Optimizing the menu of travel options for a Flexible Mobility on Demand System

**FMOD**

Bilge Atasoy, Takuro Ikeda, Moshe Ben-Akiva

November 12, 2014

*INFORMS Annual Meeting*
Agenda

- Motivation and background
- Concept of FMOD
- Modeling framework
- Simulation experiments
- Conclusions and future directions
Motivation and background

• Personalized services using smartphone apps are emerging for taxi:
  – Uber, Lyft, SideCar, GoMyWay, etc.

• Why not apply similar technologies to also DRT and fixed route public transportation?
Concept of FMOD

- **Real-time** transportation system
- **Personalized** demand responsive system that gives the traveler an optimized menu
- **Dynamic allocation** of vehicles to services
Concept of FMOD (cont.)

- **Taxi**: Flexible route, flexible schedule, private

- **Shared-taxi**: Flexible route, flexible schedule, shared

- **Mini-bus**: Fixed route, flexible schedule, shared
Concept of FMOD (cont.)

Supply Demand

FMOD Server

optimization

Request:
Origin: A, Destination: B
Preferred Departure Time: 8:00 – 8:30
Preferred Arrival Time: 8:45 – 9:00

Offer:
taxi: DT: 8:25/AT: 8:45, $20
shared-taxi: DT: 8:27/AT: 8:57, $10
as the 4th passenger
mini-bus: DT: 8:14/AT: 8:59, $5
as the 6th passenger

Choice:
service: shared-taxi
DT: 8:27/AT: 8:57, $10
Modeling framework

- **Product** $p_{n,m,l}$
  - A service ($m$) on a vehicle ($n$) departing at a certain time period ($l$)

- **Feasible product** $p_{n,m,l} \in F$
  - A product that satisfies the capacity and scheduling constraints
    - Vehicle capacity
    - Existing schedule
    - Preferred time window
      - Maximum schedule delay

- **Offer**
  - A list of feasible products presented to the customer
    (max 1 product for each service)
Modeling framework (cont.)

Phase 1. Feasible product set generation
Set of feasible products to be offered to the customer taking into account:
– Capacity constraints
– Scheduling constraints based on the request

Phase 2. Assortment optimization
Optimized list to be offered to the customer from the feasible set
– Maximize operator’s profit and/or consumer surplus based on a choice model
Demand model

• A logit model in order to represent the choice probabilities for the FMOD services and the reject option

• Utility functions are defined by:
  – Price of the service
  – In-vehicle travel time
  – Out-vehicle travel time (for mini-bus)
  – Schedule delay

\[
\text{Prob}_{n,m,l}(x) = \frac{x_{n,m,l} \exp (\mu V_{n,m,l})}{\exp (\mu V_{\text{reject}}) + \sum_{n' \in N} \sum_{m' \in M} \sum_{l' \in L} x_{n',m',l'} \exp (\mu V_{n',m',l'})}
\]
Assortment optimization model

• Myopic version (current request only)
• Maximize expected profit

$$\max \sum_{n \in N} \sum_{m \in M} \sum_{l \in L} r_{n,m,l} \text{Prob}_{n,m,l}(x)$$

• At most one product for each service

$$\sum_{n \in N} \sum_{l \in L} x_{n,m,l} \leq 1 \quad \forall m \in M$$

• Among the set of feasible products

$$x_{n,m,l} \in \{0, 1\} \quad \forall p_{n,m,l} \in F$$
Assortment optimization model

- Formulated as a mixed integer linear problem
- Transformation with a new decision variable for choice probability
- Myopic vs dynamic

- Different versions of the model are considered:
  - maximize consumer surplus (logsum)
  - maximize profit
  - maximize profit + consumer surplus: total benefit
Simulation experiments

Case study

- Simulation time: 24 hours
- Network
  - Hino city in Tokyo (approx. 9km × 8km)
- Supply
  - Fleet size: 60
  - Bus line: actual route
- Demand
  - 5000 requests / day
  - OD: station, hospital etc. (population density)
  - VOT: from $6/h to $30/h
- Fare
  - Taxi: $5 (base) + $0.5 (per 320m)
  - Shared-taxi: 50% of taxi fare
  - Bus: $3 (flat)
- Operator Cost
  - $200 / day / vehicle + $0.2 per km
Simulation experiments

Snapshots

Red: Taxi, Green: Shared taxi, Blue: Mini-bus, Yellow: empty

Off-peak (AM 6:00)
Taxi is dominant

Peak (AM 8:00)
Shared taxi / Mini-bus are dominant
Simulation experiments
Comparison of models

T: taxi, S: shared-taxi, B: mini-bus
Simulation experiments
Main findings

• The offer given by FMOD is significantly affected by the objective function.

• Total benefit case compared to profit maximization:
  – Significant increase in consumer surplus without much decrease in profit

• Dynamic allocation of vehicles provides significant improvements over static allocation
Conclusions and future directions

- FMOD has a potential to increase operator’s profit and improve passenger satisfaction

- Ongoing and further research directions include:
  - Field test
  - Estimation of future demand
  - Real life conditions (e.g. traffic)
  - Learning the behavior of customer through repeated visits
Conclusions and future directions

\[
\begin{align*}
\max & \sum_{n \in N} \sum_{m \in M} \sum_{l \in L} r_{n,m,l} \omega_{n,m,l} \\
& + \sum_{m \in M} \sum_{l \in L} \tilde{r}_{m,l} E[\text{Dem}_m,l | \text{Dem}_m,l \leq \tilde{z}_{m,l}] \\
\text{s.t.} & \sum_{n \in N} \sum_{m \in M} \sum_{l \in L} \omega_{n,m,l} = 1 - \omega_{\text{reject}} \\
& \sum_{n \in N} \sum_{l \in L} \frac{\omega_{n,m,l}}{\exp(\mu V_{n,m,l})} \leq \frac{\omega_{\text{reject}}}{\exp(\mu V_{\text{reject}})} \quad \forall m \in M \\
& \omega_{n,m,l} \leq 0 \\
& 0 \leq \frac{\omega_{n,m,l}}{\exp(\mu V_{n,m,l})} \leq \frac{\omega_{\text{reject}}}{\exp(\mu V_{\text{reject}})} \quad \forall p_{n,m,l} \notin F, n \in N, m \in M, l \in L \\
& x_{n,m,l} \geq \omega_{n,m,l} \quad \forall n \in N, m \in M, l \in L \\
& \tilde{z}_{m,l} \leq \sum_{n} z_{n,m,l} \quad \forall m \in M, l \in L \\
& z_{n,m,l} \leq \text{Cap}_{n,m,l} - x_{n,m,l} \quad \forall n \in N, m \in M, l \in L \\
& x_{n,m,l} \in \{0, 1\} \quad \forall n \in N, m \in M, l \in L \\
& z_{n,m,l} \geq 0 \quad \forall n \in N, m \in M, l \in L \\
& \tilde{z}_{m,l} \geq 0 \quad \forall m \in M, l \in L 
\end{align*}
\]
Thank you for your attention!

batasoy@mit.edu
Feasible product set generation

- Insertion of a new schedule

<table>
<thead>
<tr>
<th>Schedule Block</th>
<th>SB₁</th>
<th>SB₂</th>
<th>SB₃</th>
<th>SB₄</th>
<th>SB₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service type</td>
<td>Shared</td>
<td>empty</td>
<td>Mini-bus</td>
<td>empty</td>
<td>Mini-bus</td>
</tr>
<tr>
<td>[Stops]</td>
<td>a</td>
<td>b</td>
<td>g</td>
<td>d</td>
<td>f</td>
</tr>
<tr>
<td>Location</td>
<td>c</td>
<td>e</td>
<td>h</td>
<td>e</td>
<td>i</td>
</tr>
<tr>
<td>Arrival Time</td>
<td>d</td>
<td>f</td>
<td>i</td>
<td>f</td>
<td>j</td>
</tr>
<tr>
<td>Departure time</td>
<td>-</td>
<td>g</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Boarding passengers</td>
<td>1</td>
<td>2</td>
<td>4,5</td>
<td>6,7</td>
<td>-</td>
</tr>
<tr>
<td>Alighting passengers</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Schedule Block</th>
<th>SB₁</th>
<th>SB₂</th>
<th>SB₃</th>
<th>SB₄</th>
<th>SB₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service type</td>
<td>Shared</td>
<td>empty</td>
<td>taxi</td>
<td>empty</td>
<td>Mini-bus</td>
</tr>
<tr>
<td>[Stops]</td>
<td>a</td>
<td>b</td>
<td>e</td>
<td>d</td>
<td>f</td>
</tr>
<tr>
<td>Location</td>
<td>c</td>
<td>e</td>
<td>f</td>
<td>e</td>
<td>i</td>
</tr>
<tr>
<td>Arrival Time</td>
<td>d</td>
<td>f</td>
<td>g</td>
<td>f</td>
<td>j</td>
</tr>
<tr>
<td>Departure time</td>
<td>-</td>
<td>g</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Boarding passengers</td>
<td>1</td>
<td>2</td>
<td>4,5</td>
<td>6,7</td>
<td>-</td>
</tr>
<tr>
<td>Alighting passengers</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Feasible product set generation (cont.)

- Insertion to the existing schedule

<table>
<thead>
<tr>
<th>[Schedule Block]</th>
<th>SB₁</th>
<th>SB₂</th>
<th>SB₃</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Service type</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[Stops]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Location</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arrival Time</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Departure time</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boarding passengers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alighting passengers</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SB₁</th>
<th>SB₂</th>
<th>SB₃</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Location</strong></td>
<td><strong>Shared</strong></td>
<td><strong>Mini-bus</strong></td>
</tr>
<tr>
<td>a b c d</td>
<td>s₁ s₂ s₃ s₄</td>
<td>g h i j</td>
</tr>
<tr>
<td>9:00 9:15 9:30 9:50</td>
<td>d</td>
<td>- 15:00 15:10 15:20</td>
</tr>
<tr>
<td>1 2 - -</td>
<td>12:30</td>
<td>14:50 15:01 15:11 -</td>
</tr>
<tr>
<td>- - 1 2</td>
<td>-</td>
<td>4,5 6,7 - -</td>
</tr>
<tr>
<td>9:15 9:30 9:40 9:50</td>
<td>-</td>
<td>- 4 6 5,7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SB₁</th>
<th>SB₂</th>
<th>SB₃</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Location</strong></td>
<td><strong>Shared</strong></td>
<td><strong>Mini-bus</strong></td>
</tr>
<tr>
<td>a b c d</td>
<td>s₁ s₂ s₃ s₄</td>
<td>g h i j</td>
</tr>
<tr>
<td>9:00 9:15 9:30 9:50</td>
<td>d</td>
<td>- 15:00 15:10 15:20</td>
</tr>
<tr>
<td>1 2 - -</td>
<td>12:30</td>
<td>14:50 15:01 15:11 -</td>
</tr>
<tr>
<td>- - 1 2</td>
<td>-</td>
<td>4,5 6,7 - -</td>
</tr>
<tr>
<td>9:15 9:30 9:40 9:50</td>
<td>-</td>
<td>- 4 6 5,7</td>
</tr>
</tbody>
</table>
Transformation of the model

\[
\begin{align*}
\max & \sum_{n \in N} \sum_{m \in M} \sum_{l \in L} r_{n,m,l} \omega_{n,m,l} \\
\text{s.t.} & \sum_{n \in N} \sum_{m \in M} \sum_{l \in L} \omega_{n,m,l} = 1 - \omega_{\text{reject}} \\
& \sum_{n \in N} \sum_{l \in L} \frac{\omega_{n,m,l}}{\exp(\mu V_{n,m,l})} \leq \frac{\omega_{\text{reject}}}{\exp(\mu V_{\text{reject}})} \\
& 0 \leq \frac{\omega_{n,m,l}}{\exp(\mu V_{n,m,l})} \leq \frac{\omega_{\text{reject}}}{\exp(\mu V_{\text{reject}})}
\end{align*}
\]

\[\forall m \in M\]

\[\forall p_{n,m,l} \in F\]
Additional simulation results

Added value of dynamic allocation

x-axis: % change in consumer surplus with respect to FMOD-CS
y-axis: % profit in profit compared to FMOD-CS
Dynamic programming

- Dynamic programming Bellman equation:

\[
J_c(S_c) = \max_{x \in F_c} \{ E[R_c(x)] + E[J_{c+1}(S_{c+1})] \}
\]

where expected profit is:

\[
E[R_c(x)] = \sum_{n \in N} \sum_{m \in M} \sum_{l \in L} r_{n,m,l} \text{Prob}_{n,m,l}(x)
\]

- For each possible choice of the customer the state will be updated differently for the next customer.
Dynamic programming (cont.)

• The dynamic programming equation can be solved with the reference point being the last customer:
  – Given the feasible set of products we know what we will offer to the last customer: we will offer the best product we have at hand!

$$J_C(S_C) = \max_{x \in F_C} \{ E[R_C(x)] \}$$
Dynamic programming (cont.)

- The recursive function can be demonstrated as below
  - We know what we will decide at the last stage (last customer request)
  - The recursive function will enable to store the best decisions at each state for all possible feasible set of products

```python
function MAXPROFIT(c, Sc)
    if c==C then
        return E[R_C(x)]
    else
        return E[R_c(x)] + MAXPROFIT(c + 1, Sc+1)
    end if
end function
```