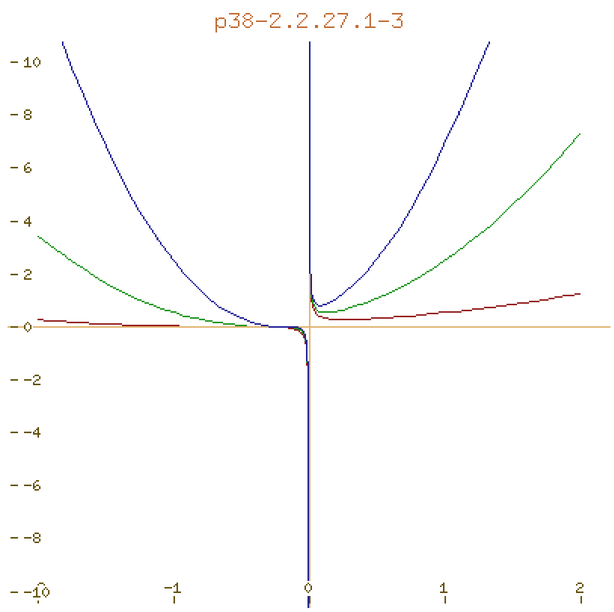
$$y = \frac{c(a+bx)^3}{x}$$

$$xy - b^3cx^3 - 3ab^2cx^2 - 3a^2bcx - a^3c = 0$$

— p38-2.2.27.1-3 —

```
)clear all
f(x,a,b,c) == c*(a+b*x)^3/x

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p38-2.2.27.1-3")
graph22271:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,4.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22272:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,6.0,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22273:=getGraph(viewport3,1)

putGraph(viewport1,graph22272,2)
putGraph(viewport1,graph22273,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

————-



Page 38 2.2.28

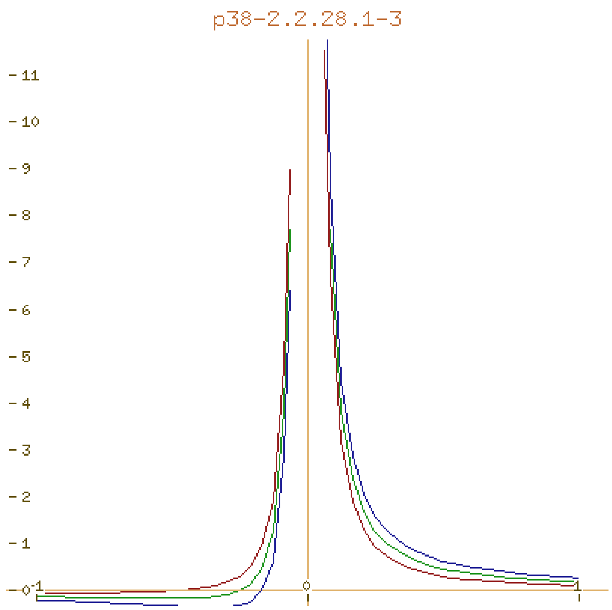
$$y = \frac{c(a+bx)}{x^2}$$

$$x^2y - bcx - ca = 0$$

— **p38-2.2.28.1-3** —

```
)clear all
f(x,a,b,c) == c*(a+b*x)/x^2

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.04),x=-1..1,adaptive==true,unit==[1.0,1.0],_
                title=="p38-2.2.28.1-3")
graph22281:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,4.0,0.04),x=-1..1,adaptive==true,unit==[1.0,1.0])
graph22282:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,6.0,0.04),x=-1..1,adaptive==true,unit==[1.0,1.0])
graph22283:=getGraph(viewport3,1)

putGraph(viewport1,graph22282,2)
```

```
putGraph(viewport1,graph22283,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 38 2.2.29

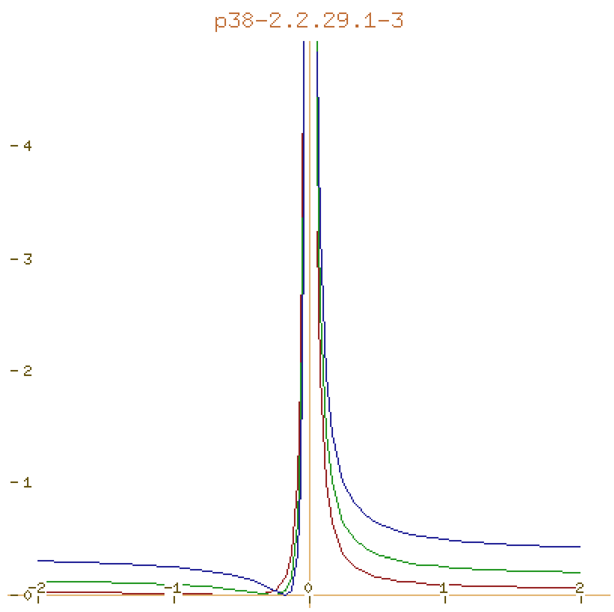
$$y = \frac{c(a+bx)^2}{x^2}$$

$$x^2y - b^2cx^2 - 2abcx - a^2c = 0$$

— **p38-2.2.29.1-3** —

```
)clear all
f(x,a,b,c) == c*(a+b*x)^2/x^2

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
               title=="p38-2.2.29.1-3")
graph22291:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,1.0,4.0,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22292:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,6.0,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22293:=getGraph(viewport3,1)

putGraph(viewport1,graph22292,2)
putGraph(viewport1,graph22293,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 38 2.2.30

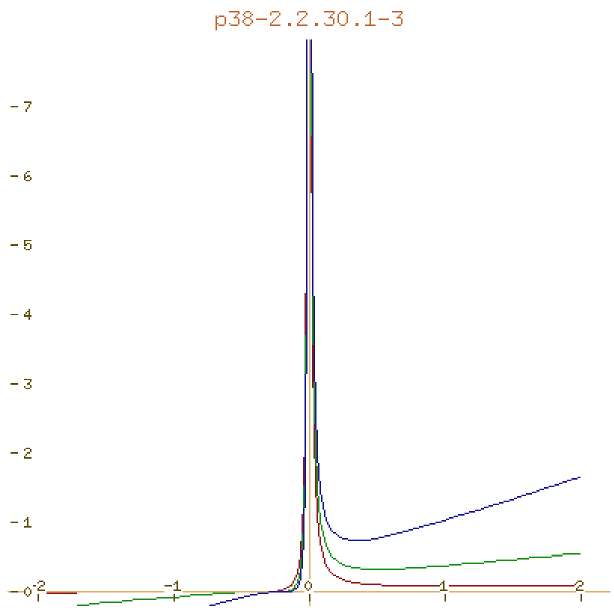
$$y = \frac{c(a+bx)^3}{x^2}$$

$$x^2y - b^3cx^3 - 3ab^2cx^2 - 3a^2bcx - a^3c = 0$$

— **p38-2.2.30.1-3** —

```
)clear all
f(x,a,b,c) == c*(a+b*x)^3/x^2

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.003),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p38-2.2.30.1-3")
graph22301:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,4.0,0.003),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22302:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,6.0,0.003),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22303:=getGraph(viewport3,1)

putGraph(viewport1,graph22302,2)
putGraph(viewport1,graph22303,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 40 2.2.31

$$y = \frac{c(a + bx)}{x^3}$$

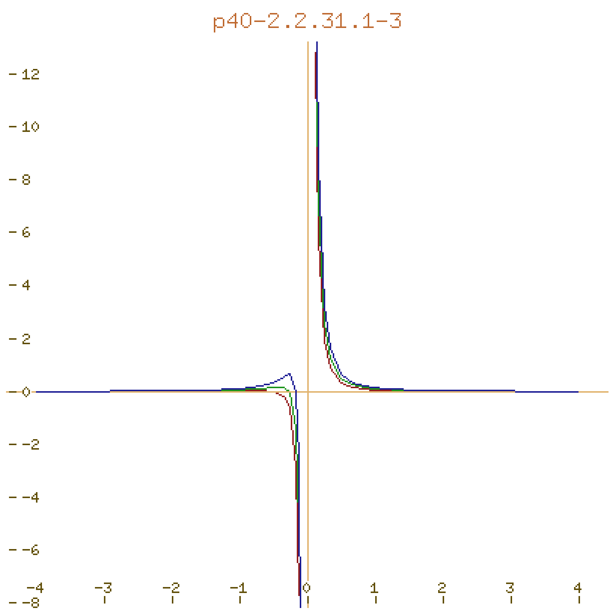
$$x^3y - bcx - ca = 0$$

— **p40-2.2.31.1-3** —

```
)clear all
f(x,a,b,c) == c*(a+b*x)/x^3

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.02),x=-4..4,adaptive==true,unit==[1.0,1.0],_
                title=="p40-2.2.31.1-3")
graph22311:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,4.0,0.02),x=-4..4,adaptive==true,unit==[1.0,1.0])
graph22312:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,6.0,0.02),x=-4..4,adaptive==true,unit==[1.0,1.0])
graph22313:=getGraph(viewport3,1)

putGraph(viewport1,graph22312,2)
```

```
putGraph(viewport1,graph22313,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

——–



Page 40 2.2.32

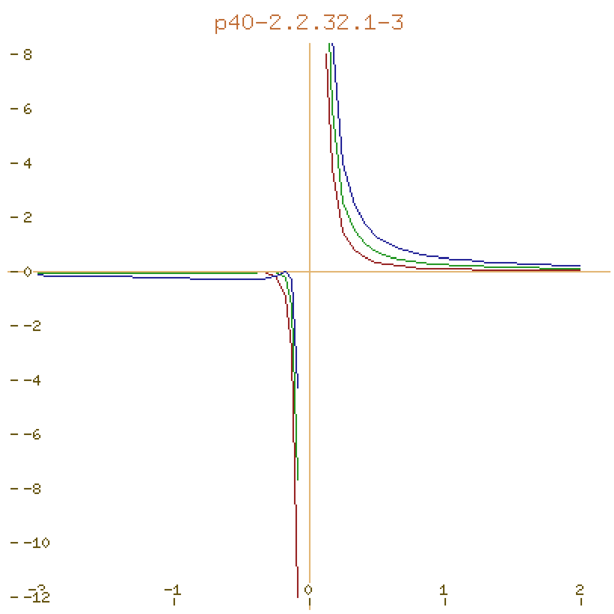
$$y = \frac{c(a+bx)^2}{x^3}$$

$$x^3y - b^2cx^2 - 2abcx - a^2c = 0$$

— **p40-2.2.32.1-3** —

```
)clear all
f(x,a,b,c) == c*(a+b*x)^2/x^3

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
               title=="p40-2.2.32.1-3")
graph22321:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,1.0,4.0,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22322:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,6.0,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22323:=getGraph(viewport3,1)

putGraph(viewport1,graph22322,2)
putGraph(viewport1,graph22323,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 40 2.2.33

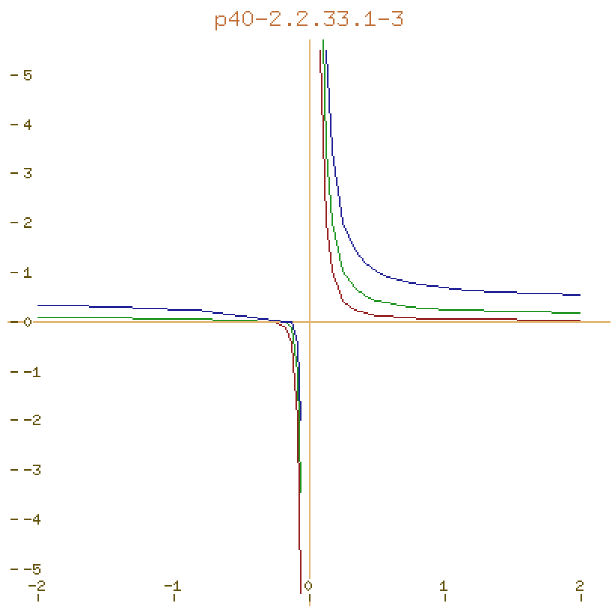
$$y = \frac{c(a+bx)^3}{x^3}$$

$$x^3y - b^3cx^3 - 3ab^2cx^2 - 3a^2bcx - a^3c = 0$$

— p40-2.2.33.1-3 —

```
)clear all
f(x,a,b,c) == c*(a+b*x)^3/x^3

lineColorDefault(red())
viewport1:=draw(f(x,1.0,2.0,0.002),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p40-2.2.33.1-3")
graph22331:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,1.0,4.0,0.002),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22332:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,6.0,0.002),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph22333:=getGraph(viewport3,1)

putGraph(viewport1,graph22332,2)
putGraph(viewport1,graph22333,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Functions with $a^2 + x^2$ and x^m

Page 42 2.3.1

$$y = \frac{c}{(a^2 + b^2)}$$

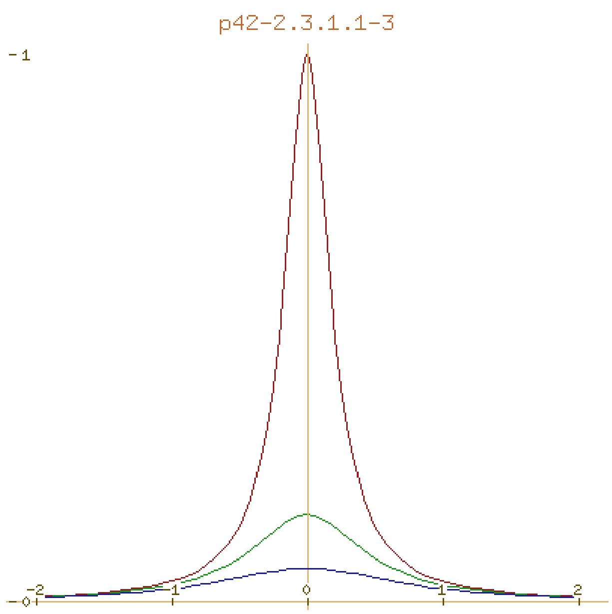
$$a^2y + x^2y - c = 0$$

— p42-2.3.1.1-3 —

```
)clear all
f(x,a,c) == c/(a^2+x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.04),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p42-2.3.1.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,0.04),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.04),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)
```

```
putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 42 2.3.2

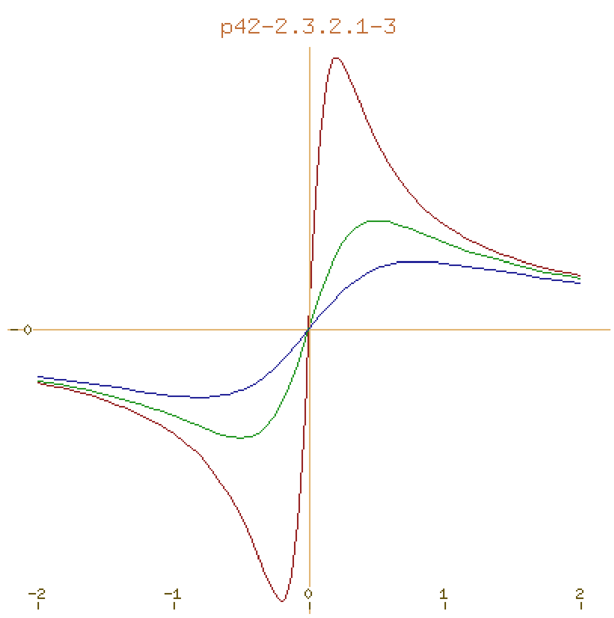
$$y = \frac{cx}{(a^2 + b^2)}$$

$$a^2y + x^2y - cx = 0$$

— **p42-2.3.2.1-3** —

```
)clear all
f(x,a,c) == c*x/(a^2+x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.3),x=-2..2,adaptive==true,unit==[1.0,1.0],_
```

```
                title=="p42-2.3.2.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,0.3),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.3),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



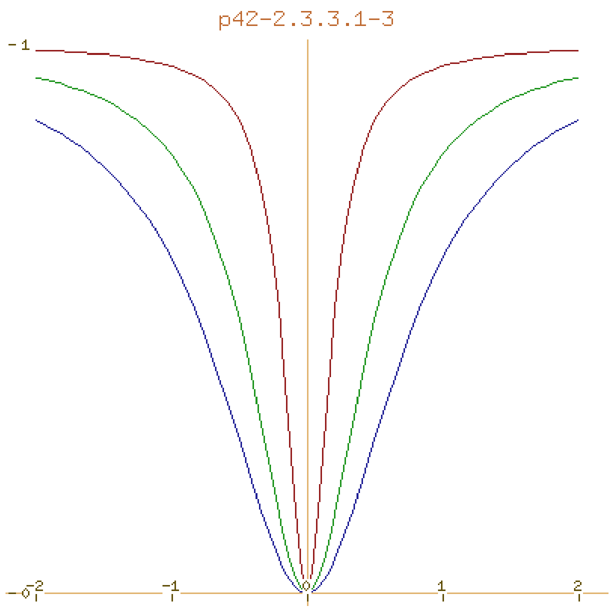
Page 42 2.3.3

$$y = \frac{cx^2}{(a^2 + b^2)}$$

$$a^2y + x^2y - cx^2 = 0$$

— p42-2.3.3.1-3 —

```
)clear all
f(x,a,c) == c*x^2/(a^2+x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p42-2.3.3.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 42 2.3.4

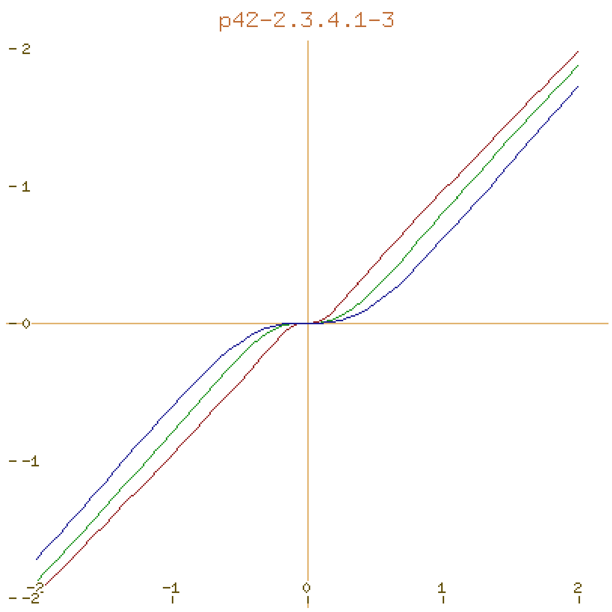
$$y = \frac{cx^3}{(a^2 + b^2)}$$

$$a^2y + x^2y - cx^3 = 0$$

— **p42-2.3.4.1-3** —

```
)clear all
f(x,a,c) == c*x^3/(a^2+x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p42-2.3.4.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
```

```
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 44 2.3.5

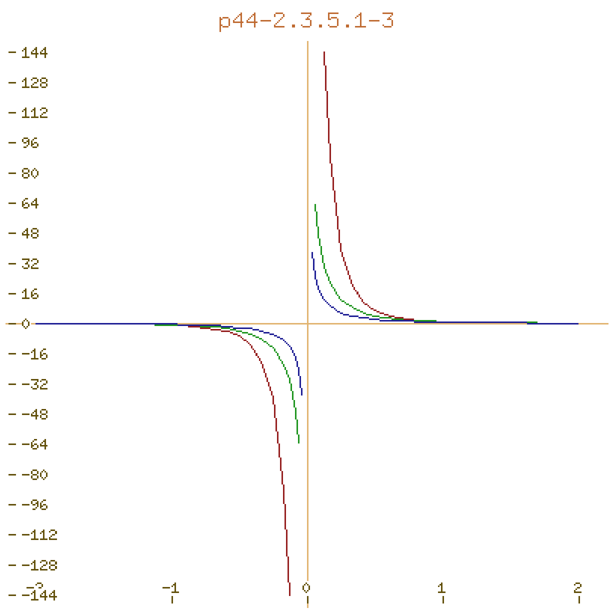
$$y = \frac{c}{x(a^2 + b^2)}$$

$$a^2xy + x^3y - c = 0$$

— p44-2.3.5.1-3 —

```
)clear all
f(x,a,c) == c/(x*(a^2+x^2))

lineColorDefault(red())
viewport1:=draw(f(x,0.2,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p44-2.3.5.1-3")
graph1:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 44 2.3.6

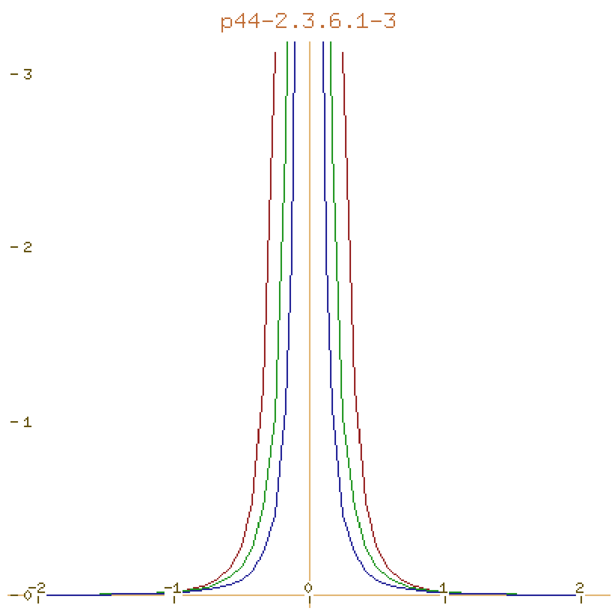
$$y = \frac{c}{x^2(a^2+b^2)}$$

$$a^2x^2y + x^4y - c = 0$$

— p44-2.3.6.1-3 —

```
)clear all
f(x,a,c) == c/(x^2*(a^2+x^2))

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p44-2.3.6.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 44 2.3.7

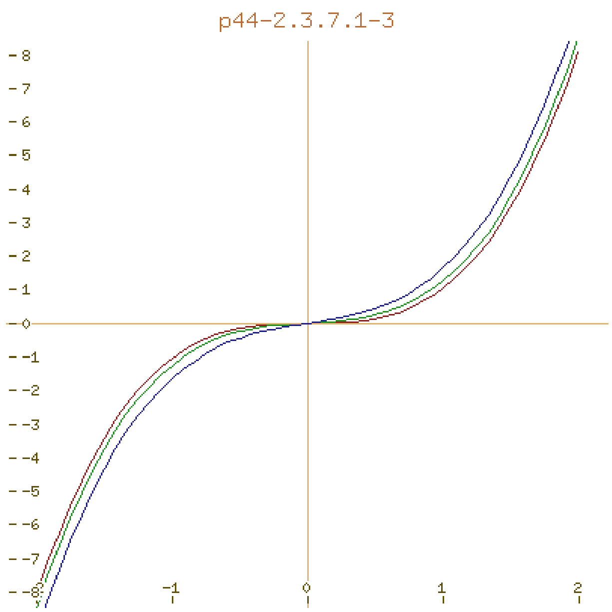
$$y = cx(a^2 + x^2)$$

$$y - a^2cx - cx^3 = 0$$

— **p44-2.3.7.1-3** —

```
)clear all
f(x,a,c) == c*x*(a^2+x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p44-2.3.7.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
```

```
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 44 2.3.8

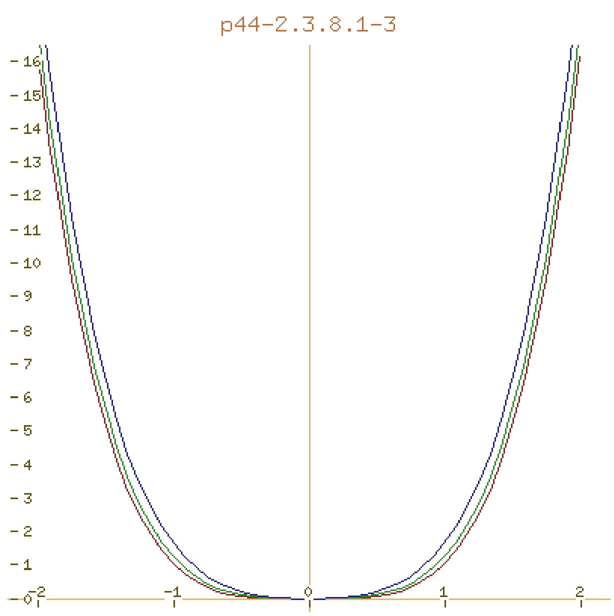
$$y = cx^2(a^2 + x^2)$$

$$y - a^2cx^2 - cx^4 = 0$$

— **p44-2.3.8.1-3** —

```
)clear all
f(x,a,c) == c*x^2*(a^2+x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p44-2.3.8.1-3")
graph1:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Functions with $a^2 - x^2$ and x^m

Page 46 2.4.1

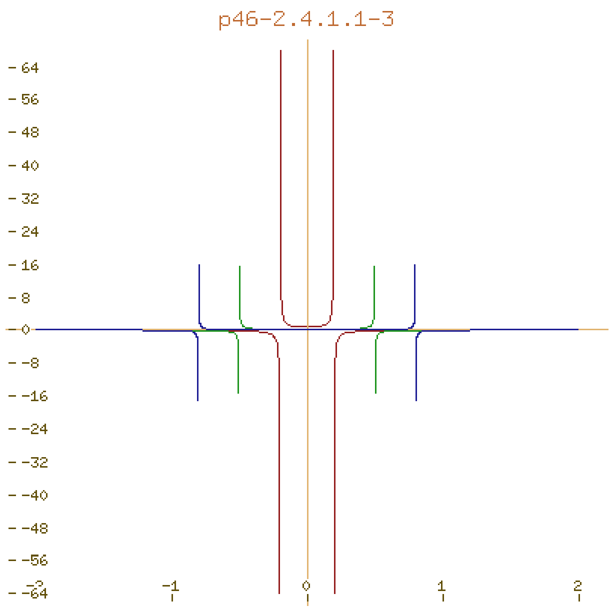
$$y = \frac{c}{(a^2 - x^2)}$$

$$a^2y - x^2y - c = 0$$

— p46-2.4.1.1-3 —

```
)clear all
f(x,a,c) == c/(a^2-x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.03),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p46-2.4.1.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,0.03),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.03),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 46 2.4.2

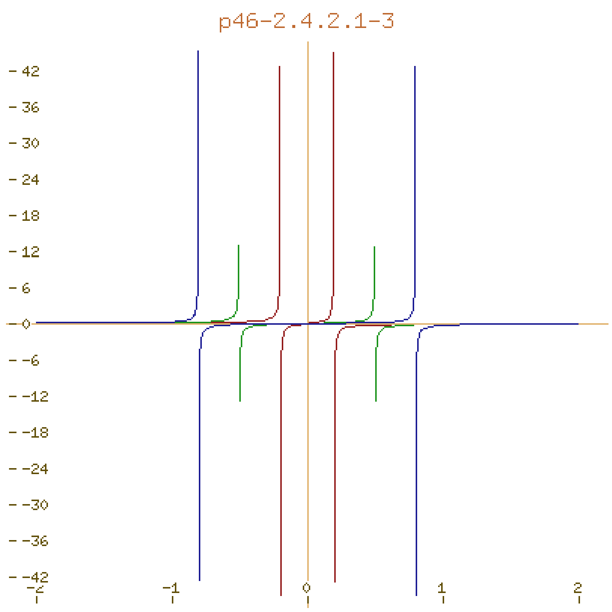
$$y = \frac{cx}{(a^2 - x^2)}$$

$$a^2y - x^2y - cx = 0$$

— **p46-2.4.2.1-3** —

```
)clear all
f(x,a,c) == c*x/(a^2-x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p46-2.4.2.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
```

```
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 46 2.4.3

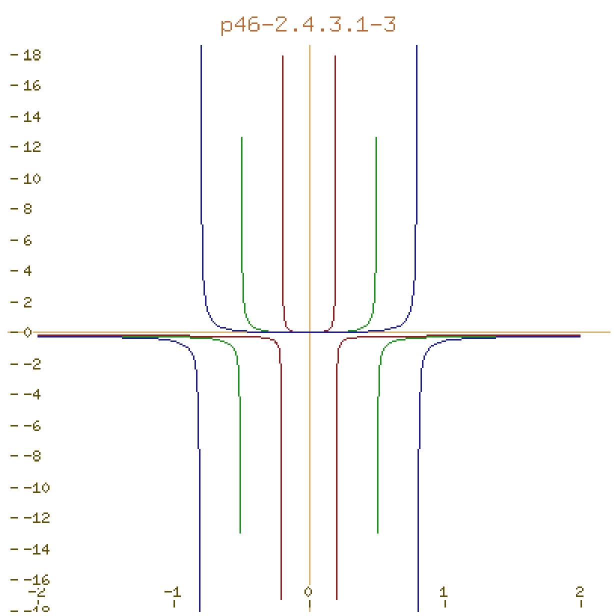
$$y = \frac{cx^2}{(a^2 - x^2)}$$

$$a^2y - x^2y - cx^2 = 0$$

— p46-2.4.3.1-3 —

```
)clear all
f(x,a,c) == c*x^2/(a^2-x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p46-2.4.3.1-3")
```

```
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 46 2.4.4

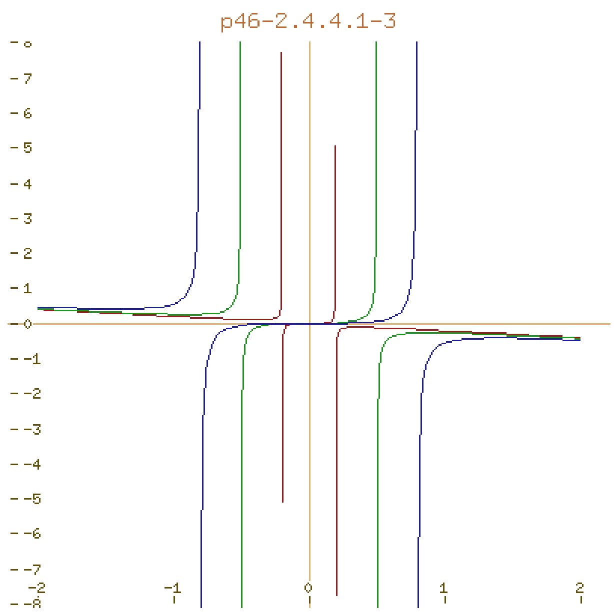
$$y = \frac{cx^3}{(a^2 - x^2)}$$

$$a^2y - x^2y - cx^3 = 0$$

— p46-2.4.4.1-3 —

```
)clear all
f(x,a,c) == c*x^3/(a^2-x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p46-2.4.4.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

————



Page 48 2.4.5

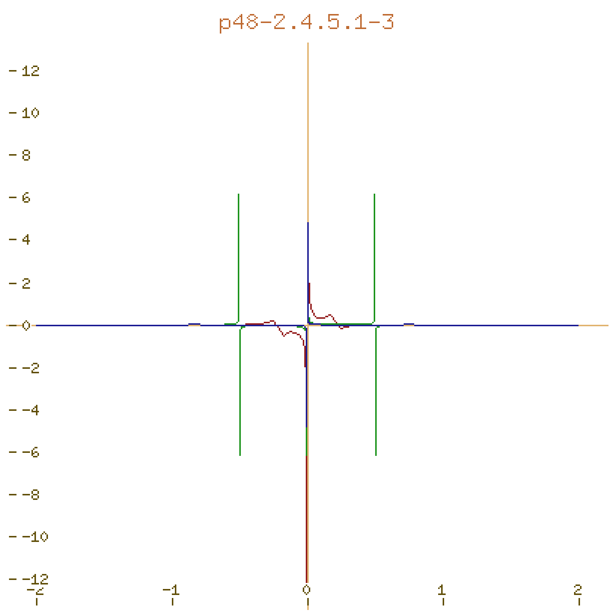
$$y = \frac{c}{x(a^2 - x^2)}$$

$$a^2xy - x^3y - c = 0$$

— **p48-2.4.5.1-3** —

```
)clear all
f(x,a,c) == c/(x*(a^2-x^2))

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.001),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p48-2.4.5.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,0.001),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.001),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
```

```
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 48 2.4.6

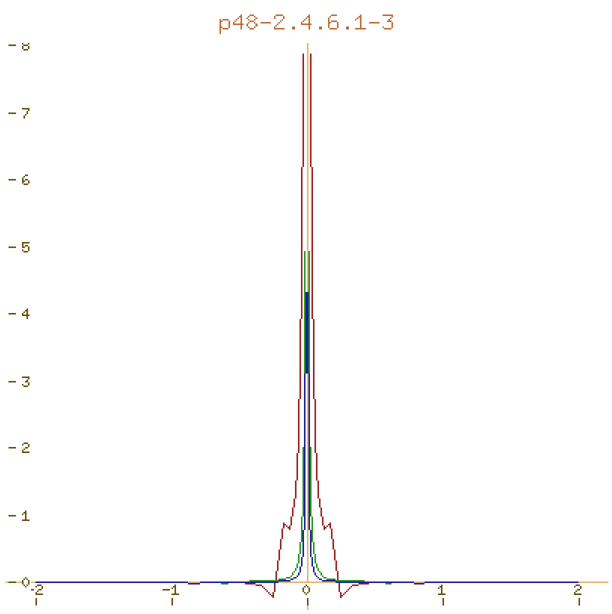
$$y = \frac{c}{x^2(a^2 - x^2)}$$

$$a^2x^2y - x^4y - c = 0$$

— **p48-2.4.6.1-3** —

```
)clear all
f(x,a,c) == c/(x^2*(a^2-x^2))

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.0003),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p48-2.4.6.1-3")
graph1:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,0.5,0.0003),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.0003),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

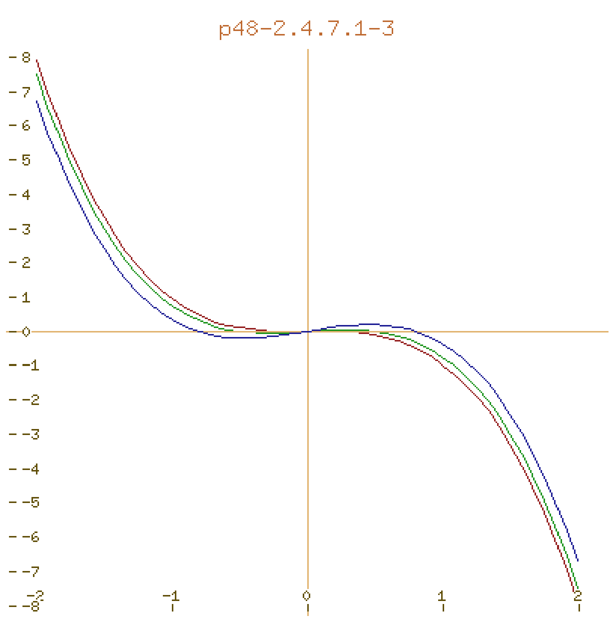


Page 48 2.4.7

$$y = cx(a^2 - x^2)$$
$$y - a^2cx + cx^3 = 0$$

— **p48-2.4.7.1-3** —

```
)clear all
f(x,a,c) == c*x*(a^2-x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p48-2.4.7.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 48 2.4.8

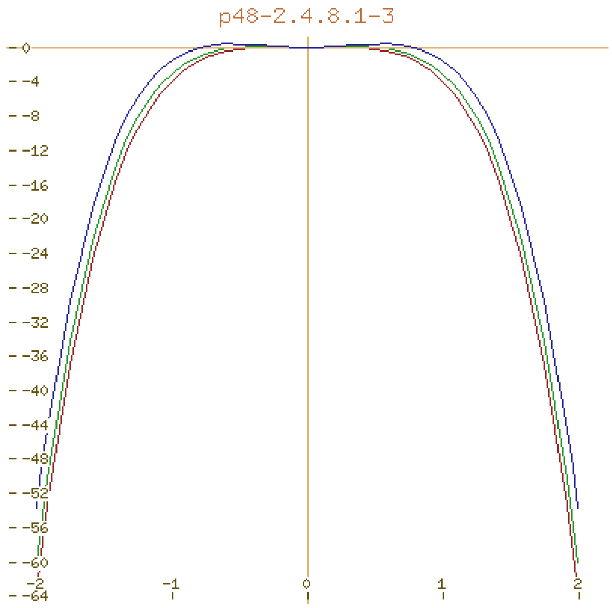
$$y = cx^2(a^2 - x^2)$$

$$y - a^2cx^2 + cx^4 = 0$$

— p48-2.4.8.1-3 —

```
)clear all
f(x,a,c) == c*x^2*(a^2-x^2)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,4.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p48-2.4.8.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,4.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,4.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Functions with $a^3 + x^3$ and x^m

Page 50 2.5.1

$$y = \frac{c}{(a^3 + x^3)}$$

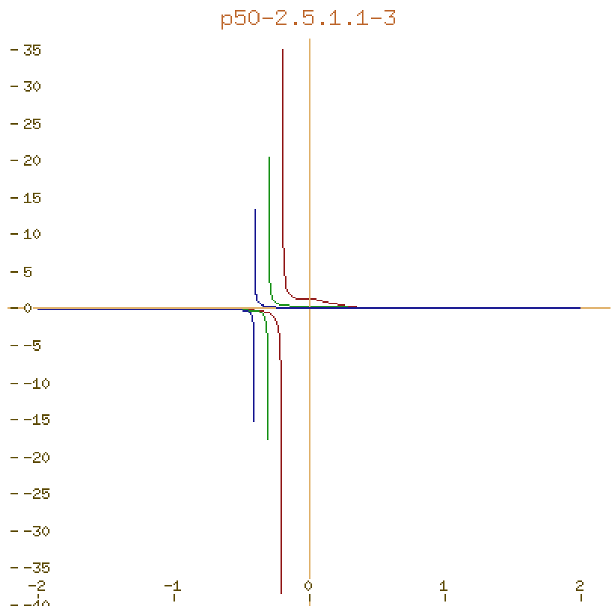
$$a^3y + x^3y - c = 0$$

— **p50-2.5.1.1-3** —

```
)clear all
f(x,a,c) == c/(a^3+x^3)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p50-2.5.1.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.3,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.4,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)
```

```
putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———–



Page 50 2.5.2

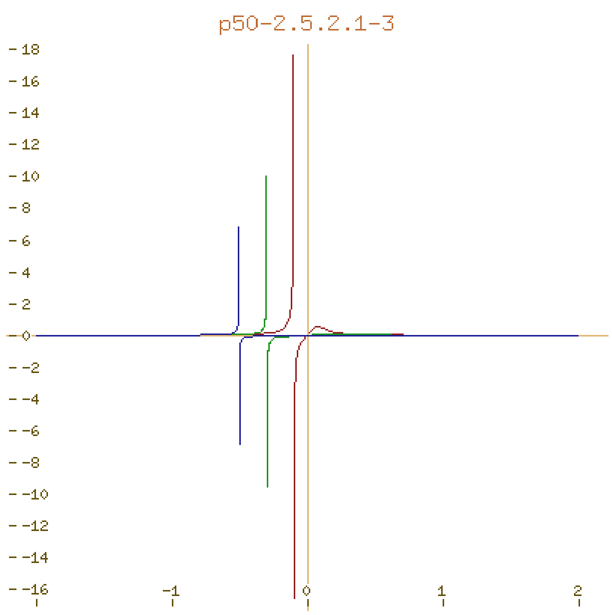
$$y = \frac{cx}{(a^3 + x^3)}$$

$$a^3y + x^3y - cx = 0$$

— **p50-2.5.2.1-3** —

```
)clear all
f(x,a,c) == c*x/(a^3+x^3)

lineColorDefault(red())
viewport1:=draw(f(x,0.1,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
```

```
               title=="p50-2.5.2.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.3,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 50 2.5.3

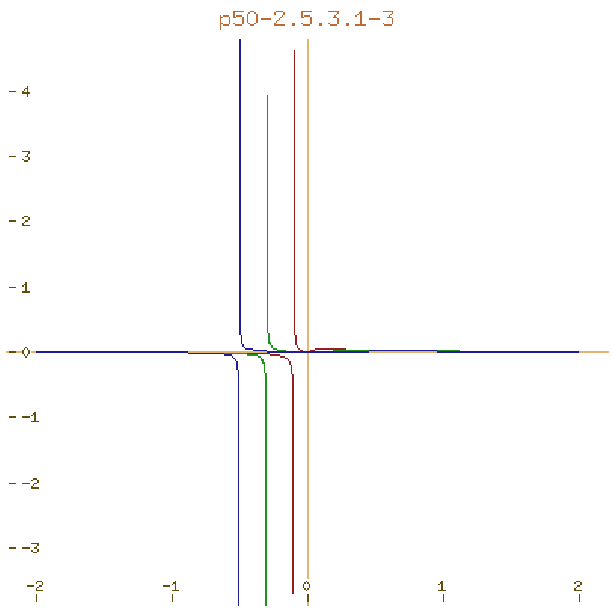
$$y = \frac{cx^2}{(a^3 + x^3)}$$

$$a^3y + x^3y - cx^2 = 0$$

— p50-2.5.3.1-3 —

```
)clear all
f(x,a,c) == c*x^2/(a^3+x^3)

lineColorDefault(red())
viewport1:=draw(f(x,0.1,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p50-2.5.3.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.3,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———



Page 50 2.5.4

$$y = \frac{cx^3}{(a^3 + x^3)}$$

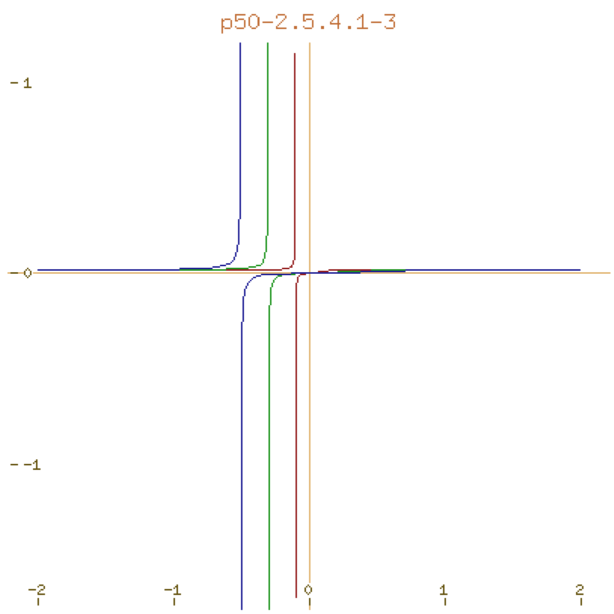
$$a^3 y + x^3 y - cx^3 = 0$$

— **p50-2.5.4.1-3** —

```
)clear all
f(x,a,c) == c*x^3/(a^3+x^3)

lineColorDefault(red())
viewport1:=draw(f(x,0.1,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p50-2.5.4.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.3,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,0.02),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
```

```
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———



Page 50 2.5.5

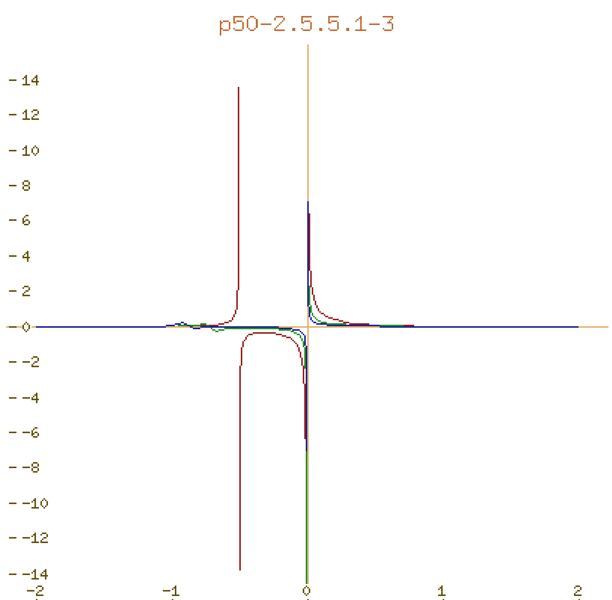
$$y = \frac{c}{x(a^3 + x^3)}$$

$$a^3xy + x^4y - c = 0$$

— **p50-2.5.5.1-3** —

```
)clear all
f(x,a,c) == c/(x*(a^3+x^3))

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p50-2.5.5.1-3")
graph1:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,0.7,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.9,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

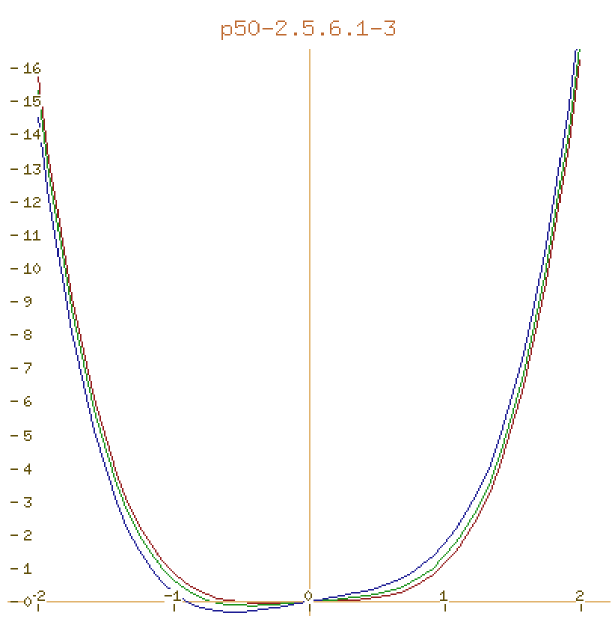


Page 50 2.5.6

$$y = cx(a^3 + x^3)$$
$$y - a^3cx - cx^4 = 0$$

— p50-2.5.6.1-3 —

```
)clear all
f(x,a,c) == c*x*(a^3+x^3)

lineColorDefault(red())
viewport1:=draw(f(x,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p50-2.5.6.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.7,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.9,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Functions with $a^3 - x^3$ and x^m

Page 52 2.6.1

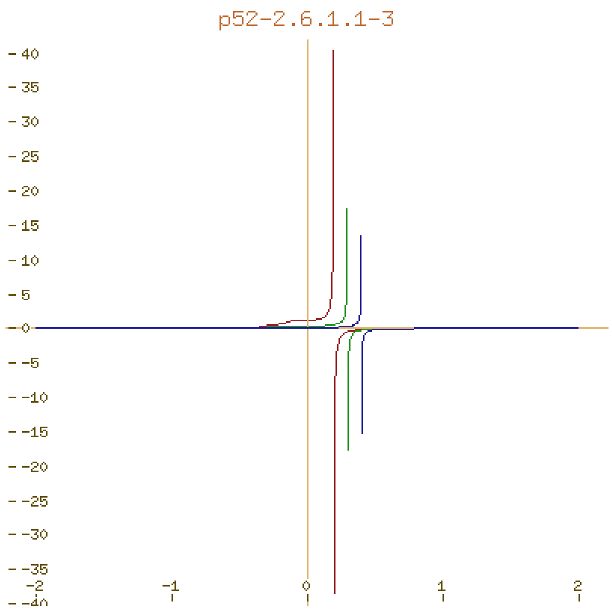
$$y = \frac{c}{(a^3 - x^3)}$$

$$a^3y - x^3y - c = 0$$

— **p52-2.6.1.1-3** —

```
)clear all
f(x,a,c) == c/(a^3-x^3)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p52-2.6.1.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.3,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.4,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 52 2.6.2

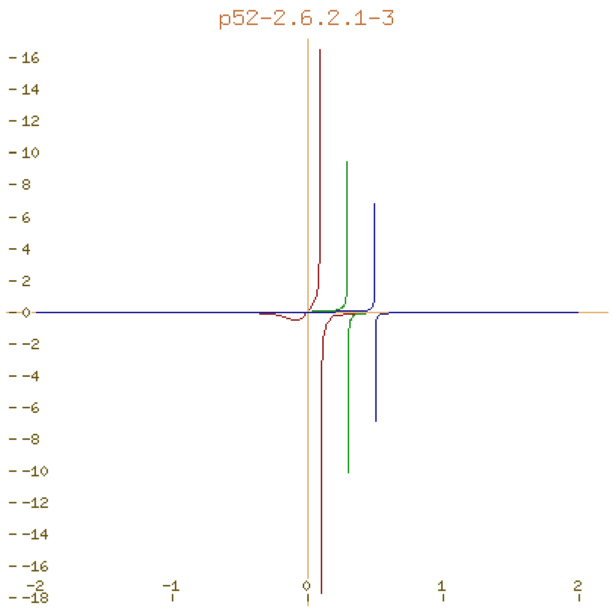
$$y = \frac{cx}{(a^3 - x^3)}$$

$$a^3y - x^3y - cx = 0$$

— **p52-2.6.2.1-3** —

```
)clear all
f(x,a,c) == c*x/(a^3-x^3)

lineColorDefault(red())
viewport1:=draw(f(x,0.1,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p52-2.6.2.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.3,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
```

```
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 52 2.6.3

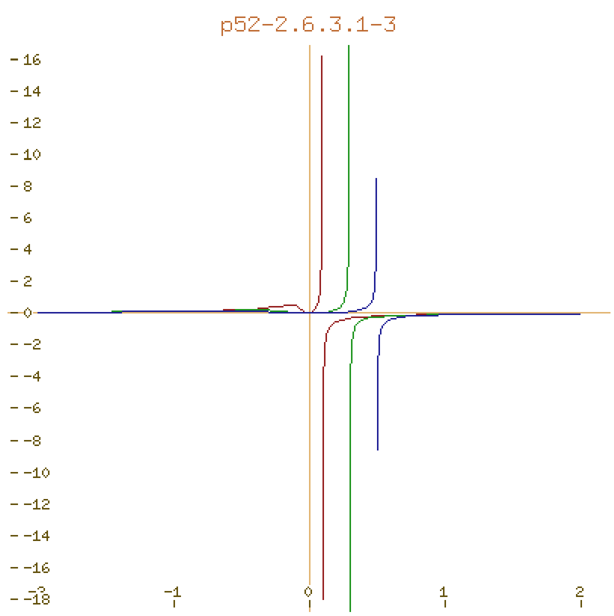
$$y = \frac{cx^2}{(a^3 - x^3)}$$

$$a^3y - x^3y - cx^2 = 0$$

— p52-2.6.3.1-3 —

```
)clear all
f(x,a,c) == c*x^2/(a^3-x^3)

lineColorDefault(red())
viewport1:=draw(f(x,0.1,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p52-2.6.3.1-3")
```

```
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.3,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 52 2.6.4

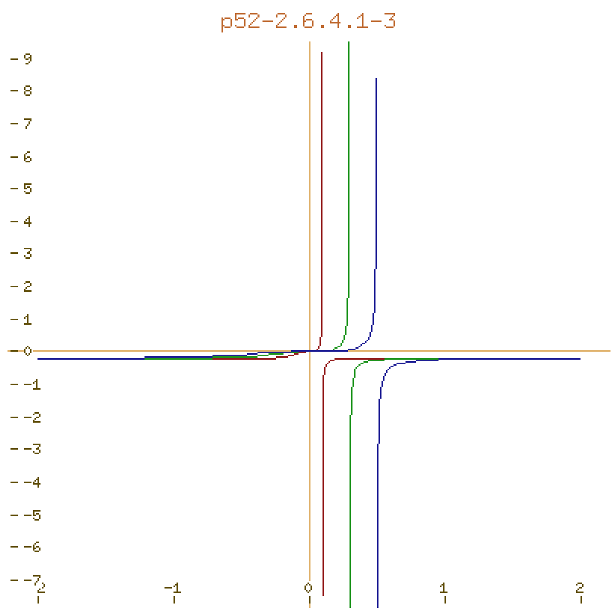
$$y = \frac{cx^3}{(a^3 - x^3)}$$

$$a^3y - x^3y - cx^3 = 0$$

— p52-2.6.4.1-3 —

```
)clear all
f(x,a,c) == c*x^3/(a^3-x^3)

lineColorDefault(red())
viewport1:=draw(f(x,0.1,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p52-2.6.4.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.3,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,0.2),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 52 2.6.5

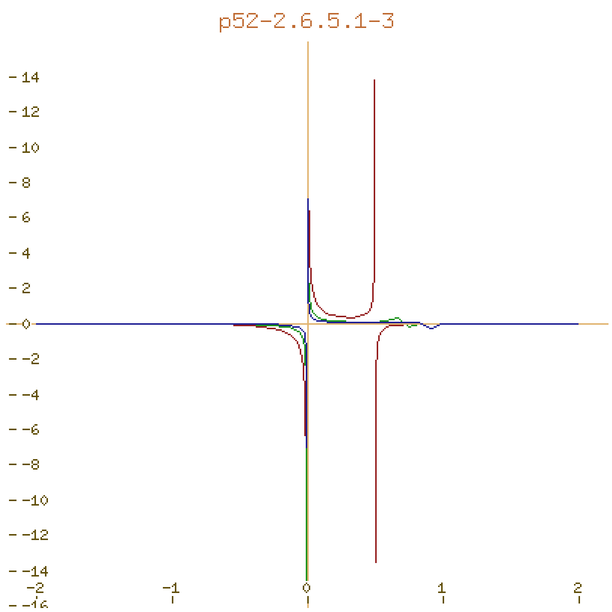
$$y = \frac{c}{x(a^3 - x^3)}$$

$$a^3xy - x^4y - c = 0$$

— **p52-2.6.5.1-3** —

```
)clear all
f(x,a,c) == c/(x*(a^3-x^3))

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
               title=="p52-2.6.5.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.7,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.9,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
```

```
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 52 2.6.6

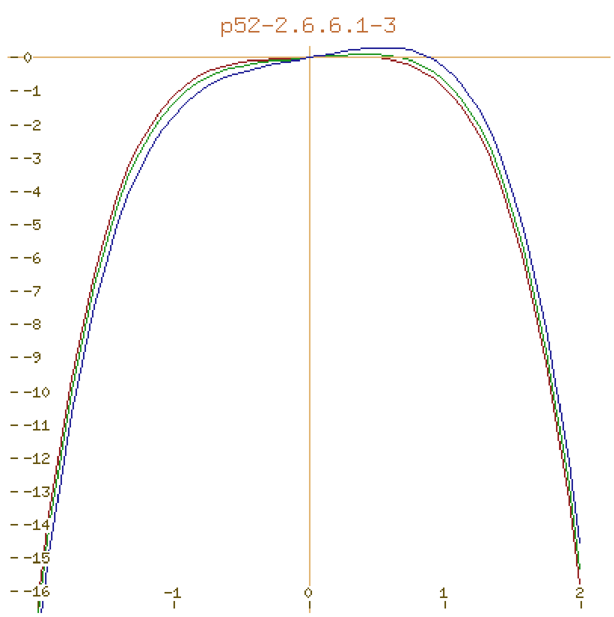
$$y = cx(a^3 - x^3)$$

$$y - a^3cx + cx^4 = 0$$

— **p52-2.6.6.1-3** —

```
)clear all
f(x,a,c) == c*x*(a^3-x^3)

lineColorDefault(red())
viewport1:=draw(f(x,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p52-2.6.6.1-3")
graph1:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,0.7,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.9,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Functions with $a^4 + x^4$ and x^m

Page 54 2.7.1

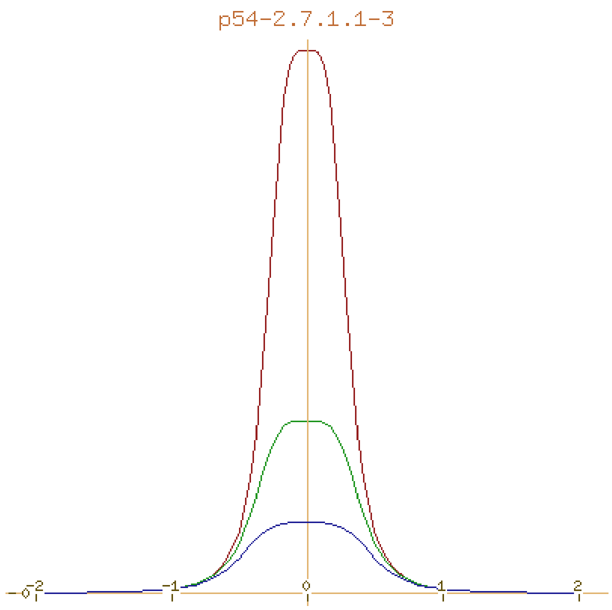
$$y = \frac{c}{(a^4 + x^4)}$$

$$a^4y + x^4y - c = 0$$

— p54-2.7.1.1-3 —

```
)clear all
f(x,a,c) == c/(a^4+x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.3,0.007),x=-2..2,adaptive==true,unit==[1.0,1.0],_
               title=="p54-2.7.1.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.4,0.007),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,0.007),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 54 2.7.2

$$y = \frac{cx}{(a^4 + x^4)}$$

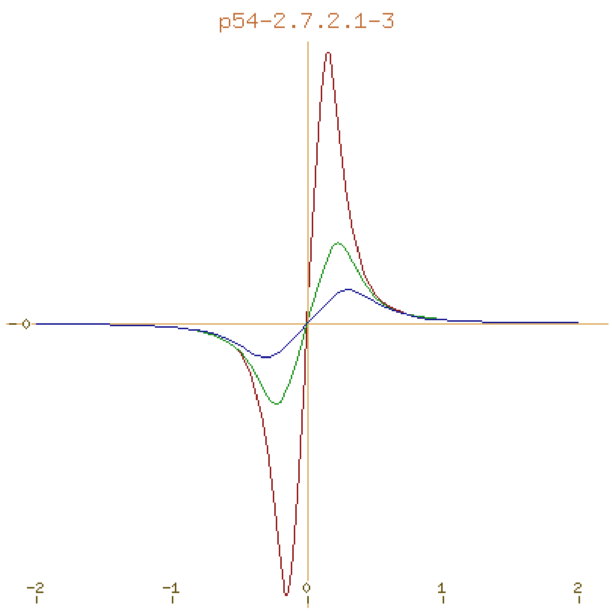
$$a^4y + x^4y - cx = 0$$

— **p54-2.7.2.1-3** —

```
)clear all
f(x,a,c) == c*x/(a^4+x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p54-2.7.2.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.3,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.4,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
```

```
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

————



Page 54 2.7.3

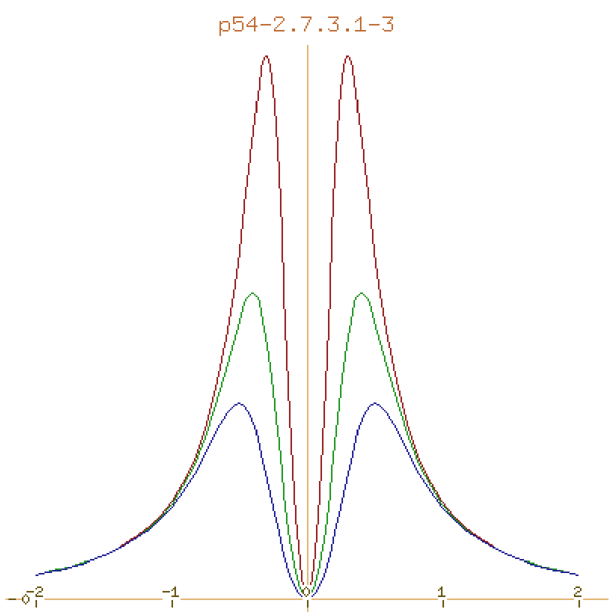
$$y = \frac{cx^2}{(a^4 + x^4)}$$

$$a^4y + x^4y - cx^2 = 0$$

— **p54-2.7.3.1-3** —

```
)clear all
f(x,a,c) == c*x^2/(a^4+x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.3,0.15),x=-2..2,adaptive==true,unit==[1.0,1.0],_
               title=="p54-2.7.3.1-3")
```

```
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.4,0.15),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.5,0.15),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 54 2.7.4

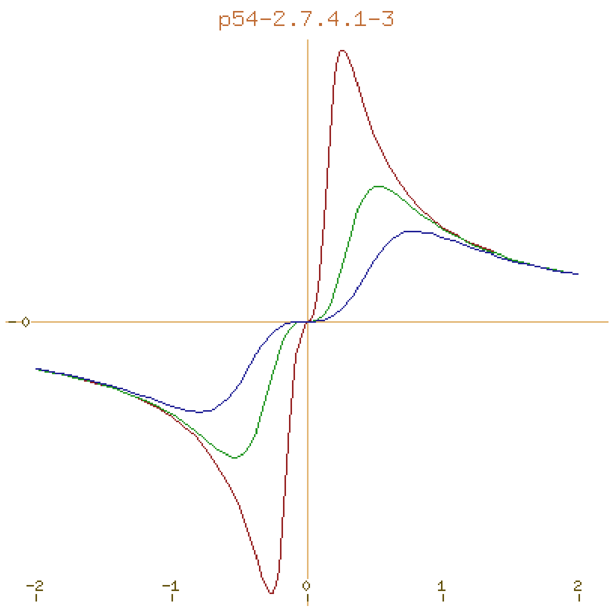
$$y = \frac{cx^3}{(a^4 + x^4)}$$

$$a^4y + x^4y - cx^3 = 0$$

— p54-2.7.4.1-3 —

```
)clear all
f(x,a,c) == c*x^3/(a^4+x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.25),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p54-2.7.4.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.4,0.25),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.6,0.25),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 54 2.7.5

$$y = \frac{cx^4}{(a^4 + x^4)}$$

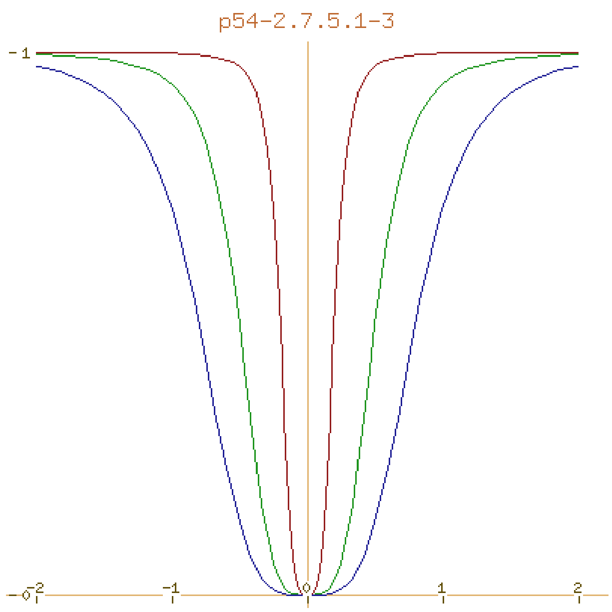
$$a^4y + x^4y - cx^4 = 0$$

— **p54-2.7.5.1-3** —

```
)clear all
f(x,a,c) == c*x^4/(a^4+x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p54-2.7.5.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
```

```
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

————-



Page 54 2.7.6

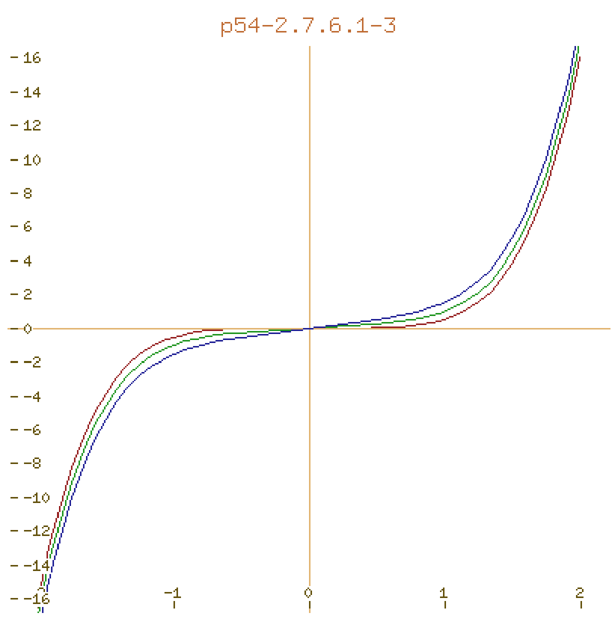
$$y = cx(a^4 + x^4)$$

$$y - a^4cx - cx^5 = 0$$

— **p54-2.7.6.1-3** —

```
)clear all
f(x,a,c) == c*x*(a^4+x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.5,0.5),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p54-2.7.6.1-3")
graph1:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,1.0,0.5),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.2,0.5),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Functions with $a^4 - x^4$ and x^m

Page 56 2.8.1

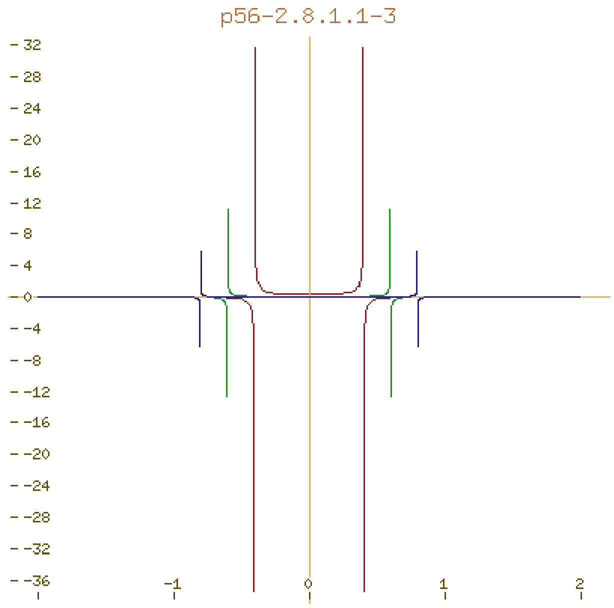
$$y = \frac{c}{(a^4 - x^4)}$$

$$a^4y - x^4y - c = 0$$

— p56-2.8.1.1-3 —

```
)clear all
f(x,a,c) == c/(a^4-x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.4,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
               title=="p56-2.8.1.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.6,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

————



Page 56 2.8.2

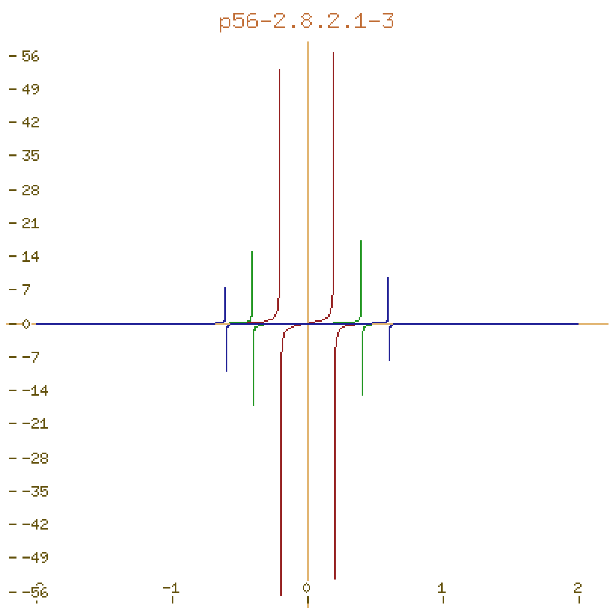
$$y = \frac{cx}{(a^4 - x^4)}$$

$$a^4y - x^4y - cx = 0$$

— **p56-2.8.2.1-3** —

```
)clear all
f(x,a,c) == c*x/(a^4-x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p56-2.8.2.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.4,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.6,0.01),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
```

```
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 56 2.8.3

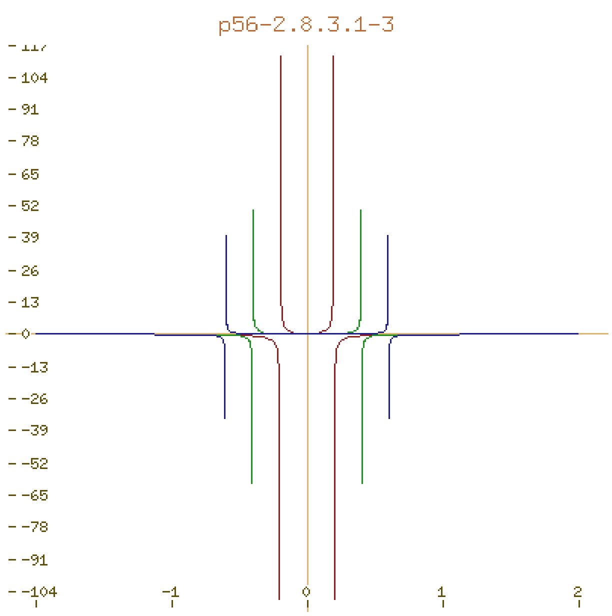
$$y = \frac{cx^2}{(a^4 - x^4)}$$

$$a^4y - x^4y - cx^2 = 0$$

— p56-2.8.3.1-3 —

```
)clear all
f(x,a,c) == c*x^2/(a^4-x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p56-2.8.3.1-3")
```

```
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.4,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.6,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 56 2.8.4

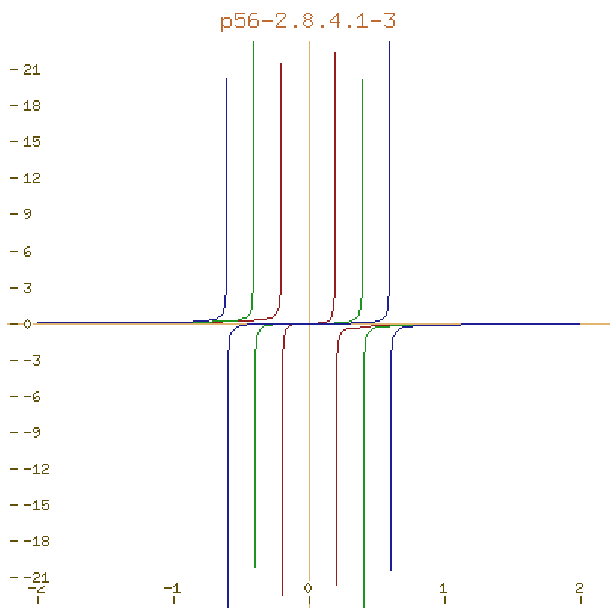
$$y = \frac{cx^3}{(a^4 - x^4)}$$

$$a^4y - x^4y - cx^3 = 0$$

— p56-2.8.4.1-3 —

```
)clear all
f(x,a,c) == c*x^3/(a^4-x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p56-2.8.4.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.4,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.6,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 56 2.8.5

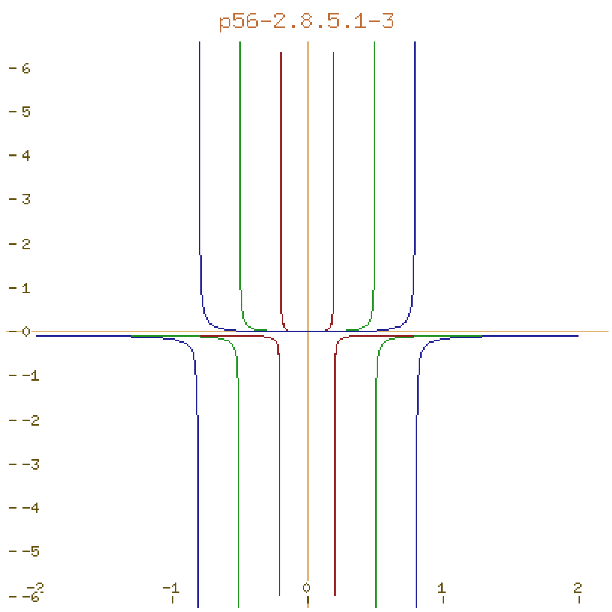
$$y = \frac{cx^4}{(a^4 - x^4)}$$

$$a^4y - x^4y - cx^4 = 0$$

— **p56-2.8.5.1-3** —

```
)clear all
f(x,a,c) == c*x^4/(a^4-x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.2,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p56-2.8.5.1-3")
graph1:=getGraph(viewport1,1)

lineColorDefault(green())
viewport2:=draw(f(x,0.5,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,0.8,0.1),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
```

```
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 56 2.8.6

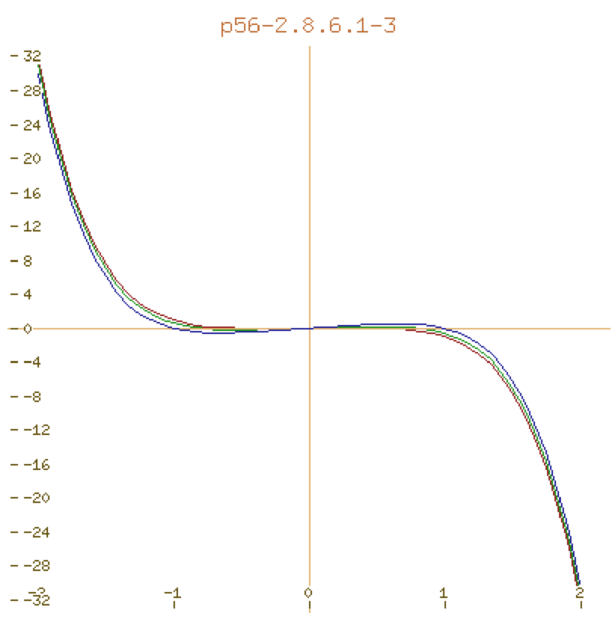
$$y = cx(a^4 - x^4)$$

$$y - a^4cx + cx^5 = 0$$

— p56-2.8.6.1-3 —

```
)clear all
f(x,a,c) == c*x*(a^4-x^4)

lineColorDefault(red())
viewport1:=draw(f(x,0.4,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0],_
                title=="p56-2.8.6.1-3")
graph1:=getGraph(viewport1,1)
```

```
lineColorDefault(green())
viewport2:=draw(f(x,0.8,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

lineColorDefault(blue())
viewport3:=draw(f(x,1.0,1.0),x=-2..2,adaptive==true,unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Functions with $(a + bx)^{1/2}$ and x^m

Page 58 2.9.1

Parabola

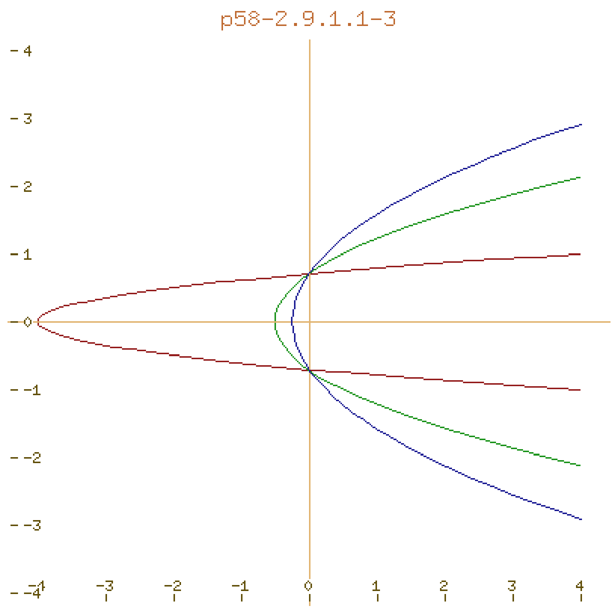
$$y = c(a + bx)^{1/2}$$

$$y^2 - bc^2x - ac^2 = 0$$

— p58-2.9.1.1-3 —

```
)clear all
f1(x,y) == y^2 - x/8 - 1/2

lineColorDefault(red())
viewport1:=draw(f1(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0],title=="p58-2.9.1.1-3")
graph1:=getGraph(viewport1,1)

f2(x,y) == y^2 - x - 1/2

lineColorDefault(green())
viewport2:=draw(f2(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == y^2 - 2*x - 1/2

lineColorDefault(blue())
viewport3:=draw(f3(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-



Page 58 2.9.2

Trisectrix of Catalan... fails "singular pts in region of sketch"

$$y = cx(a + bx)^{1/2}$$

$$y^2 - bc^2x^3 - ac^2x^2 = 0$$

— p58-2.9.2.1-3 —

```
)clear all
f1(x,y) == y^2 - x^3/2 - x^2/2

lineColorDefault(red())
viewport1:=draw(f1(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0],title=="p58-2.9.2.1-3")
graph1:=getGraph(viewport1,1)

f2(x,y) == y^2 - x^3 - x^2/2

lineColorDefault(green())
viewport2:=draw(f2(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == y^2 - 2*x^3 - x^2/2
```

```
lineColorDefault(blue())
viewport3:=draw(f3(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

————-

Page 58 2.9.3

fails with "singular pts in region of sketch"

$$y = cx^2(a + bx)^{1/2}$$

$$y^2 - bc^2x^5 - ac^2x^4 = 0$$

— **p58-2.9.3.1-3** —

```
)clear all
f1(x,y) == y^2 - x^5/2 - x^4/2

lineColorDefault(red())
viewport1:=draw(f1(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0],title=="p58-2.9.3.1-3")
graph1:=getGraph(viewport1,1)

f2(x,y) == y^2 - x^5 - x^4/2

lineColorDefault(green())
viewport2:=draw(f2(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == y^2 - 2*x^5 - x^4/2

lineColorDefault(blue())
viewport3:=draw(f3(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)
```

```
putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

——–

Page 58 2.9.4

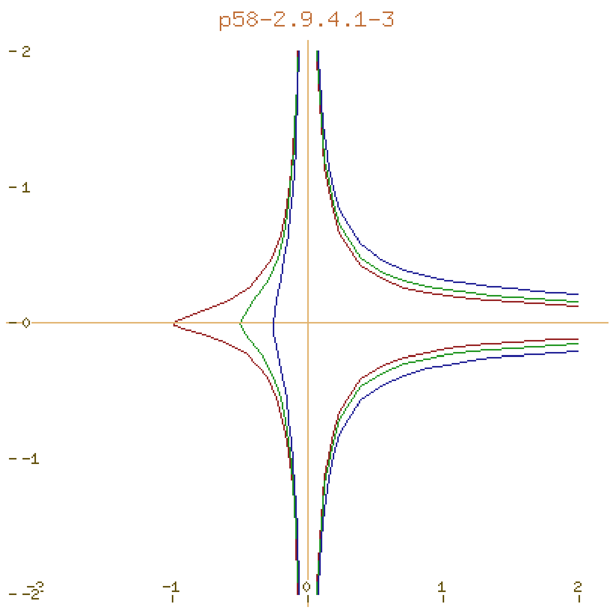
$$y = c(a + bx)^{1/2}/x$$

$$x^2y^2 - c^2bx - c^2a = 0$$

— **p58-2.9.4.1-3** —

```
)clear all
f1(x,y) == x^2*y^2 - x/50 - 1/50

lineColorDefault(red())
viewport1:=draw(f1(x,y)=0,x,y,range==[-2.0..2.0,-2.0..2.0],adaptive==true,_
                unit==[1.0,1.0],title=="p58-2.9.4.1-3")
graph1:=getGraph(viewport1,1)

f2(x,y) == x^2*y^2 - x/25 - 1/50

lineColorDefault(green())
viewport2:=draw(f2(x,y)=0,x,y,range==[-2.0..2.0,-2.0..2.0],adaptive==true,_
                unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == x^2*y^2 - 2*x/25 -1/50

lineColorDefault(blue())
viewport3:=draw(f3(x,y)=0,x,y,range==[-2.0..2.0,-2.0..2.0],adaptive==true,_
                unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
```

```
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 58 2.9.5

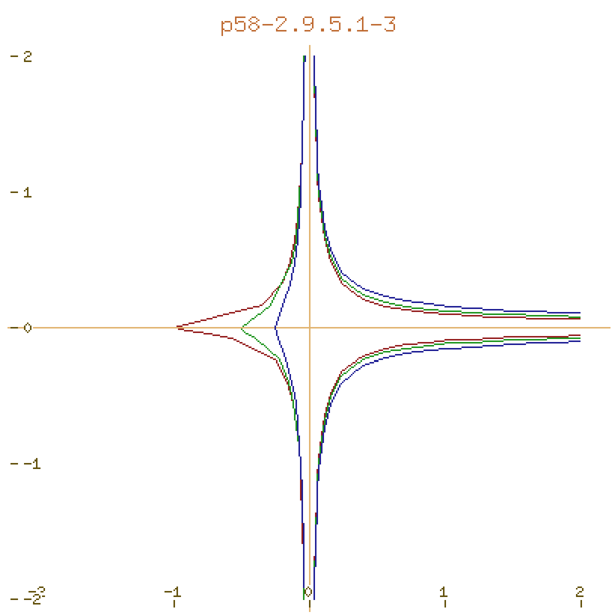
$$y = c(a + bx)^{1/2}/x^2$$

$$x^4y^2 - c^2bx - c^2a = 0$$

— **p58-2.9.5.1-3** —

```
)clear all
f1(x,y) == x^2*y^2 - x/200 - 1/200

lineColorDefault(red())
viewport1:=draw(f1(x,y)=0,x,y,range==[-2.0..2.0,-2.0..2.0],adaptive==true,_
              unit==[1.0,1.0],title=="p58-2.9.5.1-3")
graph1:=getGraph(viewport1,1)

f2(x,y) == x^2*y^2 - x/100 - 1/200
```

```
lineColorDefault(green())
viewport2:=draw(f2(x,y)=0,x,y,range==[-2.0..2.0,-2.0..2.0],adaptive==true,_
                unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == x^2*y^2 - x/50 - 1/200

lineColorDefault(blue())
viewport3:=draw(f3(x,y)=0,x,y,range==[-2.0..2.0,-2.0..2.0],adaptive==true,_
                unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 58 2.9.6

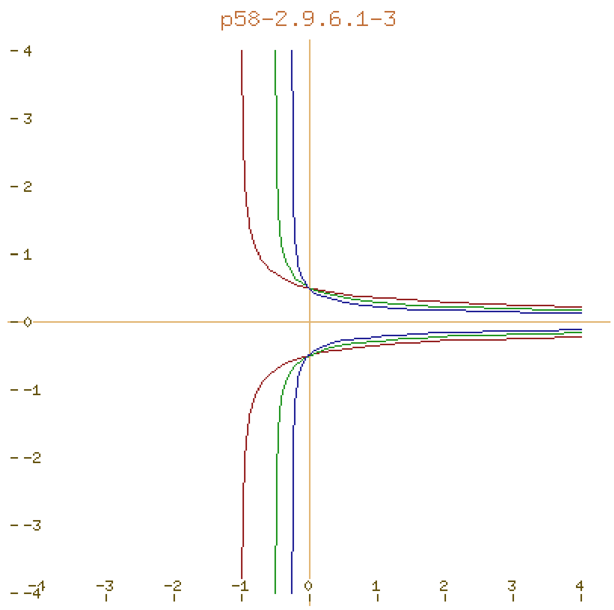
$$y = c/(a + bx)^{1/2}$$

$$ay^2 + bxy^2 - c^2 = 0$$

— p58-2.9.6.1-3 —

```
)clear all
f1(x,y) == (x + 1)*y^2 - 1/4

lineColorDefault(red())
viewport1:=draw(f1(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0],title=="p58-2.9.6.1-3")
graph1:=getGraph(viewport1,1)

f2(x,y) == (2*x+1)*y^2 - 1/4

lineColorDefault(green())
viewport2:=draw(f2(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == (4*x+1)*y^2 - 1/4

lineColorDefault(blue())
viewport3:=draw(f3(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———



Page 60 2.9.7

singular pts in region of sketch

$$y = \frac{cx}{(a+bx)^{1/2}}$$

$$ay^2 + bxy^2 - c^2x^2 = 0$$

— **p60-2.9.7.1-3** —

```
)clear all
f1(x,y) == (x + 1)*y^2 - x^2

lineColorDefault(red())
viewport1:=draw(f1(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0],title=="p60-2.9.7.1-3")
graph1:=getGraph(viewport1,1)

f2(x,y) == (2*x+1)*y^2 - x^2

lineColorDefault(green())
viewport2:=draw(f2(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == (4*x+1)*y^2 - x^2
```

```
lineColorDefault(blue())
viewport3:=draw(f3(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-

Page 60 2.9.8

singular pts in region of sketch

$$y = \frac{cx^2}{(a+bx)^{1/2}}$$

$$ay^2 + bxy^2 - c^2x^4 = 0$$

— **p60-2.9.8.1-3** —

```
)clear all
f1(x,y) == (x + 1)*y^2 - x^4

lineColorDefault(red())
viewport1:=draw(f1(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0],title=="p60-2.9.8.1-3")
graph1:=getGraph(viewport1,1)

f2(x,y) == (2*x+1)*y^2 - x^4

lineColorDefault(green())
viewport2:=draw(f2(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == (4*x+1)*y^2 - x^4

lineColorDefault(blue())
viewport3:=draw(f3(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)
```

```
putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

———-

Page 60 2.9.9

$$y = \frac{c}{x(a+bx)^{1/2}}$$

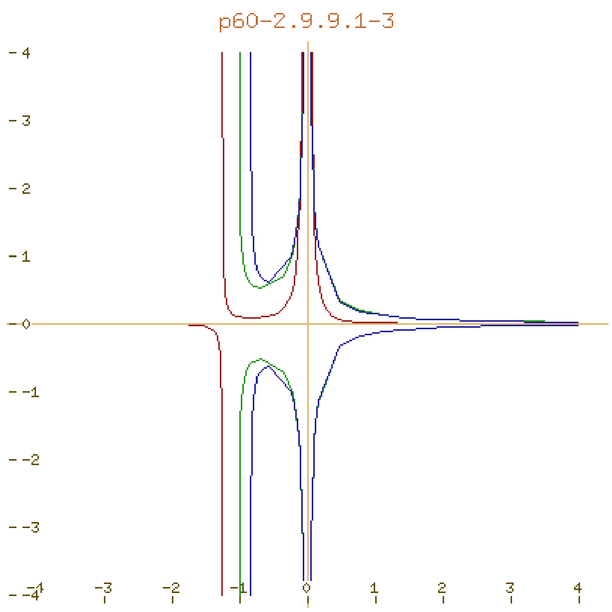
$$ax^2y^2 + bx^3y^2 - c^2 = 0$$

— p60-2.9.9.1-3 —

```
)clear all
f1(x,y) == (4/5*x^3 + x^2)*y*2 - 1/25

lineColorDefault(red())
viewport1:=draw(f1(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0],title=="p60-2.9.9.1-3")
graph1:=getGraph(viewport1,1)

f2(x,y) == (x^3 + x^2)*y^2 - 1/25

lineColorDefault(green())
viewport2:=draw(f2(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == (6/5*x^3 + x^2)*y^2 - 1/25

lineColorDefault(blue())
viewport3:=draw(f3(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
```

```
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

——



Page 60 2.9.10

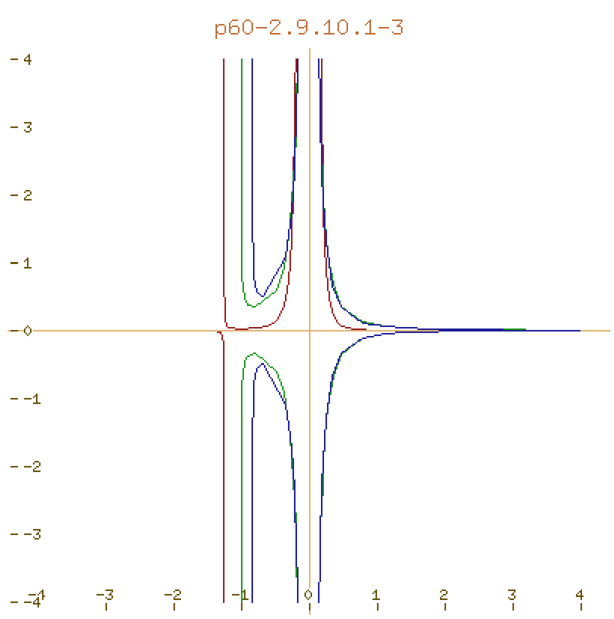
$$y = \frac{c}{x^2(a+bx)^{1/2}}$$

$$ax^4y^2 + bx^5y^2 - c^2 = 0$$

— **p60-2.9.10.1-3** —

```
)clear all
f1(x,y) == (4/5*x^5 + x^4)*y*2 - 1/100

lineColorDefault(red())
viewport1:=draw(f1(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0],title=="p60-2.9.10.1-3")
graph1:=getGraph(viewport1,1)

f2(x,y) == (x^5 + x^4)*y^2 - 1/100
```

```
lineColorDefault(green())
viewport2:=draw(f2(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == (6/5*x^5 + x^4)*y^2 - 1/100

lineColorDefault(blue())
viewport3:=draw(f3(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```



Page 60 2.9.11

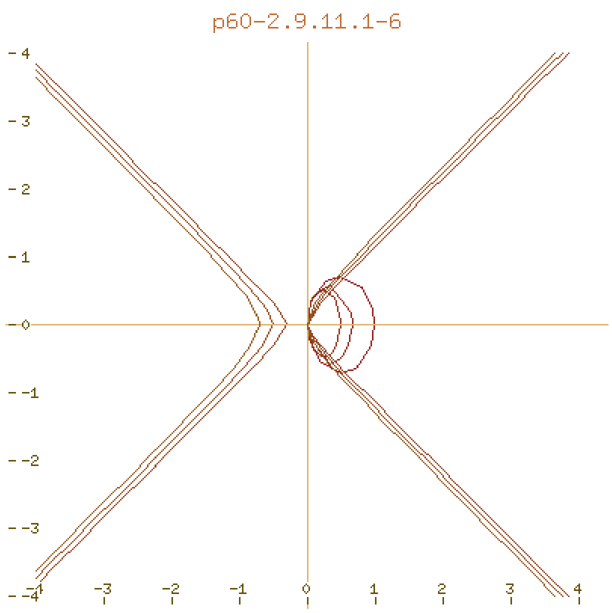
$$y = \frac{cx^{1/2}}{(a+bx)^{1/2}}$$

$$y^2 - ac^2x - bc^2x^2 = 0$$

— p60-2.9.11.1-6 —

```
)clear all
f1(x,y) == y^2 + 2*x^2 - 2*x

lineColorDefault(color(1))
viewport1:=draw(f1(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
               unit==[1.0,1.0],title=="p60-2.9.11.1-6")
graph1:=getGraph(viewport1,1)

f2(x,y) == y^2 + 3*x^2 - 2*x

lineColorDefault(color(2))
viewport2:=draw(f2(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
               unit==[1.0,1.0])
graph2:=getGraph(viewport2,1)

f3(x,y) == y^2 + 4*x^2 - 2*x

lineColorDefault(color(3))
viewport3:=draw(f3(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
               unit==[1.0,1.0])
graph3:=getGraph(viewport3,1)

f4(x,y) == y^2 - x^2 - 3*x/10

lineColorDefault(color(4))
viewport4:=draw(f4(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
               unit==[1.0,1.0])
graph4:=getGraph(viewport4,1)

f5(x,y) == y^2 - x^2 - x/2

lineColorDefault(color(5))
viewport5:=draw(f5(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
               unit==[1.0,1.0])
graph5:=getGraph(viewport5,1)

f6(x,y) == y^2 - x^2 -7*x/10

lineColorDefault(color(6))
```

```
viewport6:=draw(f6(x,y)=0,x,y,range==[-4.0..4.0,-4.0..4.0],adaptive==true,_
                unit==[1.0,1.0])
graph6:=getGraph(viewport6,1)

putGraph(viewport1,graph2,2)
putGraph(viewport1,graph3,3)
putGraph(viewport1,graph4,4)
putGraph(viewport1,graph5,5)
putGraph(viewport1,graph6,6)
units(viewport1,1,"on")
points(viewport1,1,"off")
points(viewport1,2,"off")
points(viewport1,3,"off")
makeViewport2D(viewport1)
```

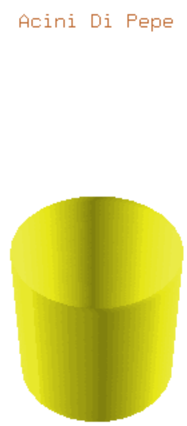


Chapter 5

Pasta by Design[4]

This is a book that combines a taxonomy of pasta shapes with the Mathematica equations that realize those shapes in three dimensions. We implemented examples from this book as a graphics test suite for Axiom.

5.1 Acini Di Pepe

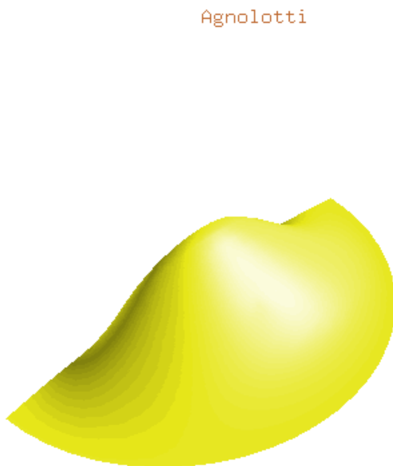


— **Acini Di Pepe** —

```
X(i,j) == 15*cos(i/60*%pi)
Y(i,j) == 15*sin(i/60*%pi)
Z(i,y) == j
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..120,j=0..30,_
          style=="smooth",title=="Acini Di Pepe",colorFunction==cf)
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
```

The smallest member of the *postine minute* (tiny pasta) family, *acini de pepe* (peppercorns) are most suited to consommes (clear soups), with the occasional addition of croutons and diced greens. Made of durum wheat flour and eggs, acini di pepe are commonly used in the Italian-American “wedding soup”, a broth of vegetables and meat.

5.2 Agnolotti



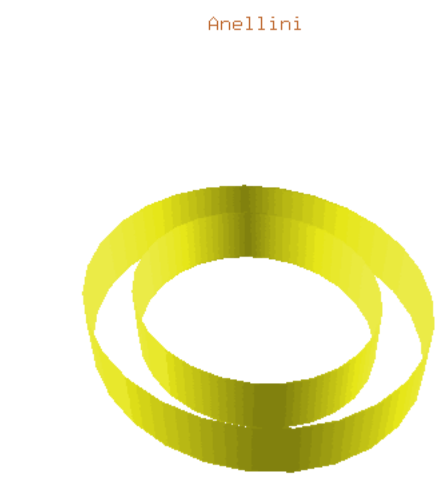
— **Agnolotti** —

```
X(i,j) == (10*sin((i/120)*%pi)^(0.5) + _
            (1/400)*sin(((3*j)/10)*%pi)) * _
           cos(((19*j)/2000)*%pi+0.03*%pi)
Y(i,j) == (10*sin((i/120)*%pi) + _
            (1/400)*cos(((30*j)/10)*%pi)) * _
           sin(((19*j)/2000)*%pi+0.03*%pi)
Z(i,j) == 5*cos((i/120)*%pi)^5 * sin((j/100)*%pi) - _
           5*sin((j/100)*%pi) * cos((i/120)*%pi)^200
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..60,j=0..100,_
           style=="smooth",title=="Agnolotti",colorFunction==cf)
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,0.6,0.6,0.6)
```

———-

These shell-like *ravioli* from Piedmont, northern italy, are fashioned from small pieces of flattened dough made of wheat flour and egg, and are often filled with braised veal, port, vegetables or cheese. The true agnolotto should feature a crinkled edge, cut using a fluted pasta wheel. Recommended with melted butter and sage

5.3 Anellini

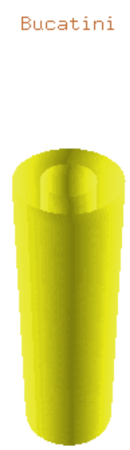


— **Anellini** —

```
X(i,j) == cos(0.01*i*%pi)
Y(i,j) == 1.1*sin(0.01*i*%pi)
Z(i,j) == 0.05*j
canvas := createThreeSpace()
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
makeObject(surface(X(i,j),Y(i,j),Z(i,j)),i=0..200,j=0..8,space==canvas,_
          colorFunction==cf,style=="smooth")
makeObject(surface(X(i,j)/1.4,Y(i,j)/1.4,Z(i,j)),i=0..200,j=0..8,_
          space==canvas,colorFunction==cf,style=="smooth")
vp:=makeViewport3D(canvas,style=="smooth",title=="Anellini")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
zoom(vp,1.1,1.1,1.1)
```

The diminutive *onellini* (small rings) are part of the extended *postine minute* (tiny pasta) clan. Their thickness varies between only 1.15 and 1.20 mm, and the are therefore usually found in light soups together with croutons and thinly sliced vegetables. This pasta may also be found served in a *timballo* (baked pasta dish).

5.4 Bucatini



— **Bucatini** —

```
X(i,j) == 0.3*cos(i/30*%pi)
Y(i,j) == 0.3*sin(i/30*%pi)
Z(i,j) == j/45
canvas := createThreeSpace()
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
makeObject(surface(X(i,j),Y(i,j),Z(i,j)),i=0..60,j=0..90,space==canvas,_
           colorFunction==cf,style=="smooth")
makeObject(surface(X(i,j)/2,Y(i,j)/2,Z(i,j)),i=0..60,j=0..90,space==canvas,_
           colorFunction==cf,style=="smooth")
vp:=makeViewport3D(canvas,style=="smooth",title=="Bucatini")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
zoom(vp,2.0,2.0,2.0)
```

Bucatini (pierced) pasta is commonly served as a *pastasciutta* (pasta boiled, drained, and dished up with a sauce, rather than in broth). Its best known accompaniment is *amatriciana*: a hearty traditional sauce made with dried port, Pecorino Romano and tomato sauce, and named after the medieval town of Amatrice in central Italy.

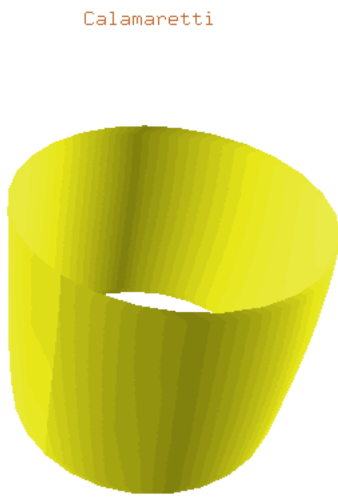
5.5 Buccoli

— **Buccoli** —

```
X(i,j) == (0.7 + 0.2*sin(21*j/250 * %pi))*cos(i/20*%pi)
Y(i,j) == (0.7 + 0.2*sin(21*j/250 * %pi))*sin(i/20*%pi)
Z(i,j) == 39.0*i/1000. + 1.5*sin(j/50*%pi)
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..200,j=0..25,_
          style=="smooth",title=="Buccoli",colorFunction==cf)
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

A spiral-shaped example from the *pasta corta* (short pasta) family, and of rather uncertain pedigree, *buccoli* are suitable in a mushroom and sausage dish. They are also excellent with a tomato aubergine, pesto, and ricotta salad.

5.6 Calamaretti

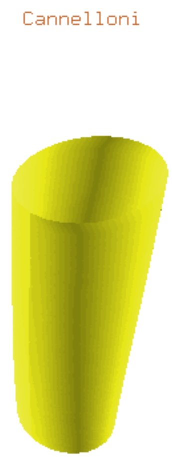


— Calamaretti —

```
X(i,j) == cos(i/75*%pi) + 0.1*cos(j/40*%pi) + 0.1*cos(i/75*%pi + j/40*%pi)
Y(i,j) == 1.2*sin(i/75*%pi) + 0.2*sin(j/40*%pi)
Z(i,j) == j/10
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..20,_
          style=="smooth",title=="Calamaretti",colorFunction==cf)
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,1.1,1.1,1.1)
```

Literally "little squids", *calamaretti* are small ring-shaped pasta cooked as *pastasciutta* (pasta boiled and drained) then dished up with a tomato-, egg-, or cheese-based sauce. Their shape means that *calamaretti* hold both chunky and thin sauces equally well. Fittingly, they are often served with seafood.

5.7 Cannelloni



— Cannelloni —

```
X(i,j) == (1+j/100)*cos(i*%pi/55) + 0.5*cos(j*%pi/100) + _
          0.1*cos(i*%pi/55+j*%pi/125)
Y(i,j) == 1.3*sin(i*%pi/55) + 0.3*sin(j*%pi/100)
Z(i,j) == 7.*j/50.
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..110,j=0..50,_
          style=="smooth",title=="Cannelloni",colorFunction==cf)
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

Made with wheat flour, eggs, and olive oil, *cannelloni* (big tubes) originate as strips of pasta shaped into perfect cylinders, which can be stuffed with meat, vegetables, or ricotta. The stuffed *cannelloni* are covered with a creamy besciamella sauce, a sprinkling of Parmigiano-Reggiano cheese and then oven-baked.

5.8 Cannolicchi Rigati

— Cannolicchi Rigati —

```
X(i,j) == 8*cos(i*%pi/70) + 0.2*cos(2*i*%pi/7) + 5*cos(j*%pi/100)
Y(i,j) == 8*sin(i*%pi/70) + 0.2*sin(2*i*%pi/7) + 4*sin(j*%pi/100)
Z(i,j) == 6.0*j/5.0
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..140,j=0..50,_
          style=="smooth",title=="Cannolicchi Rigati",colorFunction==cf)
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
rotate(v3d,2,7)
zoom(v3d,3.0,3.0,3.0)
```

Known as “little tubes”, *cannolicchi* exist both in a *rigati* (grooved) and *lisci* (smooth) form. These hollow *pasta corta* (short pasta) come in various diameters and are often served with seafood. *Cannolicchi* hail from Campania in southern Italy.

5.9 Capellini

— **Capellini** —

```
X(i,j) == 0.05*cos(2*i*%pi/15) + 0.6*cos(j*%pi/100)
Y(i,j) == 0.05*sin(2*i*%pi/15) + 0.5*sin(j*%pi/100)
Z(i,j) == 7.0*j/100.0
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..15,j=0..100,_
          style=="smooth",title=="Capellini",colorFunction==cf)
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

An extra-fine rod-like pasta *capellini* (thin hair) may be served in a light broth, but also combine perfectly with butter, nutmeg, or lemon. This variety (or its even more slender relative, *capelli d'angelo* (angel hair) is sometimes used to form the basis of an unusual sweet pasta dish, made with lemons and almonds, called *torta ricciolina*.

5.10 Cappelletti

— **Cappelletti** —

```
X(i,j) == (0.1 + sin(((3*i)/160)*%pi)) * cos(((2.3*j)/120)*%pi)
Y(i,j) == (0.1 + sin(((3*i)/160)*%pi)) * sin(((2.3*j)/120)*%pi)
Z(i,j) == 0.1 + (1/400.)*j + (0.3 - 0.231*(i/40.)) * cos((i/20.)*%pi)
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..40,j=0..120,_
          style=="smooth",title=="Cappelletti",colorFunction==cf)
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,1.1,1.1,1.1)
```

This pasta is customarily served as the first course of a traditional north Italian Christmas meal, dished up in a chicken brodo (broth). Typically, it is the children of a houseold who prepare the cappelletti (little hats) on Christmas Eve, filling the pasta parcels (made from wheat flour and fresh eggs) with mixed meats or soft cheeses, such as ricotta.

5.11 Casarecce

— **Casarecce** —

```
X(i,j) == _
  if (i <= 30)_
   then 0.5*cos(j*%pi/30)+0.5*cos((2*i+j+16)/40*%pi) _
   else cos(j*%pi/40)+0.5*cos(j*%pi/30)+0.5*sin((2*i-j)/40*%pi)
Y(i,j) == _
  if (i <= 30)_
   then 0.5*sin(j*%pi/30)+0.5*sin((2*i+j+16)/40*%pi) _
   else sin(j*%pi/40)+0.5*sin(j*%pi/30)+0.5*cos((2*i-j)/40*%pi)
Z(i,j) == j/4.0
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..60,j=0..60,_
          style=="smooth",title=="Casarecce",colorFunction==cf)
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

Easily identified by their unique s-shaped cross-section *casarecce* (home-made) are best cooked as *postasciutta* (pasta boiled, drained and dished up with a sauce). Often *casarecce* are served with a classic *ragu* and topped with a sprinkle of pepper and Parmigiano-Reggiano cheese.

5.12 Castellane

— **Castellane** —

```
X(i,j) == ((0.3*sin(j*%pi/120)*abs(cos((j+3)*%pi/6)) + _
           i^2/720.*(sin(2*j*%pi/300)^2+0.1) + 0.3)) * cos(7*i*%pi/150)
Y(i,j) == ((0.3*sin(j*%pi/120)*abs(cos((j+3)*%pi/6)) + _
           i^2/720.*(sin(2*j*%pi/300)^2+0.1) + 0.3)) * sin(7*i*%pi/150)
Z(i,j) == 12*cos(j*%pi/120)
cf(x:DFLOAT,y:DFLOAT):DFLOAT == 1.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..60,j=0..120,_
          style=="smooth",title=="Castellane",colorFunction==cf)
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
viewpoint(v3d,0.0,0.0,45.0)
viewpoint(v3d,5,5,0)
zoom(v3d,1.5,1.5,1.5)
```

The manufacturer Bailla has recently created this elegant pasta shape. Accordking to its maker, they were originally called *paguri* (hermit crabs) but renamed *castellane* (castle dwellers). The sturdy form and rich nutty taste of *castellane* stand up to hearty meals and full-flavoured sauces.

5.13 Cavatappi

— **Cavatappi** —

```
X(i,j) == (3+2*cos(i*%pi/35)+0.1*cos(2*i*%pi/7))*cos(j*%pi/30)
Y(i,j) == (3+2*cos(i*%pi/35)+0.1*cos(2*i*%pi/7))*sin(j*%pi/30)
Z(i,j) == 3+2*sin(i*%pi/35)+0.1*sin(2*i*%pi/7)+j/6.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..70,j=0..150,_
          style=="smooth",title=="Cavatappi")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

Perfect with chunky sauces made from lamb or pork, *cavatappi* (corkscrews) are 36 mm-long, hollow helicoidal tubes. As well as an accompaniment to creamy sauces, such as *boscaiola* (woodsman's) sauce, they are also often used in oven-baked cheese-topped dishes, or in salads with pesto.

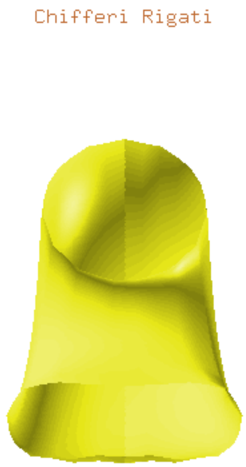
5.14 Cavatelli

— **Cavatelli** —

```
A(i) == 0.5*cos(i*%pi/100)
B(i,j) == j/60.*sin(i*%pi/100)
X(i,j) == 3*(1-sin(A(i)*2*%pi))*cos(A(i)*%pi+0.9*%pi)
Y(i,j) == 3*sin(A(i)*2*%pi)*sin(A(i)*%pi+0.63*%pi)
Z(i,j) == 4*B(i,j)*(5-sin(A(i)*%pi))
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..200,j=0..30,_
          style=="smooth",title=="Cavatelli")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
rotate(v3d,5,50)
zoom(v3d,1.5,1.5,1.5)
```

Popular in the south of Italy, and related in shape to the longer twisted *casareccia*, *cavatelli* can be served *alla puttanesca* (with a sauce containing chilli, garlic, capers, and anchovies). They can also be added to a salad with olive oil, sauteed crushed garlic and a dusting of soft cheese.

5.15 Chifferi Rigati



— **Chifferi Rigati** —

```
X(i,j) == (0.45+0.3*cos(i*%pi/100)+0.005*cos(2*i*%pi/5)) * cos(j*%pi/45) + _
          0.15*(j/45.0)^10*cos(i*%pi/100)^3
Y(i,j) == (0.35+j/300.0)*sin(i*%pi/100) + 0.005*sin(2*i*%pi/5)
Z(i,j) == (0.4+0.3*cos(i*%pi/100))*sin(j*%pi/45)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..200,j=0..45,_
          style=="smooth",title=="Chifferi Rigati")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
rotate(v3d,90,180)
```

This pasta - available in both *rigoti* (grooved) and *lisci* (smooth) forms - is typically cooked in broth or served in *rogu alla bolognese*, though *chifferi rigati* also make an excellent addition to salads with carrot, red pepper and courgette. *Chifferi rigoti* bear a resemblance to, and the term is a transliteration of, the Austrian 'kipfel' sweet.

5.16 Colonne Pompeii

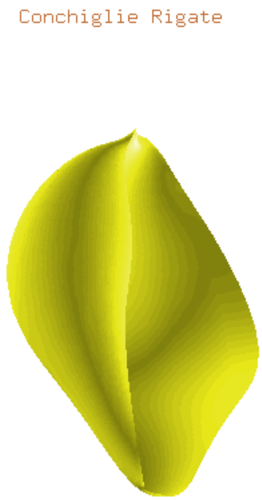
— Colonne Pompeii —

```
X0(i,j) == _
  if (j <= 50) _
   then 2*cos(i*%pi/20) _
   else 2*cos(i*%pi/20)*cos(-j*%pi/25)
Y0(i,j) == _
  if (j <= 50) _
   then 0.0 _
   else 2*cos(i*%pi/20)*sin(-j*%pi/25) + 3*sin((j-50)*%pi/200)
Z0(i,j) == _
  if (j <= 50) _
   then sin(i*%pi/20)+12 _
   else sin(i*%pi/20)+6.0*j/25.0
X1(i,j) == _
  if (j <= 200) _
   then 2*cos(i*%pi/20)*cos(-j*%pi/25+2*%pi/3) _
   else 2*cos(i*%pi/20)*sin(-28*%pi/3)
Y1(i,j) == _
  if (j <= 200) _
   then 2*cos(i*%pi/20)*sin(-j*%pi/20 + 2*%pi/3) + 3*sin(j*%pi/200) _
   else 2*cos(i*%pi/20)*sin(-28*%pi/3)
Z1(i,j) == _
  if (j <= 200) _
   then 12+sin(i*%pi/20)+6.0*j/25.0 _
   else sin(i*%pi/20)+60
X2(i,j) == _
  if (j <= 200) _
```

```
   then 2*cos(i*%pi/20)*cos(-j*%pi/25+4*%pi/3) _
   else 2*cos(i*%pi/20)*sin(-28*%pi/3)
Y2(i,j) == _
  if (j <= 200) _
   then 2*cos(i*%pi/200)*sin(-j*%pi/25+4*%pi/3)+3*sin(j*%pi/200) _
   else 2*cos(i*%pi/20)*sin(-28*%pi/3)
vsp:=createThreeSpace()
makeObject(surface(X0(i,j),Y0(i,j),Z0(i,j)),i=0..10,j=0..250,space==vsp)
makeObject(surface(X1(i,j),Y1(i,j),Z1(i,j)),i=0..10,j=0..250,space==vsp)
makeObject(surface(X2(i,j),Y2(i,j),Z1(i,j)),i=0..10,j=0..250,space==vsp)
vp:=makeViewport3D(vsp,title=="Colonne Pompeii",style=="smooth")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
zoom(vp,3.0,3.0,3.0)
```

This ornate pasta (originally from Campania, southern Italy) is similar in shape to *fusilloni* (a large *fusilli*) but is substantially longer. *Colonne Pompeii* (columns of Pompeii) are best served with a seasoning of fresh basil, pine nuts, finely sliced garlic and olive oil, topped with a sprinkling of freshly grated Parmigiano-Reggiano.

5.17 Conchiglie Rigate



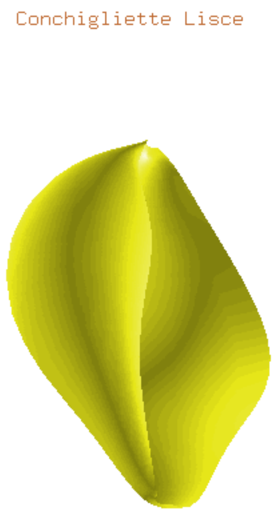
— **Conchiglie Rigate** —

```
A(i,j) == 0.25*sin(j*%pi/250)*cos((6*j+25)/25*%pi)
B(i,j) == ((40.0-i)/40.0)*(0.3+sin(j*%pi/250))*%pi
C(i,j) == 2.5*cos(j*%pi/125)+2*sin((40-i)*%pi/80)^10 * _
          sin(j*%pi/250)^10*sin(j*%pi/125+1.5*%pi)
X(i,j) == A(i,j)+cos(j*%pi/125)+(5+30*sin(j*%pi/250))*sin(B(i,j)) * _
          sin(i/40*(0.1*(1.1+sin(j*%pi/250)^5))*%pi)
Y(i,j) == A(i,j)+(5+30*sin(j*%pi/250))*cos(B(i,j)) * _
          sin(i/40*(0.1*(1.1+sin(j*%pi/250)^5))*%pi) + C(i,j)
Z(i,j) == 25.0*cos(j*%pi/250)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..40,j=0..250,_
          style=="smooth",title=="Conchiglie Rigate")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
rotate(v3d,45,45)
zoom(v3d,1.5,1.5,1.5)
```

———-

Shaped like their namesake, *conchiglie* (shells) exist in both *rigate* (grooved) and *lisce* (smooth) forms. Suited to light tomato sauces, ricotta cheese or *pesto genovese*, *conchiglie* hold flavourings in their grooves and cunningly designed shell. Smaller versions are used in soups, while larger shells are more commonly served with a sauce.

5.18 Conchigliette Lisce



— **Conchigliette Lisce** —

```
A(i,j) == (60.0-i)/60.0*(0.5+sin(j*%pi/60))*%pi
B(i,j) == i/60.0*(0.1*(1.1+sin(j*%pi/60)^5))*%pi
C(i,j) == 2.5*cos(j*%pi/30)+2*sin((60-i)*%pi/120)^10 * _
          sin(j*%pi/60)^10*sin((j+45)*%pi/30)
X(i,j) == (5+30*sin(j*%pi/60))*sin(A(i,j))*sin(B(i,j))+cos(j*%pi/30)
Y(i,j) == (5+30*sin(j*%pi/60))*cos(A(i,j))*sin(B(i,j))+C(i,j)
Z(i,j) == 25*cos(j*%pi/60)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..60,j=0..60,_
          style=="smooth",title=="Conchigliette Lisce")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
rotate(v3d,45,45)
zoom(v3d,1.5,1.5,1.5)
```

———-

Typically found in central and southern Italy (notably Campania), *conchigliette lisce* (small smooth shells) can be served in soups such as *minestrone*. Alternatively, these shells can accompany a meat- or vegetable-based sauce.

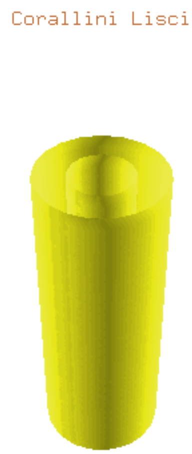
5.19 Conchiglioni Rigate

— Conchiglioni Rigate —

```
A(i,j) == 0.25*sin(j*%pi/200)*cos((j+4)*%pi/4)
B(i,j) == i/40.0*(0.1+0.1*sin(j*%pi/200)^6)*%pi
C(i,j) == 2.5*cos(j*%pi/100)+3*sin((40-i)*%pi/80)^10 * _
          sin(j*%pi/200)^10*sin((j-150)*%pi/100)
X(i,j) == A(i,j)+(10+30*sin(j*%pi/200)) * _
          sin((40.0-i)/40*(0.3+sin(j*%pi/200)^3)*%pi) * _
          sin(B(i,j))+cos(j*%pi/100)
Y(i,j) == A(i,j)+(10+30*sin(j*%pi/200)) * _
          cos((40.0-i)/40*(0.3+sin(j*%pi/200)^3)*%pi) * _
          sin(B(i,j))+C(i,j)
Z(i,j) == 30.0*cos(j*%pi/200)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..40,j=0..200,_
          style=="smooth",title=="Conchiglioni Rigati")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
rotate(v3d,45,45)
zoom(v3d,1.5,1.5,1.5)
```

The shape of *conchiglioni rigate* (large ribbed shells) is ideal for holding sauces and fillings (either fish or meat based) and the pasta can be baked in the oven, or placed under a grill and cooked as a gratin. *Conchiglioni rigati* are often served in the Italian-American dish *pasta primavera* (pasta in spring sauce) alongside crisp spring vegetables.

5.20 Corallini Lisci



— Corallini Lisci —

```
X(i,j) == 0.8*cos(i*%pi/50)
Y(i,j) == 0.8*sin(i*%pi/50)
Z(i,j) == 3.0*j/50.0
vsp:=createThreeSpace()
makeObject(surface(X(i,j),Y(i,j),Z(i,j)),i=0..140,j=0..70,space==vsp)
makeObject(surface(X(i,j)/2,Y(i,j)/2,Z(i,j)),i=0..140,j=0..70,space==vsp)
vp:=makeViewport3D(vsp,style=="smooth",title=="Corallini Lisci")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
zoom(vp,2.0,2.0,2.0)
```

Members of the *postine minute* (tiny pasta) group, *corallini lisci* (small smooth coral) are so called because their pierced appearance resembles the coral beads worn as jewelry in Italy. Their small size (no larger than 3.5 mm in diameter) means that *corallini* are best cooked in broths, such as Tuscan white bean soup.

5.21 Creste Di Galli

— Creste Di Galli —

```
A(i)   == ((1+sin((1.5+i)*%pi))/2)^5
B(i,j) == 0.3*sin(A(i/140.)*%pi+0.5*%pi)^1000*cos(j*%pi/70.0)
C(i,j) == 0.3*cos(A(i/140.)*%pi)^1000*sin(j*%pi/70.0)
X(i,j) == (0.5+0.3*cos(A(i/140.)*2*%pi))*cos(j*%pi/70.0) + _
          0.15*(j/70.0)^10*cos(A(i/140.0)*2*%pi)^3 + B(i,j)
Y(i,j) == 0.35*sin(A(i/140.0)*2*%pi) + 0.15*j/70.0*sin(A(i/140.0)*2*%pi)
Z(i,j) == (0.4+0.3*cos(A(i/140.0)*2*%pi))*sin(j*%pi/70.0) + C(i,j)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..140,j=0..70,_
          style=="smooth",title=="Creste Di Galli")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,1.5,1.5,1.5)
```

Part of the *pasta ripiena* (filled pasta) family, *creste di galli* (coxcombs) are identical to *galletti* except for the crest, which is smooth rather than crimped. They may be stuffed, cooked and served in a simple *marinara* (mariner's) sauce, which contains tomato, garlic, and basil.

5.22 Couretti

— **Couretti** —

```
X(i,j) == 2*cos(i*%pi/150)-cos(i*%pi/75)-sin(i*%pi/300)^150 - _
          (abs(cos(i*%pi/300)))^5
Y(i,j) == 2*sin(i*%pi/150)-sin(i*%pi/75)
Z(i,j) == j/10.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..300,j=0..10,_
          style=="smooth",title=="Couretti")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
```

A romantically shaped scion of the *postine minute* (tiny pasta) clan, *cuoretti* (tiny hearts) are minuscule. In fact, along with *acini di pepe*, they are one of the smallest forms of pasta. Like all *postine* they may be served in soup, such as cream of chicken.

5.23 Ditali Rigati

— **Ditali Rigati** —

```
X(i,j) ==       cos(i*%pi/100) + 0.03*cos((7*i)*%pi/40) + 0.25*cos(j*%pi/50)
Y(i,j) == 1.1 * sin(i*%pi/100) + 0.03*sin((7*i)*%pi/40) + 0.25*sin(j*%pi/50)
Z(i,j) == j/10.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..200,j=0..25,_
          style=="smooth",title=="Ditali Rigati")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,1.5,1.5,1.5)
```

Another speciality of the Campania region of southern Italy, *ditali rigati* (grooved thimbles) are compact and typically less than 10 mm long. Like other *pastine*, they are usually found in soups such as *pasta e patate*. Their stocky shape makes them a sustaining winter snack, as well as an excellent addition to salads.

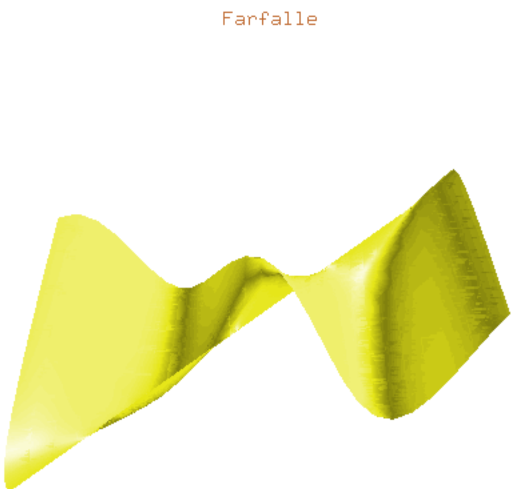
5.24 Fagottini

— **Fagottini** —

```
A(i,j) == (0.8 + sin(i/100*%pi)^8 - 0.8 * cos(i/25*%pi))^1.5 + _
           0.2 + 0.2 * sin(1/100*%pi)
B(i,j) == (0.9 + cos(i/100*%pi)^8 - 0.9 * cos(i/25*%pi + 0.03*%pi))^1.5 + _
           0.3 * cos(i/100*%pi)
C(i,j) == 4 - ((4*j)/500)*(1+cos(i/100*%pi)^8 - 0.8*cos(i/25*%pi))^1.5
X(i,j) == cos(i/100*%pi) * _
          (A(i,j) * sin(j/100*%pi)^8 + _
           0.6 * (2 + sin(i/100*%pi)^2) * sin(j/50*%pi)^2)
Y(i,j) == sin(i/100*%pi) * _
          (B(i,j) * sin(j/100*%pi)^8 + _
           0.6 * (2 + cos(i/100*%pi)^2) * sin(j/50*%pi)^2)
Z(i,j) == (1 + sin(j/100*%pi - 0.5*%pi)) * _
           (C(i,j) * ((4*j)/500) * _
            (1 + sin(i/100*%pi)^8 - 0.8 * cos(i/25*%pi))^1.5)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..200,j=0..50,_
          style=="smooth",title=="Fagottini")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
```

A notable member of the pasta ripiena (filled pasta) family, fagottini (little purses) are made from circles of durum wheat dough. A spoonful of ricotta, steamed vegetables or even stewed fruit is placed on the dough, and the corners are then pinched together to form a bundle. These packed dumplings are similar to ravioli, only larger.

5.25 Farfalle



— **Farfalle** —

```
A(i)   == sin((7*i+16)*%pi/40)
B(i,j) == (7.0*j/16.0)+4*sin(i*%pi/80)*sin((j-10)*%pi/120)
C(i,j) == 10*cos((i+80)*%pi/80)*sin((j+110)*%pi/100)^9
D(i,j) == (7.0*j/16.0)-4*sin(i*%pi/80)-A(i)*sin((10-j)*%pi/20)
-- E(i,j) was never defined. We guess at a likely function - close but wrong
E(i,j) == _
  if ((20 <= i) and (i <= 60)) _
   then 7*sin((i+40)*%pi/40)^3*sin((2*j*%pi)/10+1.1*%pi)^9 _
   else C(i,j)
F(i) == _
  if ((20 <= i) and (i <= 60)) _
   then 7*sin((i+40)*%pi/40)^3*sin((j+110)*%pi/100)^9 _
   else C(i,j)
X(i,j) == (3.0*i)/8.0+F(i)
Y(i,j) == _
  if ((10 <= j) and (j <= 70)) _
   then B(i,j)-4*sin(i*%pi/80)*sin((70-j)*%pi/120) _
   else if (j <= 10) _
         then D(i,j) _
         else E(i,j)
Z(i,j) == 3*sin((i+10)*%pi/20)*sin(j*%pi/80)^1.5
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..80,j=0..80,_
          style=="smooth",title=="Farfalle")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,0.5,0.5,0.5)
```

A mixture of durum-wheat flour, eggs, and water, *farfalle* (butterflies) come from the Emilia-Romagnaand Lombardy regions of northern Italy. They are best served in a rich *carbonara* sauce (made with cream, eggs, and bacon). Depending on se, *farfalle* might be accompanied by green peas and chicken or ham.

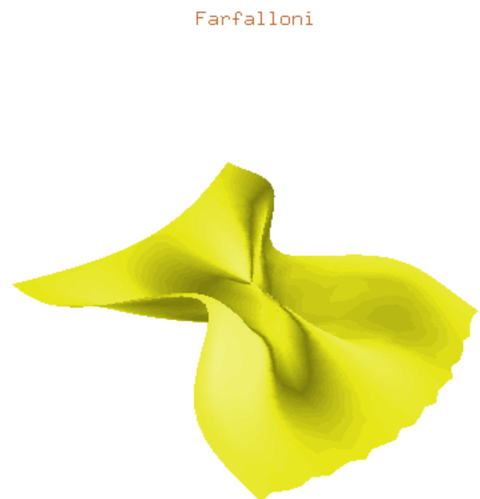
5.26 Farfalline

— **Farfalline** —

```
A(i) == 30*cos(i*%pi/125)+0.5*cos((6*i)*%pi/25)
B(i) == 30*sin(i*%pi/125)+0.5*sin((6*i)*%pi/25)
X(i,j) == cos(3*A(i)*%pi/100)
Y(i,j) == 0.5*sin((3*A(i))*%pi/100)*(1+sin(j*%pi/100)^10)
Z(i,j) == B(i)*j/500.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..250,j=0..50,_
          style=="smooth",title=="Farfalline")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
rotate(v3d,45,45)
zoom(v3d,1.5,1.5,1.5)
```

The small size of this well-known member of the *postine minute* (tiny pasta) lineage means that *farfalline* (tiny butterflies), are suitable for light soups, such as *pomodori e robiolo* (a mixture of tomato and soft cheese). A crimped pasta cutter and a central pinch create the iconic shape.

5.27 Farfalloni



— **Farfalloni** —

```
A(i,j) == 10*cos((i+70)*%pi/70)*sin((2*j)*%pi/175+1.1*%pi)^9
B(j)   == 0.3*sin((6-j)*%pi/7+0.4*%pi)
C(i,j) == _
  if ((17 <= i) and (i <= 52)) _
   then 7*sin((i+35)*%pi/35)^3*sin((2*j*%pi)/175+1.1*%pi)^9 _
   else A(i,j)
D(i,j) == (j/2.0)+4*sin(i*%pi/70)*sin((j-10)*%pi/100) - _
          4*sin(i*%pi/70)*sin((60-j)*%pi/100)
E(i,j) == (j/2.0)+4*sin(i*%pi/70)+0.3*sin((2*i+2.8)*%pi/7)*sin((j-60)*%pi/20)
F(i,j) == (j/2.0)-4*sin(i*%pi/70)-0.3*sin((2*i+2.8)*%pi/7)*sin((10-j)*%pi/20)
X(i,j) == (3.0*i)/7.0+C(i,j)
Y(i,j) == _
  if ((10 <= j) and (j <= 60)) _
   then D(i,j) _
   else if (j <= 10) _
         then F(i,j) _
         else E(i,j)
Z(i,j) == 3*sin((2*i+17.5)*%pi/35.)*sin(j*%pi/70)^1.5
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..70,j=0..70,_
          style=="smooth",title=="Farfalloni")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

Like *farfalle*, *farfalloni* (large butterflies) are well matched by a tomato- or butter-based sauce with peas and ham. They are also perfect with marrow vegetables such as roast courgette or pureed pumpkin, topped with Parmigiano-Reggiano and a sprinkling of *noce moscata* (nutmeg).

5.28 Festonati

— Festonati —

```
X(i,j) == 5*cos(i*%pi/50)+0.5*cos(i*%pi/50)*(1+sin(j*%pi/100)) + _
          0.5*cos((i+25)*%pi/25)*(1+sin(j*%pi/5))
Y(i,j) == 5*sin(i*%pi/50)+0.5*sin(i*%pi/50)*(1+sin(j*%pi/100)) + _
          0.5*cos(i**%pi/25)*(1+sin(j*%pi/5))
Z(i,j) == j/2.0+2*sin((3*i+25)*%pi/50)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..100,j=0..100,_
          style=="smooth",title=="Festonati")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

This smooth member of the *pasta corta* (short pasta) family is named after 'festoons' (decorative lengths of fabric with the rippled profile of a garland). *Festonati* can be served with grilled aubergine or home-grown tomatoes, topped with grated scamorza, fresh basil, olive oil, garlic, and red chilli flakes.

5.29 Fettuccine

— **Fettuccine** —

```
X(i,j) == 1.8*sin((4*i)*%pi/375)
Y(i,y) == 1.6*cos((6*i)*%pi/375)*sin((3*i)*%pi/750)
Z(i,j) == i/75.0 + j/20.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..10,_
          style=="smooth",title=="Fettuccine")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
```

This famous *pasta lunga* (long pasta) is made with durum-wheat flour, water, and in the case of *fettuccine alluovo*, eggs ideally within days of laying. *Fettuccine* (little ribbons) hail from the Lazio region. Popular in many dishes, they are an ideal accompaniment to *Alfredo* sauce, a rich mix of cream, parmesan, garlic, and parsley.

5.30 Fiocchi Rigati

— **Fiocchi Rigati** —

```
A(i,j) == 10*cos((i+80)*%pi/80)*sin((j+110)*%pi/100)^9
B(i,j) == 35.0*j/80.0+4*sin(i*%pi/80)*sin((j-10)*%pi/120)
X(i,j) == _
  if ((20 <= i) and (i <= 60)) _
   then 7*sin((i+40)*%pi/40)^3*sin((j+110)*%pi/100)^9 + 30.0*i/80.0 _
   else A(i,j) + 30.0*i/80.0
Y(i,j) == B(i,j)-4*sin(i*%pi/80)*sin((70-j)*%pi/120)
Z(i,j) == 3*sin((1+10)*%pi/20)*sin(j*%pi/80)^1.5-0.7*((sin(3*j*%pi/8)+1)/2)^4
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..80,j=0..80,_
           style=="smooth",title=="Fiocchi Rigati")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

————

A distant relative of the *farfalle* family, *fiocchi rigati* (grooved flakes) are smaller than either *farfalloni* or *farfalle*, but larger than *farfalline*. Their corrugated surface collects more sauce than a typical *farfalle*. For a more unusual disk, *fiocchi rigati* can be served in a tomato and vodka sauce.

5.31 Fisarmoniche

— **Fisarmoniche** —

```
X(i,j) == (1.5+3*(i/70.0)^5+4*sin(j*%pi/200)^50)*cos(4*i*%pi/175)
Y(i,j) == (1.5+3*(i/70.0)^5+4*sin(j*%pi/200)^50)*sin(4*i*%pi/175)
Z(i,j) == j/50.0+cos(3*i*%pi/14)*sin(j*%pi/1000)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..70,j=0..1000,_
          style=="smooth",title=="Fisarmoniche")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

——–

Named after the accordion - whose bellows their bunched profiles recall - *fisarmoniche* are perfect for capturing thick sauces, which cling to their folds. This sturdy pasta is said to have been invented in the fifteenth century, in the Italian town of Loreto in the Marche, east central Italy.

5.32 Funghini

— **Funghini** —

```
A(i,j) == 5*cos(i*%pi/150)+0.05*cos(i*%pi/3)*sin(j*%pi/60)^2000
B(i,j) == j/30.0*(5*sin(i*%pi/150)+0.05*sin(i*%pi/3))
C(i,j) == j/10.0*(2*sin(i*%pi/150)+0.05*sin(i*%pi/3))
D(i,j) == _
  if (i <= 150) _
   then B(i,j) _
   else if (j <= 10) _
         then C(i,j) _
         else 2*sin(i*%pi/150)+0.05*sin(i*%pi/6)
X(i,j) == 0.05*cos(A(i,j)*%pi/5)+0.3*cos(A(i,j)*%pi/5)*sin(3*D(i,j)*%pi/50)^2
Y(i,j) == 0.01*sin(A(i,j)*%pi/5)+0.3*sin(A(i,j)*%pi/5)*sin(3*D(i,j)*%pi/50)^2
Z(i,j) == 0.25*sin((D(i,j)+3)*%pi/10)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..300,j=0..30,_
           style=="smooth",title=="Funghini")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
```

The modest dimensions of this *pastine minute* (tiny pasta) make *funghini* (little mushrooms) especially suitable for soups, such as a *minestrone* made from chopped and sauteed celeriac.

5.33 Fusilli

— **Fusilli** —

```
X(i,j) == 6*cos((3*i+10)*%pi/100)*cos(j*%pi/25)
Y(i,j) == 6*sin((3*i+10)*%pi/100)*cos(j*%pi/25)
Z(i,j) == (3.0*i)/20.0+2.5*cos((j+12.5)*%pi/25)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..200,j=0..25,_
           style=="smooth",title=="Fusilli")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

———-

A popular set from the *pasta corta* (short pasta) family, *fusilli* (little spindles) were originally made by quickly wrapping a *spaghetto* around a large needle. Best served as *pastasciutta* (pasta boild and drained) with a creamy sauce containing slices of spicy sausage.

5.34 Fusilli al Ferretto

— Fusilli al Ferretto —

```
A(i,j) == 6.0*i/7.0+15*cos(j*%pi/20)
X(i,j) == (3+1.5*sin(i*%pi/140)^0.5*sin(j*%pi/20))*sin(13*i*%pi/280) + _
          5*sin(2*A(i,j)*%pi/135)
Y(i,j) == (3+1.5*sin(i*%pi/140)^0.5*sin(j*%pi/20))*cos(13*i*%pi/280)
Z(i,j) == A(i,j)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..140,j=0..40,_
           style=="smooth",title=="Fusilli al Ferretto")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

To create this Neapolitan variety of *fusilli*, a small amount of durum-wheat flour is kneaded and placed along a *ferretto* (small iron stick) that is then rolled between the hands to create a thick irregular twist of dough. The shape is removed and left to dry on a wicker tray known as a *spasa*. *Fusilli al ferretto* are best dished up with a lamb *ragu*.

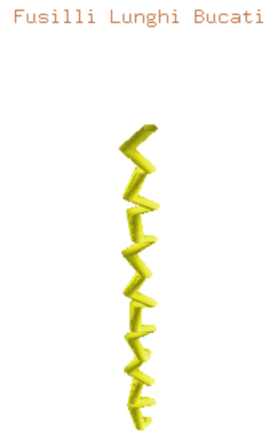
5.35 Fusilli Capri

— **Fusilli Capri** —

```
X(i,j) == 6*cos(j*%pi/50)*cos((i+2.5)*%pi/25)
Y(i,j) == 6*cos(j*%pi/50)*sin((i+2.5)*%pi/25)
Z(i,j) == 2.0*i/3.0 + 14*cos((j+25)*%pi/50)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..50,_
           style=="smooth",title=="Fusilli Capri")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

A longer and more compact regional adaptation of *fusilli*, *fusilli Capri* are suited to a hearty *ragu* of lamb or por sausages, or may also be combined with rocket and lemon to form a lighter dish.

5.36 Fusilli Lunghi Bucati



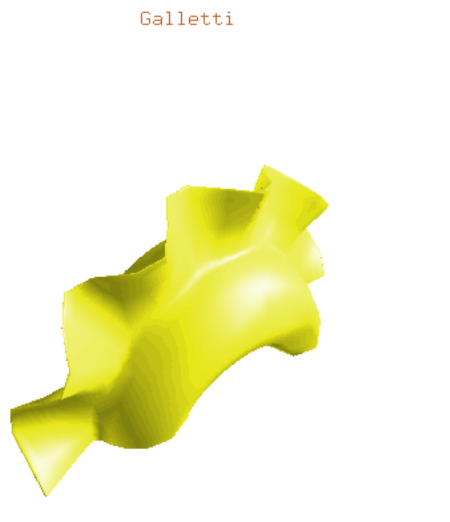
— **Fusilli Lunghi Bucati** —

```
A(i,j) == 10+cos(i*%pi/10)+2*cos((j+10)*%pi/10)+10*cos((j+140)*%pi/160)
B(i,j) == 20+cos(i*%pi/10)+2*cos((j+10)*%pi/10)
C(i,j) == (j+10.0)*%pi/10.0
D(i,j) == i*%pi/10.0
E(i,j) == 7+20*sin((j-20)*%pi/160)
F(i,j) == 70*(0.1-(j-180.0)/200.0)
X(i,j) == _
  if ((20 <= j) and (j <= 180)) _
   then A(i,j) _
   else if (j <= 20) _
         then cos(D(i,j))+2*cos(C(i,j)) _
         else B(i,j)
Y(i,j) == _
  if ((20 <= j) and (j <= 180)) _
   then sin(D(i,j))+2*sin(C(i,j)) _
   else sin(D(i,j))+2*sin(C(i,j))
Z(i,j) == _
  if ((20 <= j) and (j <= 180)) _
   then E(i,j) _
   else if (j <= 20) _
         then ((7.0*j)/20.0) _
         else F(i,j)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..20,j=0..200,_
           style=="smooth",title=="Fusilli Lunghi Bucati")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
```

```
zoom(v3d,2.5,2.5,2.5)
```

A distinctive member of the extended *fusilli* clan, *fusilli lunghi bucati* (long pierced *fusilli*) originated in Campania, southern Italy, and have a spring-like profile. Like all *fusilli* they are traditionally consumed with a meat-based *ragu*, but may also be combined with thick vegetable sauces and baked in an oven.

5.37 Galletti



— **Galletti** —

```
A(i)   == ((1+sin(i*%pi+1.5*%pi))/2)^5
B(i,j) == 0.4*sin(A(i/140)*%pi+0.5*%pi)^1000*cos(j*%pi/70)
C(i,j) == 0.15*sin(A(i/140)*%pi+0.5*%pi)^1000*cos(j*%pi/7)
D(i,j) == 0.4*cos(A(i/140)*%pi)^1000*sin(j*%pi/70)
X(i,j) == (0.5+0.3*cos(A(i/140)*2*%pi))*cos(j*%pi/70) + _
          0.15*(j/70.0)^10*cos(A(i/140)*2*%pi)^3 + B(i,j)
Y(i,j) == 0.35*sin(A(i/140)*2*%pi)+0.15*(j/70.0)*sin(A(i/140)*2*%pi)+C(i,j)
Z(i,j) == (0.4+0.3*cos(A(i/140)*2*%pi))*sin(j*%pi/70) + D(i,j)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..140,j=0..70,_
           style=="smooth",title=="Galletti")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,1.5,1.5,1.5)
```

According to their maker, Barilla, the origin of *galletti* (small cocks) is uncertain, but their shape recalls that of the *chifferi* with the addtion of an undulating crest. *Galletti* are usually served in tomato sauces, but combine wqually well with a *boscaiola* (woodsman's) sauce of mushrooms.

5.38 Garganelli

— **Garganelli** —

```
A(i,j) == (i-25.0)/125.0*j
B(i,j) == (i-25.0)/125.0*(150.0-j)
C(i,j) == _
  if ((j <= 75) or (i <= 25)) _
   then A(i,j) _
   else if ((J >= 75) or (i <= 25)) _
         then B(i,j) _
         else if ((j >= 75) or (i >= 25)) _
               then B(i,j) _
               else A(i,j)
X(i,j) == 0.1*cos(j*%pi/3)+(3+sin(C(i,j)*%pi/60))*cos(7*C(i,j)*%pi/60)
Y(i,j) == 0.1*sin(j*%pi/3)+(3+sin(C(i,j)*%pi/60))*sin(7*C(i,j)*%pi/60)
Z(i,j) == 6.0*j/25.0+C(i,j)/4.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..50,j=0..150,_
           style=="smooth",title=="Garganelli")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
viewpoint(v3d,-5,0,0)
```

A grooved *pasta corta* (short pasta), similar to *maccheroni* but with pointed slanting ends, *garganelli* are shaped like the gullet of a chicken ('*garganel*' in the northern Italian Emiliano-Romagnolo dialect). Traditionally cooked in broth, *garganelli* are also sometimes served in hare sauce with chopped bacon.

5.39 Gemelli

— **Gemelli** —

```
X(i,j) == 6*cos(j*1.9*%pi/50+0.55*%pi)*cos(3.0*i/25.0)
Y(i,j) == 6*cos(j*1.9*%pi/50+0.55*%pi)*sin(3.0*i/25.0)
Z(i,j) == 8*sin(j*1.9*%pi/50+0.55*%pi)+3.0*i/4.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..100,j=0..50,_
           style=="smooth",title=="Gemelli")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

To create a *gemello*, a single pasta strand is twisted into a spiral with a deceptively dual appearance. In the south of Italy *gemelli* (twins) are served with tomato, mozzarella, and basil, while in the northwest they are preferred with pesto and green beans, or in salads.

5.40 Gigli

— **Gigli** —

```
X(i,j) == (0.8-0.6*sin(j*%pi/80)^0.5)*cos(i*%pi/50)+0.08*sin(j*%pi/40)
Y(i,j) == (0.8-0.6*sin(j*%pi/80)^0.5)*sin(i*%pi/50)+0.08*sin(j*%pi/40)
Z(i,j) == 1.1*j/40.0+0.7*(1-sin((150-i)*%pi/300))^2
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..40,_
           style=="smooth",title=="Gigli")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

With their fluted edges and cone-like shape, *gigli* resemble small bells (*campanelle*) or lilies (*gigli*), after which they are named. Another more recent design, *gigli* are shaped to capture thick meaty sauces.

5.41 Giglio Ondulato

— **Giglio Ondulato** —

```
A(i,j) == 0.6+0.03*((40.0-j)/40.0)^10*cos((4*i+75)*%pi/15) - _
          0.5*sin(j*%pi/80)^0.6
B(i,j) == sin(2*i*%pi/75)+(i/150.0)^10*(0.08*sin(j*%pi/40)+0.03*sin(j*%pi/5))
X(i,j) == A(i,j)*cos(2*i*%pi/75)+(i/150.0)^10 * _
          (0.08*sin(j*%pi/40)+0.03*cos(j*%pi/5))
Y(i,j) == (0.6+0.03*((40.0-j)/40.0)^10*sin(4*i*%pi/15)-0.5*sin(j*%pi/80)^0.6)*_
          B(i,j)
Z(i,j) == 1.1*j/40.0+0.7*(1-sin((150.0-i)*%pi/300.0))
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..40,_
           style=="smooth",title=="Giglio Ondulato")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

Identical to *gigli* but with crenellated edges, *giglio ondulato* are made of durum-wheat flour and water.

5.42 Gnocchetti Sardi

— **Gnocchetti Sardi** —

```
A(j)   == 0.8+3*sin(j*%pi/150)^0.8
B(i,j) == cos((2*j+7.5)*%pi/15)
X(i,j) == A(j)*cos(i*%pi/50)+0.2*cos(i*%pi/50)*sin(j*%pi/150)*B(i,j)
Y(i,j) == A(j)*sin(i*%pi/50)+0.2*sin(i*%pi/50)*sin(j*%pi/150)*B(i,j)
Z(i,j) == 13.0*cos(j*%pi/150)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..50,j=0..150,_
           style=="smooth",title=="Gnocchetti Sardi")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,1.5,1.5,1.5)
```

————

As their name suggests, *gnocchetti* are simply small *gnocchi* (or 'dumplings'). They go well with ricotta or Pecorino Romano cheese, or served with thick sauces such as a veal *ragu*. *Gnocchetti* originated in Sardinia, a Mediterranean island to the west of Italy.

5.43 Gnocchi

— Gnocchi —

```
A(i,j) == i/40.0*sin(j*%pi/130)
B(i,j) == abs(cos((j+13)*%pi/26))
X(i,j) == 0.2*cos(i*1.3*%pi/40)*sin(j*%pi/130)*B(i,j) + _
          A(i,j)*cos(i*1.3*%pi/40)
Y(i,j) == 0.2*sin(i*1.3*%pi/40)*sin(j*%pi/130)*B(i,j) + _
          A(i,j)*sin(i*1.3*%pi/40)
Z(i,j) == 1.5*cos(j*%pi/130)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..40,j=0..130,_
           style=="smooth",title=="Gnocchi")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,1.5,1.5,1.5)
```

Members of an extended family, *gnocchi* (dumplings) often resemble a semi-open grooved shell. Their preparation and ingredients (including potato, durum wheat, buckwheat and semolina) vary according to type. *Gnocchi* are often added to a sauce made from fontina cheese.

5.44 Gramigna

— **Gramigna** —

```
X(i,j) == (0.5+5.6*(j/150.0)^2+0.3*cos(2*i*%pi/25))*cos(2.1*j*%pi/150)
Y(i,j) == 0.3*sin(2*i*%pi/25)
Z(i,j) == (0.5+3.2*(j/150.0)^2+0.3*cos(2*i*%pi/25))*sin(2.1*j*%pi/150)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..25,j=0..150,_
          style=="smooth",title=="Gramigna")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

A speciality of the northern Italian region of Emilia-Romagna, *gramigna* (little weed) are traditionally served with a chunky sauce of sausages, or accompanied by the world-famous *ragu alla bolognese*. Alternatively, *gramigna* are sometimes presented *alla pomodoro* (with a light tomato sauce).

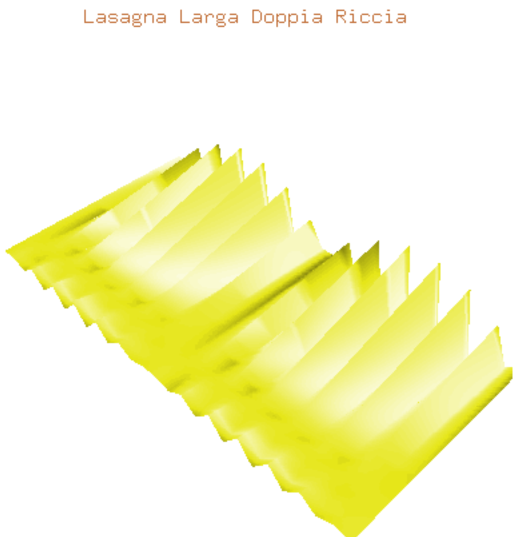
5.45 Lancette

— **Lancette** —

```
A(i,j) == 0.4*sin(5*j*%pi/9)
B(i,j) == sin((j+45)*%pi/90)
C(i,j) == (3.0*i-75.0)/5.0*sin(j*%pi/90)-(1-sin(i*%pi/50))^25*A(i,j)*B(i,j)
D(i,j) == _
  if (i <= 25) _
   then C(i,j) _
   else 30.0*(i-25.0)/50.0*sin(j*%pi/90)+sin((i-25)*%pi/50)*A(i,j)*B(i,j)
X(i,j) == 4*cos(3*D(i,j)*%pi/50)
Y(i,j) == 4*sin(3*D(i,j)*%pi/50)*(0.3+(1-sin(j*%pi/90))^0.6)
Z(i,j) == j/3.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..50,j=0..90,_
           style=="smooth",title=="Lancette")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

No longer than 15mm, *lancette* or 'hands' (of a clock) belong to the *postine minute* (tiny pasta) clan. They are delicious in consommes with a sprinkling of croutons and chopped greens. *Lancette* are also an excellent addition to mushroom or chicken soups.

5.46 Lasagna Larga Doppia Riccia



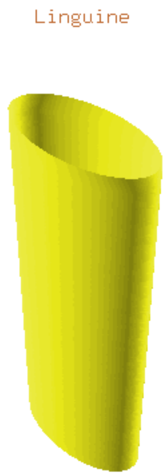
— **Lasagna Larga Doppia Riccia** —

```
X(i,j) == _
  if ((8 <= i) and (i <= 42)) _
   then 5.0/6.0+(5.0*i-40.0)/34.0 _
   else if (i <= 8) _
         then 5.0*i/48.0 _
         else 5.0/6.0*(7+(i-42.0)/8)
Y(i,j) == j/15.0
Z(i,j) == _
  if ((8 <= i) and (i <= 42)) _
   then 0.0 _
   else if (i <= 8) _
         then (8.0-i)/32.0*cos((j+3)*%pi/6) _
         else 0.25*(i-42)/8.0*cos((j+9)*%pi/6)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..50,j=0..150,_
           style=="smooth",title=="Lasagna Larga Doppia Riccia")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

———-

As their name suggests, *lasagna larga doppia riccia* (large doubly curled lasagna) have two long undulating edges. The curls give the pasta a variable consistency when cooked, and help them to collect sauce. *Lasagne* are excellent with ricotta and a *ragu napoletano*, or cooked *al forno* (at the oven) with a creamy *besciamella* sauce.

5.47 Linguine



— Linguine —

```
X(i,j) == cos(i*%pi/75)
Y(i,j) == 2*sin(i*%pi/75)
Z(i,j) == j/5.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..50,_
          style=="smooth",title=="Linguine")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

The thinnest member of the *bavette* (little dribble) family, *linguine* (little tongues) are best accompanied by fresh tomato, herbs, a drop of olive oil, garlic, anchovies, and hot peppers. *Linguine* may also be served with shellfish sauces, or white sauces of cream and soft cheese, flavoured with lemon, saffron, or ginger.

5.48 Lumaconi Rigati

— **Lumaconi Rigati** —

```
A(i,j) == 0.45+0.01*cos(i*%pi/3)+j/300.0*abs(cos(i*%pi/240))*cos(i*%pi/120)^20
B(i,j) == j/300.0*abs(cos(i*%pi/240))*sin(i*%pi/120) + _
          0.125*(j/60.0)^6*sin(i*%pi/120)
X(i,j) == (0.4*cos(i*%pi/120)+A(i,j))*cos(j*%pi/60) + _
          0.48*(j/60.0)^6*sin((i+60)*%pi/120)^3
Y(i,j) == 0.5*sin(i*%pi/120)+0.01*sin(i*%pi/3)+B(i,j)
Z(i,j) == (0.45+0.4*cos(i*%pi/120))*sin(j*%pi/60)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..240,j=0..60,_
           style=="smooth",title=="Lumaconi Rigati")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
rotate(v3d,90,180)
```

Originally from Campania and Liguria, *lumaconi rigati* (big ribbed snails) can be stuffed with a wide range of fillings, including spinach and ricotta cheese. Like cannelloni, they can then be covered in *besciamella* and cooked in the oven. Smaller members of the family (*lumache*) are also available.

5.49 Maccheroni

— **Maccheroni** —

```
X(i,j) == 8*cos(i*%pi/75)+0.2*cos(4*i*%pi/15)+5*cos(j*%pi/100)
Y(i,j) == 8*sin(i*%pi/75)+0.2*sin(4*i*%pi/15)+4*sin(j*%pi/100)
Z(i,j) == 6.0*j/5.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..50,_
           style=="smooth",title=="Maccheroni")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

———-

The origin of the name *maccheroni* is uncertain. It is used generically to describe a hollow *pasta corta* (short pasta) that is moade of durum-wheat flour and perhasp eggs. The pasta can be served *con le sarde* (with sardines) - a dish enhanced by a touch of fennel.

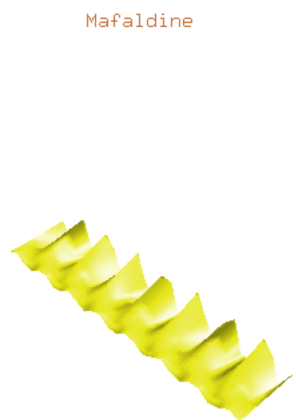
5.50 Maccheroni Alla Chitarra

— **Maccheroni Alla Chitarra** —

```
X(i,j) == 0.05*cos(i*%pi/60)^3 + 0.05*cos(i*%pi/60)
Y(i,j) == 0.05*sin(i*%pi/60)^3 + 0.05*sin(i*%pi/60)
Z(i,j) == j/60.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..120,j=0..60,_
          style=="smooth",title=="Maccheroni Alla Chitarra")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

Coming from the central Italian region of Abruzzo, *maccheroni alla chitarra* (guitar *maccheroni*) have a square cross-section, produced when a thin sheet of pasta is pressed through a frame of closely ranged wires (or *chitarra*) that lends the pasta its name. Usually seved with a mutton *ragu*, or *pallottelle* (veal meatballs).

5.51 Mafaldine



— Mafaldine —

```
X(i,j) == 7.0*i/18.0
Y(i,j) == j/3.0
Z(i,j) == _
  if ((6 <= i) and (i <= 24)) _
   then 0.0 _
   else if (i <= 6) _
         then (6.0-i)/6.0*cos((j+5)*%pi/10) _
         else (i-24.0)/6.0*cos((j+15)*%pi/10)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..30,j=0..150,_
           style=="smooth",title=="Mafaldine")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

Named at the turn of the twentieth century after Princess Mafalda of the House of Savoy, *mafaldine* are thin flat sheets of durum-wheat flour pasta with a rippled finish on each long edge. They are generally served with meaty sauces such as *ragu napoletano* or in seafood dishes.

5.52 Manicotti

— **Manicotti** —

```
A(i,j) == -7*sin((i-100)*%pi/100)^2+0.3*sin((3*i-300)*%pi/10)
B(i,j) == -8*cos((i-100)*%pi/100)+0.3*cos((3*i-300)*%pi/10)
C(i,j) == -8*cos((i-100)*%pi/100)+22*sin((j-20)*%pi/40)
X(i,j) == _
  if (i < 100) _
   then 7*sin(i*%pi/100)^2+0.3*sin(3*i*%pi/10) _
   else A(i,j)
Y(i,j) == _
  if (i < 100) _
   then 8*cos(i*%pi/100)+0.3*cos(3*i*%pi/10) _
   else B(i,j)
Z(i,j) == _
  if (i < 100) _
   then 8*cos(i*%pi/100)+22*sin((j-20)*%pi/40) _
   else C(i,j)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..200,j=0..40,_
           style=="smooth",title=="Manicotti")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,1.5,1.5,1.5)
```

———-

One of the oldest-known durum-wheat varieties, *manicotti* (sleeves) were originally prepared by cutting dough into rectangles, which were topped with stuffing, rolled and finally baked *al forno* (at the oven). Today they are served containing a variety of cheeses and covered in a savoury sauce, like filled dinner crepes.

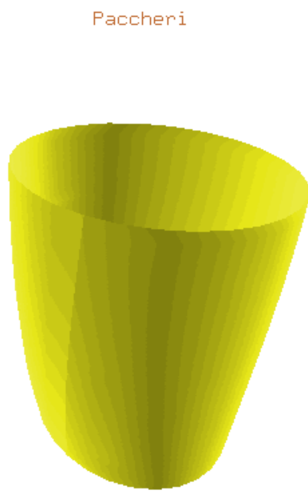
5.53 Orecchiette

— **Orecchiette** —

```
X(i,j) == 2.0*j/3.0*cos(i*%pi/75)+0.3*cos(2*i*%pi/15)
Y(i,j) == 10*sin(i*%pi/75)
Z(i,j) == 0.1*cos(i*%pi/3)+5*(0.5+0.5*cos(2*i*%pi/75))^4*cos(j*%pi/30)^2 + _
          1.5*(0.5+0.5*cos(2*i*%pi/75))^5*sin(j*%pi/30)^10
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..15,_
           style=="smooth",title=="Orecchiette")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
```

This pasta is popular in the southeastern coastal region of Puglia in Italy, where it is customarily cooked in dishes with rapini, a relative of broccoli that grows plentifully in the area. *Orecchiette* (litte ears) also pair well with other vegetables such as beans, and with salty seasonings such as anchovies, capers, or olives.

5.54 Paccheri

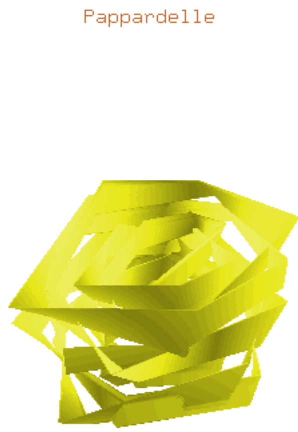


— **Paccheri** —

```
X(i,j) == (j+60.0)/60.0*cos(i*%pi/75)+0.5*cos(j*%pi/60)+cos((i+j)*%pi/75)
Y(i,j) == 2.6*sin(i*%pi/75)+0.3*sin(j*%pi/60)
Z(i,j) == 7.0*j/30.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..30,_
           style=="smooth",title=="Paccheri")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

Part of the *pasta corta* (short pasta) family, *paccheri* are ribbed pasta cylinders that (due to their large size) are recommended with aubergine, seafood, or indeed any chunky sauce. It is thought the name stems from the term *paccare*, which means 'to smack' in the southern Italian region of Campania.

5.55 Pappardelle



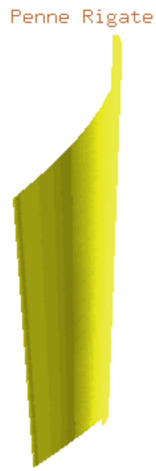
— Pappardelle —

```
A(i,j) == cos(i*%pi/80)+(3.0*j)/50.0*sin(21*i*%pi/800)
B(i,j) == sin(i*%pi/3200)^0.1*sin(i*%pi/80)
C(i,j) == 3.0*j/50.0+0.5*sin(i*%pi/200)
D(i,j) == cos(i*%pi/80)+3.0*j/50.0*sin(21*i*%pi/800)
E(i,j) == sin(i*%pi/3200)^0.5*sin(i*%pi/80)
F(i,j) == 3.0*j/50.0+0.5*sin(i*%pi/200)
vsp:=createThreeSpace()
makeObject(surface(A(i,j),B(i,j),C(i,j)),i=0..800,j=0..5,space==vsp)
makeObject(surface(0.6*D(i,j),0.8*E(i,j),0.9*F(i,j)),i=0..800,j=0..5,_
          space==vsp)
vp:=makeViewport3D(vsp,style=="smooth",title=="Pappardelle")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
```

———-

This *pasta lunga* (long pasta) is often cooked with duck, pigeon, or other game fowl. *Pappardelle* are so popular that towns in Italy hold festivals in their honour, such as the *Sagra delle Pappardelle al Cinghiale* (Feast of the *Pappardelle* and Boar) in Torre Alfina, central Italy.

5.56 Penne Rigate



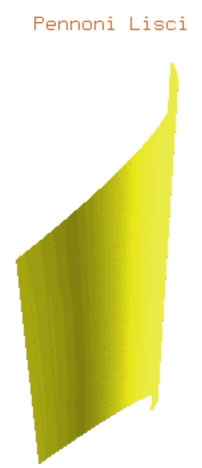
— **Penne Rigate** —

```
X(i,j) == _
  if (i < 85) _
   then 4*sin(i*%pi/85)^2+0.1*sin(6*i*%pi/17) _
   else -4*sin((i-85)*%pi/85)^2+0.1*sin(6*(i-85)*%pi/17)
Y(i,j) == _
  if (i < 85) _
   then 4*cos(i*%pi/85)+0.1*cos(6*i*%pi/17) _
   else -4*cos((i-85)*%pi/85)+0.1*cos(6*(i-85)*%pi/17)
Z(i,j) == _
  if (i < 85) _
   then 7*cos(i*%pi/85)+15*sin((j-20)*%pi/40) _
   else -7*cos((i-85)*%pi/85)+15*sin((j-20)*%pi/40)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..170,j=0..40,_
           style=="smooth",title=="Penne Rigate")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

————

A versatile pasta, *penne rigate* (grooved quills) come from Campania, in southern Italy, and belong to the *pasta corta* (short pasta) family. they can be served with spicy *arrabbiata* (angry) sauce, which gets its name from the chillies and red peppers it contains.

5.57 Pennoni Lisci



— **Pennoni Lisci** —

```
X(i,j) == _
  if (i < 100) _
   then 7*sin(i*%pi/100)^2 _
   else -7*sin((i-100)*%pi/100)^2
Y(i,j) == _
  if (i < 100) _
   then 8*cos(i*%pi/100) _
   else -8*cos((i-100)*%pi/100)
Z(i,j) == _
  if (i < 100) _
   then 12*cos(i*%pi/100)+15*sin((j-20)*%pi/40) _
   else -12*cos((i-100)*%pi/100)+15*sin((j-20)*%pi/40)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..200,j=0..40,_
           style=="smooth",title=="Pennoni Lisci")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

———-

Similar in appearance to *penne rigate*, but larger and without the grooves, *pennoni* (large quills) require a more oily sauce (perhaps containing sliced chorizo) to cling to their smooth surface.

5.58 Pennoni Rigati

— Pennoni Rigati —

```
A(i,j) == -7*sin((i-100)*%pi/100)^2+0.2*sin((3*i-300)*%pi/10)
B(i,j) == -8*cos((i-100)*%pi/100)+0.2*cos((3*i-300)*%pi/10)
C(i,j) == -12*cos((i-100)*%pi/100)+15*sin((j-20)*%pi/40)
X(i,j) == _
  if (i < 100) _
   then 7*sin(i*%pi/100)^2+0.15*sin(3*i*%pi/10) _
   else A(i,j)
Y(i,j) == _
  if (i < 100) _
   then 8*cos(i*%pi/100)+0.15*cos(3*i*%pi/10) _
   else B(i,j)
Z(i,j) == _
  if (i < 100) _
   then 12*cos(i*%pi/100)+15*sin((j-20)*%pi/40) _
   else C(i,j)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..200,j=0..40,_
           style=="smooth",title=="Pennoni Rigati")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

———

The angled trim of the *penne* pasta family makes its members easy to recognize. *Pennoni rigati* (large grooved quills) are the ridged version of *pennoni*, and can be stuffed and cooked.

5.59 Puntalette

— **Puntalette** —

```
X(i,j) == 1.4*sin(j*%pi/80)^1.2*cos(i*%pi/40)
Y(i,j) == 2.5*sin(j*%pi/80)^1.2*sin(i*%pi/40)
Z(i,j) == 8*cos(j*%pi/80)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..80,j=0..80,_
           style=="smooth",title=="Puntalette")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,1.5,1.5,1.5)
```

Another member of the *pastine minute* (tiny pasta) family, *puntalette* (tiny tips) are about 9 mm long, and no thicker than 3 mm. Like most *pastine*, they are best consumed in creamy soups, or perhaps in a salad.

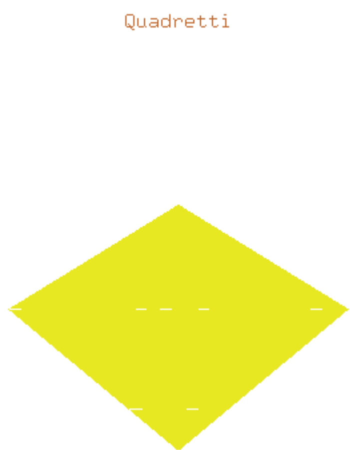
5.60 Quadrefiore

— **Quadrefiore** —

```
X(i,j) == 2*cos(i*%pi/250)*(abs(sin(3*i*%pi/250)))^20 + _
          (0.6+0.9*sin(j*%pi/50))*cos(i*%pi/250)+0.2*cos(4*j*%pi/25)
Y(i,j) == 2*sin(i*%pi/250)*(abs(sin(3*i*%pi/250)))^20 + _
          (0.6+0.9*sin(j*%pi/50))*sin(i*%pi/250)+1.5*sin(j*%pi/50)
Z(i,j) == 3.0*j/10.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..500,j=0..50,_
           style=="smooth",title=="Quadrefiore")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

An uncommon variety of *pasta corta* (short pasta), *quadrefiori* (square flowers) are sturdy, with rippled edges running down their lengths. Francis Ford Coppola, the maker and distributor, uses antique bronze moulds and wooden drying racks to achieve an authentic form and consistency.

5.61 Quadretti

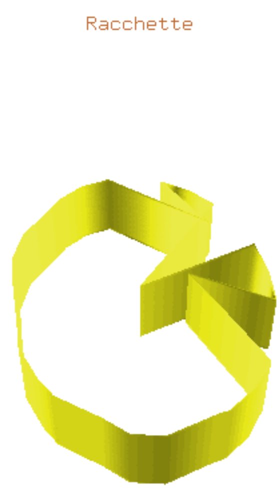


— **Quadretti** —

```
X(i,j) == 3.0*i/14.0
Y(i,j) == 3.0*j/14.0
Z(i,j) == 0.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..70,j=0..70,_
           style=="smooth",title=="Quadretti")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,1.5,1.5,1.5)
```

These flat shapes are made with durum-wheat flour, eggs, and even nutmeg. *Quadretti* (tiny squares) are a classic *pastine* prepared using the leftovers of larger pasta sheets, and can be served as part of a traditional fish broth, or in soups containing fava beans. Their small shape means that they need only be cooked for a short time.

5.62 Racchette



— **Racchette** —

```
A(i)    == sin(i*%pi/2000)^0.5
X0(i,j) == 2*cos((i+1500)*%pi/1500)+0.65*cos((i+750)*%pi/1500) + _
           2*(abs(cos(i*%pi/300)))^100*cos(i*%pi/1500)
Y0(i,j) == 2.4*sin((i+1500)*%pi/1500)+0.1*sin(i*%pi/1500) + _
           2.3*(abs(sin(i*%pi/300)))^100*sin(i*%pi/1500)
X1(i,j) == _
  if (i <= 2000) _
   then 2.1*cos((2*A(i)+1)*%pi)+0.65*cos((2*A(i)+0.5)*%pi)+_
        2.5*sin((A(i)+1.83)*%pi)^500 _
   else -2.1
Y1(i,j) == _
  if (i <= 2000) _
   then 2.5*sin((2*A(i)+1)*%pi)+0.1*sin(A(i)*2*%pi)+_
        3*sin((A(i)+1.83)*%pi)^500 _
   else 0.0
Z(i,j) == j/4.0
vsp:=createThreeSpace()
makeObject(surface(X0(i,j),Y0(i,j),Z(i,j)),i=0..3000,j=0..4,space==vsp)
makeObject(surface(X1(i,j),Y1(i,j),Z(i,j)),i=0..3000,j=0..4,space==vsp)
vp:=makeViewport3D(vsp,style=="smooth",title=="Racchette")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
```

———

Usually served in salads, *racchette* (rackets) suit crunchy pine nuts, sliced asparagus and fresh peas. Alternatively, the addition of diced watermelon or pomegranate seeds can create a light-tasking snack.

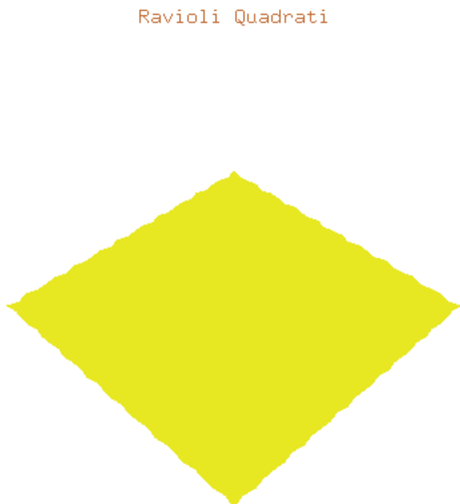
5.63 Radiatori

— **Radiatori** —

```
X(i,j) == (1.5+3*(i/70.0)^5+4*sin(j*%pi/200)^50)*cos(4*i*%pi/175)
Y(i,j) == (1.5+3*(i/70.0)^5+4*sin(j*%pi/200)^50)*sin(4*i*%pi/175)
Z(i,j) == j/50.0+cos(3*i*%pi/14)*sin(j*%pi/1000)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..70,j=0..1000,_
           style=="smooth",title=="Radiatori")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

Small and squat, *radiatori* (radiators) are named after their ruffled edge When boiled, drained, and served as a *pastasciutta* their open centre and large surface area holds thick sauces well, while the flaps sweep up and trap smaller morsels of food. This pasta is often accompanied by a lamb-, veal-, rabbit-, or pork-based *ragu*.

5.64 Ravioli Quadrati

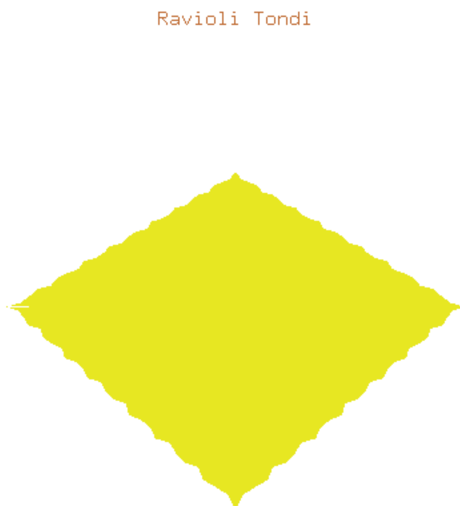


— Ravioli Quadrati —

```
X(i,j) == i/2.0+0.4*sin((j+2.5)*%pi/5) * _
          (sin(i*%pi/200)^0.2 - cos(i*%pi/200)^0.2)
Y(i,j) == j/2.0+0.4*sin((11*i+25)*%pi/50) * _
          (sin(j*%pi/200)^0.2 - cos(j*%pi/200)^0.2)
Z(i,j) == _
  if (((10 < j) and (j < 90)) or ((10 < i) and (i < 90))) _
   then 10*sin((i-10)*%pi/80)^0.6*sin((j-10)*%pi/80)^0.6 _
   else if ((10 > j) or (10 > i)) _
         then 0.0 _
         else 0.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..99,j=0..99,_
           style=="smooth",title=="Ravioli Quadrati")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

Except for the square outline, *ravioli quadrati* (square ravioli) are made in an identical fashion to *ravioli tondi* (round *ravioli*). Other variations on the theme include crescents, triangles, and hearts. Some suggest the name *ravioli* derives from the verb meaning 'to wrap', others link it with *rapa*, the Italian word for turnip.

5.65 Ravioli Tondi

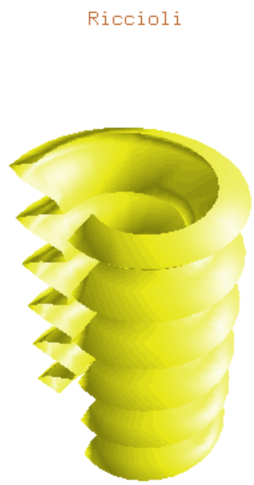


— Ravioli Tondi —

```
X(i,j) == 5.0*i/8.0+0.5*sin((j+2)*%pi/4) * _
          (sin(i*%pi/160)^0.2-cos(i*%pi/160)^0.2)
Y(i,j) == 5.0*j/8.0+0.5*sin((i+2)*%pi/4) * _
          (sin(j*%pi/160)^0.2-cos(j*%pi/160)^0.2)
Z(i,j) == _
  if (600 >= ((i-40)^2+(j-40)^2)) _
   then 0.5*sqrt(600 - (i-40)^2-(j-40)^2) _
   else 0.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..80,j=0..80,_
           style=="smooth",title=="Ravioli Tondi")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

By far the best-known *pasta ripiena* (filled pasta), *ravioli tondi* (round *ravioli*) are made by sealing a filling between two layers of dough made from wheat flour and egg. Fillings vary enormously, from lavish pairings of meat and cheese, to more deleicate centres of mushrooms, spinach, or even nettle.

5.66 Riccioli



— Riccioli —

```
X(i,j) == (2+8*sin(i*%pi/100)+9*sin((11*j+100)*%pi/400)^2)*cos(4*i*%pi/125)
Y(i,j) == (2+8*sin(i*%pi/100)+9*sin((11*j+100)*%pi/400)^2)*sin(4*i*%pi/125)
Z(i,j) == j/4.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..50,j=0..200,_
          style=="smooth",title=="Riccioli")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

A well-know *pasta corta* (short pasta), *riccioli* (curls) originated in the Emilia-Romagna region of northern Italy. Their ribbed exterior and hollow shape mean *riccioli* can retain a large quantity of sauce.

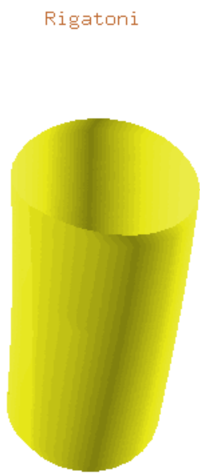
5.67 Riccioli al Cinque Sapori

— Riccioli al Cinque Sapori —

```
X(i,j) == 1.5*cos(i*%pi/20)*(1+0.5*sin(j*%pi/25)*sin(i*%pi/40) + _
          0.43*sin((j+18.75)*%pi/25)*cos(i*%pi/40))+2*cos(j*%pi/50)
Y(i,j) == 1.5*sin(i*%pi/20)^3+cos(j*%pi/25)
Z(i,j) == sin(j*%pi/100)+20*cos(j*%pi/200)^2
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..20,j=0..100,_
           style=="smooth",title=="Riccioli al Cinque Sapori")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

Riccioli and *riccioli al cinque sapori* (curls in five flavors) are both members of the *pasta corta* (small pasta) family, without being related by structural similarities. Made of durum-wheat flour, *riccioli al cinque sapori* get their color from the addition of spinach, tomato, beetroot, and turmeric, and are usually served in broth.

5.68 Rigatoni



— Rigatoni —

```
X(i,j) == 0.2*sin((7*i+15)*%pi/30)+2*cos((j+60)*%pi/120) + _
          (7+(60-j)/60.0*sin(i*%pi/240))*cos(i*%pi/120)
Y(i,j) == 0.2*sin(7*i*%pi/30)+(8+0.1*(60-j)/60+j/30*cos(i*%pi/240)) * _
          sin(i*%pi/120)
Z(i,j) == j/2.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..240,j=0..60,_
           style=="smooth",title=="Rigatoni")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

Members of the *pasta corta* (short pasta) branch, and originally from southern Italy, *rigatoni* (large ridges) are very versitile. Their robust shape holds cream or tomato sauces well, but *rigatoni* are best eaten with sausages or game meat, mushrooms, and black pepper.

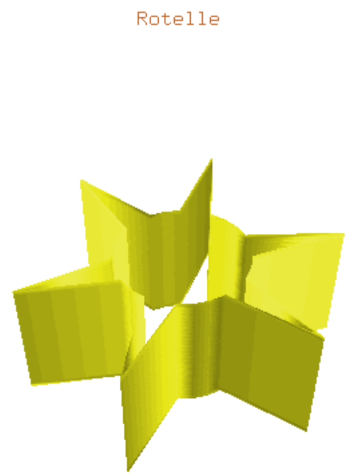
5.69 Rombi

— **Rombi** —

```
X(i,j) == _
  if ((13 <= i) and (i <= 37)) _
    then i/20.0+j/25.0-1/20.0 _
    else if (i <= 13) _
          then 3.0*i/65.0+j/25.0 _
          else 6/65.0+3.0*i/65.0+j/25.0
Y(i,j) == j/25.0
Z(i,j) == _
  if ((13 <= i) and (i <= 37)) _
   then 0.0 _
   else if (i <= 13) _
         then 0.2*((13.0-i)/13.0)*cos((2.0*j+12.5)*%pi/25) _
         else (i-37.0)/65.0*cos((2.0*j+37.5)*%pi/25)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..50,j=0..50,_
           style=="smooth",title=="Rombi")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

A lesser-known *pasta corta*, *rombi* (rhombuses) feature two curled edges like *lasagna doppia riccia*, however, they are smaller and sheared on the diagonal. Generally served with sauce and *pastasciutta* or in *brodo* (in broth), according to size.

5.70 Rotelle



— **Rotelle** —

```
X0(i,j) == 0.5*cos(i*%pi/1000)+1.5*(abs(sin(3*i*%pi/1000)))^50*cos(i*%pi/1000)
Y0(i,j) == 0.5*sin(i*%pi/1000)+1.5*(abs(sin(3*i*%pi/1000)))^50*sin(i*%pi/1000)
X1(i,j) == _
  if (i <= 666) _
   then 2*cos(3*i*%pi/1000)+0.03*cos(93*i*%pi/1000) _
   else 2.03
Y1(i,j) == _
  if (i <= 666) _
   then 2.05*sin(3*i*%pi/1000)+0.03*sin(93*i*%pi/1000) _
   else 0.0
Z(i,j) == j/5.0
vsp:=createThreeSpace()
makeObject(surface(X0(i,j),Y0(i,j),Z(i,j)),i=0..2000,j=0..5,space==vsp)
makeObject(surface(X1(i,j),Y1(i,j),Z(i,j)),i=0..2000,j=0..5,space==vsp)
vp:=makeViewport3D(vsp,style=="smooth",title=="Rotelle")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
```

———-

A modern design from the more unusual side of the *pasta corta* (short pasta) family, *rotelle* (small wheels) are constructed with spokes that help trap various flavors, making the pasta a good companion for a variety of sauces. Smaller versions can be served in salads or cooked in soups.

5.71 Saccottini

— **Saccottini** —

```
X(i,j) == cos(i*%pi/75)*(sin(j*%pi/50)+1.3*sin(j*%pi/200) + _
          3*j/1000.0*cos((i+25)*%pi/25))
Y(i,j) == sin(i*%pi/75)*(sin(j*%pi/50)+1.3*sin(j*%pi/200) + _
          0.7*(j/100.0)^2*sin(i*%pi/15))
Z(i,j) == 2*(1-(abs(cos(j*%pi/200)))^5+(j/100.0)^4.5)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..100,_
           style=="smooth",title=="Saccottini")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

Another out-and-out member of the *pasta ripiena* (filled pasta) club, *saccottini*-like *fagottini*, to which they are closely related - are made of a durum-wheat circle of dough gathered into an irregular ball-shaped bundle. *Saccottini* are usually stuffed with ricotta, meat, or steamed greens.

5.72 Sagnarelli

— **Sagnarelli** —

```
A(i) == 0.5*sin((i+1.5)*%pi/3)
B(i) == 0.05*sin((9*i+12.5)*%pi/25)
X(i,j) == i/10.0+A(j)*(cos(i*%pi/120)^100-sin(i*%pi/200)^100)
Y(i,j) == j/20.0+B(i)*(cos(j*%pi/120)^100-sin(j*%pi/200)^100)
Z(i,j) == 0.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..100,j=0..60,_
           style=="smooth",title=="Sagnarelli")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

This short and rectangular ribbon with four indented edges belongs to the *pasta lunga* (long pasta) branch. *Sagnarelli* are sometimes made with eggs and are generally served as *pastasciutta* with a meat *ragu*, or alongside vegetables such as wild asparagus.

5.73 Sagne Incannulate

— **Sagne Incannulate** —

```
X0(i,j) == (1-0.2*sin(3*j*%pi/20))*cos((i+10)*%pi/20) + _
           2*sin((i-100)*%pi/200)+(3*i)/200.0*sin(i*%pi/400)^200
Y0(i,j) == (1-0.2*sin(3*j*%pi/20))*sin((i+10)*%pi/20)+sin((i-50)/200*%pi)
X1(i,j) == -3+(1-0.1*sin(3*j*%pi/20))*sin((3*i-10)*%pi/50) + _
           cos(i*%pi/200)+3*i/200.0*sin(i*%pi/400)^5
Y1(i,j) == -5+(1-01.*sin(3*j*%pi/20))*cos((3*i+10)*%pi/50)+2*sin(i*%pi/200)
Z(i,j) == i/4.0+7/2.0*(1+cos(j*%pi/20))
vsp:=createThreeSpace()
makeObject(surface(X0(i,j),Y0(i,j),Z(i,j)),i=0..200,j=0..20,space==vsp)
makeObject(surface(X1(i,j),Y1(i,j),Z(i,j)),i=0..200,j=0..20,space==vsp)
vp:=makeViewport3D(vsp,style=="smooth",title=="Sagne Incannulate")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
zoom(vp,3.0,3.0,3.0)
```

Shaping the twisted ribbons of durum-wheat pasta known as *sagne incannulate* requires skill. Strips of dough are held at the end against a wooden board with one hand, while the palm of the other rolls the rest of the *pasta lunga* (long pasta) to form the distinctive curl. *Sagne* are best consumed with a thick sauce or a traditional *ragu*.

5.74 Scialatielli

ragu of pork or veal.

— Scialatielli —

```
X(i,j) == 0.1*cos(i*%pi/75)+0.1*cos((i+7.5)*%pi/75)^3+0.1*sin(j*%pi/50)
Y(i,j) == 0.1*cos(i*%pi/75)+0.2*sin(i*%pi/75)^3+0.1*sin(j*%pi/50)
Z(i,j) == 3.0*j/50.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..50,_
          style=="smooth",title=="Scialatielli")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

———-

Hailing from the Amalfi coast in the province of Naples, *scialatielli* are a rustic *pasta lunga* (long pasta) similar in appearance to *fettuccine* and *tagliotelle*. When they are made, milk and eggs can be added to the durum-wheat flour to lend it a golden color, *Scialatielli* are best paired with seafood, or a

5.75 Spaccatelle

— Spaccatelle —

```
X(i,j) == (0.5+5*(j/100.0)^3+0.5*cos((i+37.5)*%pi/25))*cos(2*j*%pi/125)
Y(i,j) == 0.6*sin((i+37.5)*%pi/25)
Z(i,j) == (0.5+5*(j/100.0)^3+0.5*cos((i+37.5)*%pi/25))*sin(2*j*%pi/125)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..25,j=0..100,_
          style=="smooth",title=="Spaccatelle")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

This noted speciality of Sicily belongs to the *pasta corta* (short pasta) family. *Spaccatelle* are generally elongated curves with a concave centre, and are served as a *pastasciutta* (pasta boiled and drained) with a light tomato sauce or a thick meaty *ragu*.

5.76 Spaghetti

— Spaghetti —

```
X(i,j) == 0.1*cos(i*%pi/20)
Y(i,j) == 0.1*sin(i*%pi/20)
Z(i,j) == j/10.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..40,j=0..100,_
          style=="smooth",title=="Spaghetti")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

Without a doubt, *spaghetti* (small strings) remain the best-known and most versatile *pasta lunga* (long pasta) worldwide. Above all they are known for accompanying *ragu bolognese*, a mixture containing beef, tomato, cream, onions, and pancetta. More recently, *spaghetti* have become popular in a creamy *carbonara*.

5.77 Spiralli

— Spiralli —

```
X(i,j) == (2.5+2*cos(i*%pi/50)+0.1*cos(i*%pi/5))*cos(j*%pi/30)
Y(i,j) == (2.5+2*cos(i*%pi/50)+0.1*cos(i*%pi/5))*sin(j*%pi/30)
Z(i,j) == (2.5+2*sin(i*%pi/50)+0.1*sin(i*%pi/5))+j/6.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..100,j=0..120,_
          style=="smooth",title=="Spiralli")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

The ridged and helicoidal *spirali* (spirals) are similar in shape to *cavatappi*, but are slightly larger. *Spirali* may be served with chunky sauces as *pastasciutta*, baked *al forno* (at the oven) with a thick cheese topping or added to salads.

5.78 Stellette

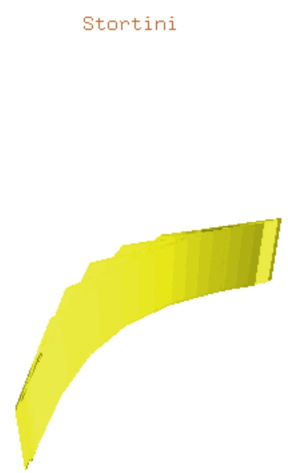
— **Stellette** —

```
X(i,j) == (3.0+3.0*i/5.0)*cos(j*%pi/75)+i/10*cos((j+15)*%pi/15)
Y(i,j) == (3.0+3.0*i/5.0)*sin(j*%pi/75)+i/10*sin(j*%pi/15)
Z(i,j) == 0.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..25,j=0..150,_
           style=="smooth",title=="Stellette")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
```

———-

A member of the *pastine minute* (tiny pasta) family, *stellette* (little stars) are only marginally larger than both *acini di pepe* and *cuoretti*. Like all *pastine*, they are best served in a light soup, perhaps flavored with portobello mushrooms or peas.

5.79 Stortini



— **Stortini** —

```
A(i,j) == 0.5*(abs(sin(2*i*%pi/125)))^3
B(i,j) == 1-abs(sin((2*i-500)*%pi/125))
C(i,j) == 1-0.5*(abs(sin(2*(i-375)*%pi/125)))^3
X(i,j) == _
  if (i <= 250) _
   then cos(2*i*%pi/125) _
   else 1.7-cos((2*i-500)*%pi/125)
Y(i,j) == _
  if (i <= 250) _
   then if (i <= 125) _
         then abs(sin(2*i*%pi/125)) _
         else A(i,j) _
   else if (i <= 375) _
         then B(i,j) _
         else C(i,j)
Z(i,j) == j/20.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..500,j=0..10,_
           style=="smooth",title=="Stortini")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

Another *pastine minute* (tiny pasta), *stortini* (little crooked pieces) are consumed in creamy soups with mushroom and celery. As a rule of thumb, smaller pasta is best in thinner soups, while larger *pastina minute* can be served with thicker varieties.

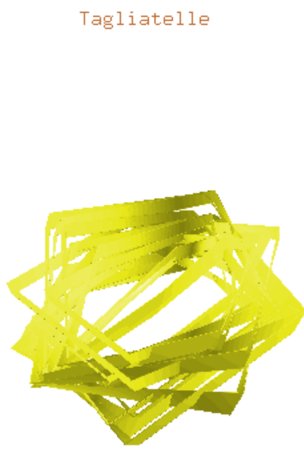
5.80 Strozzapreti

— Strozzapreti —

```
A(i,j) == 0.5*cos(j*%pi/40)+0.5*cos((j+76)*%pi/40) + _
          0.5*cos(j*%pi/30)+0.5*sin((2*i-j)*%pi/40)
B(i,j) == 0.5*sin(j*%pi/40)+0.5*sin((j+76)*%pi/40) + _
          0.5*sin(j*%pi/30)+0.5*cos((2*i-j)*%pi/40)
X(i,j) == _
  if (i <= 30) _
   then 0.5*cos(j*%pi/30)+0.5*cos((2*i+j+16)*%pi/40) _
   else A(i,j)
Y(i,j) == _
  if (i <= 30) _
   then 0.5*sin(j*%pi/30)+0.5*sin((2*i+j+16)*%pi/40) _
   else B(i,j)
Z(i,j) == j/4.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..60,j=0..60,_
           style=="smooth",title=="Strozzapreti")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

The dough used to make *strozzapreti* (or *strangolapreti*; both translate to 'priest stranglers') can be prepared with assorted flours, eggs, and even potato. *Strozzapreti* are an ideal *pastasciutta*, and can be served with a traditional meat sauce, topped with Parmigiano-Reggiano.

5.81 Tagliatelle

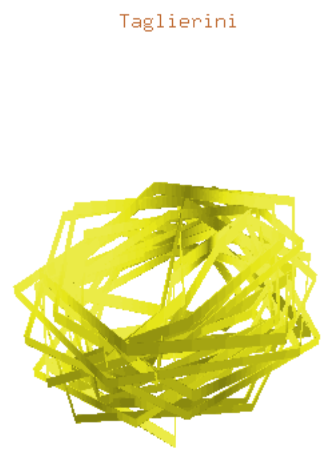


— Tagliatelle —

```
X0(i,j) == 0.4*cos(3*i*%pi/250)+j/80.0*sin(31*i*%pi/1000)
Y0(i,j) == 0.4*sin(3*i*%pi/250)*sin(i*%pi/4000)^0.1
Z0(i,j) == j/80.0+0.12*sin(9*i*%pi/1000)
X1(i,j) == 0.4*cos(3*i*%pi/250)+j/80.0*sin(31*i*%pi/1000)
Y1(i,j) == 0.4*sin(i*%pi/4000)^0.5*sin(3*i*%pi/250)
Z1(i,j) == j/80.0+0.12*sin(9*i*%pi/1000)
vsp:=createThreeSpace()
makeObject(surface(X0(i,j),Y0(i,j),Z0(i,j)),i=0..1000,j=0..4,space==vsp)
makeObject(surface(0.8*X1(i,j),Y1(i,j),0.9*Z1(i,j)),i=0..1000,j=0..4,_
           space==vsp)
makeObject(surface(X1(i,j),0.9*Y1(i,j),0.6*Z1(i,j)),i=0..1000,j=0..4,_
           space==vsp)
vp:=makeViewport3D(vsp,style=="smooth",title=="Tagliatelle")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
```

A mixture of wheat-flour and eggs, *tagliatelle* (derived from the Italian *tagliare* - 'to cut') belong to the *pasta lunga* (long pasta) family. Originally hailing from the north of Italy, *tagliatelle* are frequently served in *carbonara* sauce, but can also accompany seafood, or alternatively may form the basis of a *timballo* (baked pasta dish).

5.82 Taglierini



— **Taglierini** —

```
X0(i,j) == 0.5*cos(i*%pi/100)+0.05*cos(i*%pi/40)
Y0(i,j) == 0.5*sin(i*%pi/4000)^0.1*sin(i*%pi/100)+0.075*sin(i*%pi/40)
Z0(i,j) == 3.0*j/200.0+0.1*sin(i*%pi/125)
X1(i,j) == 0.4*cos(i*%pi/100)
Y1(i,j) == 0.4*sin(i*%pi/4000)^0.2*sin(i*%pi/100)
Z1(i,j) == 3.0*j/200.0+0.1*sin(i*%pi/125)
X2(i,j) == 0.3*cos(i*%pi/100)
Y2(i,j) == 0.3*sin(3*i*%pi/1000)*sin(i*%pi/50)
Z2(i,j) == -0.05+3.0*j/200.0+0.1*sin(i*%pi/125)
vsp:=createThreeSpace()
makeObject(surface(X0(i,j)*0.6,Y0(i,j)*0.5,Z0(i,j)),i=0..1000,j=0..2,_
           space==vsp)
makeObject(surface(X1(i,j),Y1(i,j),Z1(i,j)),i=0..1000,j=0..2,space==vsp)
makeObject(surface(0.8*X1(i,j),0.9*Y1(i,j),1.3*Z1(i,j)),i=0..1000,j=0..2,_
           space==vsp)
makeObject(surface(X2(i,j),0.9*Y2(i,j),1.5*Z2(i,j)),i=0..1000,j=0..2,_
           space==vsp)
vp:=makeViewport3D(vsp,style=="smooth",title=="Taglierini")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
```

Another coiling *pasta lunga* (long pasta), *taglierini* are of the same lineage as *tagliatelle*, but are substantially narrower - almost hair-like.

5.83 Tagliolini

— **Tagliolini** —

```
X0(i,j) == 0.5*cos(i*%pi/200)+0.05*cos(5*i*%pi/400)
Y0(i,j) == 0.5*sin(i*%pi/8000)^0.1*sin(i*%pi/200)+0.075*sin(5*i*%pi/400)
Z0(i,j) == 0.01*j+0.1*sin(i*%pi/250)
X1(i,j) == 0.4*cos(i*%pi/200)
Y1(i,j) == 0.4*sin(i*%pi/8000)^0.2*sin(i*%pi/200)
Z1(i,j) == 0.01*j+0.1*sin(i*%pi/250)
X2(i,j) == 0.3*cos(i*%pi/125)
Y2(i,j) == 0.3*sin(3*i*%pi/2000)*sin(3*i*%pi/200)
Z2(i,j) == -0.05+0.01*j+0.1*sin(i*%pi/250)
vsp:=createThreeSpace()
makeObject(surface(X0(i,j)*0.6,Y0(i,j)*0.5,Z0(i,j)),i=0..2000,j=0..1,_
          space==vsp)
makeObject(surface(X1(i,j),Y1(i,j),Z1(i,j)),i=0..2000,j=0..2,space==vsp)
makeObject(surface(0.8*X1(i,j),0.9*Y1(i,j),1.3*Z1(i,j)),i=0..2000,j=0..2,_
          space==vsp)
makeObject(surface(X2(i,j),0.9*Y2(i,j),1.5*Z2(i,j)),i=0..2000,j=0..2,_
          space==vsp)
vp:=makeViewport3D(vsp,style=="smooth",title=="Tagliolini")
colorDef(vp,yellow(),yellow())
axes(vp,"off")
```

Thinner even than *taglierini*, *tagliolini* are traditionally eaten as a starter with butter, soft cheese, or *al pomodoro* (in a light tomato sauce). Alternatively, they may be served in a chicken broth. A versatile pasta, *tagliolini* are also the main ingredient for a variety of *timballo* dishes baked *al forno* (at the oven).

5.84 Torchietti

— **Torchietti** —

```
A(i,j) == (3.0*i-60.0)*j/100.0
B(i,j) == (3.0*i-60.0)*(100.0-j)/100.0
C(i,j) == 3.0*(i-20.0)*(100.0-j)/100.0
D(i,j) == 3.0*(i-20.0)*j/100.0
E(i,j) == _
  if ((j <= 50) or (i <= 20)) _
   then A(i,j) _
   else if ((j >= 50) or (i <= 20)) _
         then B(i,j) _
         else if ((j >= 50) or (i >= 20)) _
               then C(i,j) _
               else D(i,j)
F(i,j) == (3+2.5*sin(E(i,j)*%pi/120)+3*sin(j*%pi/200)^10)
X(i,j) == F(i,j)*cos(E(i,j)*%pi/10)+0.1*cos(j*%pi/2)
Y(i,j) == F(i,j)*sin(E(i,j)*%pi/10)+0.1*sin(j*%pi/2)+3*sin(j*%pi/50)
Z(i,j) == 18.0*j/25.0+2.0*E(i,j)/5.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..40,j=0..100,_
           style=="smooth",title=="Torchietti")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

———-

The whorls of *torchietti* (tiny torches) trap chunky sauces will. Also known as *maccheroni al torchio*, they are best eaten in a tomato sauce with larger, coarsely chopped vegetables such as carrots, broccoli, and cauliflower.

5.85 Tortellini

— **Tortellini** —

```
A(i,j) == 0.2*sin(i*%pi/200)+j/400.0
B(i,j) == cos(j/60.0*(2.7+0.2*sin(i*%pi/120)^50)*%pi+1.4*%pi)
C(i,j) == sin(j/60.0*(2.7+0.2*sin(i*%pi/120)^50)*%pi+1.4*%pi)
X(i,j) == 0.5^(1+0.5*sin(i*%pi/120))*cos((11*i-60)*%pi/600) * _
          (1.35+(3+sin(i*%pi/120))*A(i,j)*B(i,j))
Y(i,j) == 0.5*sin((11*i-60)*%pi/600) * _
          (1.35+(0.6+sin(i*%pi/120))*A(i,j)*B(i,j))
Z(i,j) == 0.15+i/1200.0+0.5*(0.8*sin(i*%pi/120)+j/400.0)*sin(i*%pi/120)*C(i,j)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..120,j=0..60,_
           style=="smooth",title=="Tortellini")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
```

To prepare a *tortellino*, a teaspoon of meat, cheese, or vegetables is wrapped in a layer of dough (made of wheat flour and egg) that is then skilfully rolled and folded. These shapely members of the *pasta ripiena* (filled pasta) family are traditionally eaten in steaming soups, but are also drained and served with a local sauce as *pastasciutta*.

5.86 Tortiglioni

— **Tortiglioni** —

```
X(i,j) == 6*cos(i*%pi/75)-3.5*cos(j*%pi/100) + _
          0.15*sin((13*i/75.0+j/15.0+1.5)*%pi)
Y(i,j) == 6*sin(i*%pi/75)+0.15*sin((13*i/75.0+j/15.0)*%pi)
Z(i,j) == 11.0*j/10.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..50,_
           style=="smooth",title=="Tortiglioni")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

Another classic *pasta corta* (short pasta), *tortiglioni* originate from the Campania region of southern Italy. *Tortiglioni* (deriving from the Italian *torquere* - 'to cut') are often baked in a *timballo* or boiled, drained and served as a *pastasciutta* coupled with a strong sauce of tomato, chorizo, and black pepper.

5.87 Trenne

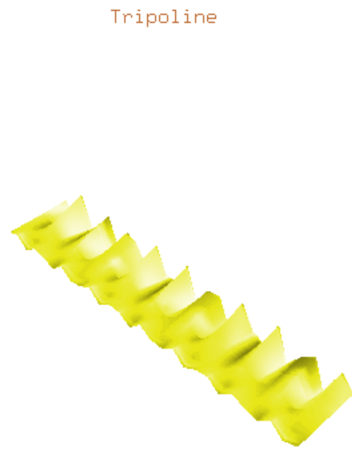
— **Trenne** —

```
X(i,j) == _
  (if (i <= 33) _
    then (3.0*i/10.0) _
     else ((300.0-3*i)/20.0))
Y(i,j) == _
 (if (i <= 33) _
   then (0.0) _
    else (if (i <= 66) _
           then ((9*i-300)/20.0) _
            else ((900-9*i)/20.0)))
Z(i,j) == _
 (if (i <= 33) _
   then (if (i <= 16) _
          then (-9.0*i/50.0+6.0*j/4.0) _
           else (-6.0+9.0*i/50.0+3.0*j/2.0) ) _
    else (if (i <= 66) _
           then (-12.0+7.0*i/20.0+6.0*j/4.0) _
            else (35.0-7.0*i/20.0+6.0*j/4.0)))
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..100,j=0..40,_
           style=="smooth",title=="Trenne")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,4.0,4.0,4.0)
```

———

A hollow triangular variety of *pasta corta* (short pasta), *trenne* (*penne* with a triangular cross-section) are extremely sturdy. They are best served with mushrooms, tomato, and spinach - or in any sauce that would normally accompany their close cousins: *penne*, *trennette*, and *ziti*.

5.88 Tripoline



— **Tripoline** —

```
X(i,j) == i/20.0
Y(i,j) == j/20.0
Z(i,j) == _
  if (i <= 30) _
   then 0.0 _
   else if (i <= 10) _
         then (10.0-i)/50.0*cos((j+2)*%pi/4) _
         else ((i-30.0)/50.0)*cos((j+15)*%pi/10)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..40,j=0..200,_
           style=="smooth",title=="Tripoline")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

————-

The ribbon-like *tripoline* are *pasta lunga* (long pasta) curled along one edge only. This wave gives the *tripoline* a varying texture after cooking, and helps them to gather extra sauce. Originally from southern Italy, they are often served with tomato and basil, or *ragu alla napoletana* and a sprinkling of Pecorino Romano.

5.89 Trofie

— **Trofie** —

```
A(i,j) == (3.0*i)/5.0+10*cos(j*%pi/25)
X(i,j) == (1+sin(i*%pi/150)+2*sin(i*%pi/150)*sin(j*%pi/25)) * _
          sin(13*i*%pi/300)
Y(i,j) == (1+sin(i*%pi/150)+2*sin(i*%pi/150)*sin(j*%pi/25)) * _
          cos(13*i*%pi/300)+5*sin(2*A(i,j)*%pi/125)
Z(i,j) == A(i,j)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..50,_
           style=="smooth",title=="Trofie")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,3.0,3.0,3.0)
```

The Ligurian version of *gnocchi*, *trofie* are made from a mixture of wheat flour, bran, water, and potatoes. Traditionally served boiled with green beans and more potatoes, they may also be paired with a simple mix of *pesto genovese*, pine nuts, salt, and olive oil.

5.90 Trottole

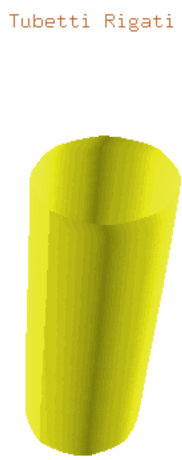
— **Trottole** —

```
A(i,j) == 0.17-0.15*sin(j*%pi/120)+0.25*((60.0-j)/60.0)^10*sin(j*%pi/30)
B(i,j) == 0.17-0.15*sin(j*%pi/120)+0.25*((60.0-j)/60.0)^10*sin(j*%pi/30)
C(i,j) == 0.25*((60.0-j)/60.0)^5*(1-sin((i-128)*%pi/160))*cos(j*%pi/30)
D(i,j) == (7.0*i)/400.0-48/25+C(i,j)+j/120.0*(1-sin((i-128)*%pi/64))
X(i,j) == _
  if (i >= 128) _
   then (A(i,j)*(1-sin((i-128)*%pi/320)))*cos(7*i*%pi/160) _
   else B(i,j)*cos(7*i*%pi/160)
Y(i,j) == _
  if (i >= 128) _
   then (A(i,j)*(1-sin((i-128)*%pi/160)))*sin(7*i*%pi/160) _
   else B(i,j)*sin(7*i*%pi/160)
Z(i,j) == _
  if (i >= 128) _
   then D(i,j) _
   else i/400.0+j/100.0+0.25*((60.0-j)/60.0)^5*cos(j*%pi/30)
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..160,j=0..60,_
           style=="smooth",title=="Trottole")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

————-

A well-formed *pasta corta* (short pasta) comprised of rings that curl up about a central stalk, *trottole* are ideal for salads. They are also delicious with pumpkin or courgette, leek, pine nuts, and a few shavings of Parmigiano-Reggiano.

5.91 Tubetti Rigati



— **Tubetti Rigati** —

```
X(i,j) == 2*cos(i*%pi/75)+0.03*sin((4*i+7.5)*%pi/15)+0.5*cos(j*%pi/60)
Y(i,j) == 2*sin(i*%pi/75)+0.03*sin(4*i*%pi/15)+0.5*sin(j*%pi/60)
Z(i,j) == j/3.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..150,j=0..30,_
           style=="smooth",title=="Tubetti Rigati")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.0,2.0,2.0)
```

The smallest members of the *pasta corta* (short pasta) clan, *tubetti rigati* (grooved tubes) were first created in Campania, southern Italy. When served *e fagioli* (with beans) they create very filling soups, but can also be served in a light *marinara* (mariner's) sauce of tomato, basil, and onions.

5.92 Ziti

— **Ziti** —

```
X(i,j) == 0.5*cos(i*%pi/35)+0.2*sin(j*%pi/70)
Y(i,j) == 0.5*sin(i*%pi/35)+0.2*sin(j*%pi/70)
Z(i,j) == j/14.0
v3d:=draw(surface(X(i,j),Y(i,j),Z(i,j)),i=0..70,j=0..70,_
          style=="smooth",title=="Ziti")
colorDef(v3d,yellow(),yellow())
axes(v3d,"off")
zoom(v3d,2.5,2.5,2.5)
```

A pasta reserved for banquets and special occasions, *ziti* ('grooms' or 'brides' in Italian dialect) originate from Sicily. Tradition has it that they should be broken by hand before being tossed into boiling water. After draining they can be served in tomato sauces with peppers or courgettes, topped with cheese like Provolone.

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Chapter 6

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