Equation (50)

I think we have an issue of clarity, which actually comes from the equation above, eq (49). Here, the meaning of the $i$ index is unclear. Because $g$ is a function of all components, $i$, the $\sum_i$ refers to a sum over components, which is most clearly seen with the $\sum_i \mu_i^\Theta c_i$ term. In the next term, because $\kappa$ is indexed by $i$ and $j$, they take on meaning of the coordinate directions there. However, there should still be concentration gradient terms from each component, $i$. A more clear way to write this equation is

$$g = \bar{g}\left(\{g(c_i)\}\right) + \sum_i \left(\mu_i^\Theta c_i + \frac{1}{2} \sum_j \sum_k (\partial_j \tilde{c}_i) \kappa_{jk} (\partial_k \tilde{c}_i)\right)$$

(1)

where $i$ indexes components and $j$ and $k$ index directions and the partial derivative with respect to direction $j$ is denoted $\partial_j$.

Then, when we want the chemical potential of a particular component $\ell$, we have that

$$\frac{\partial g}{\partial c_{\ell}} = \frac{\partial \bar{g}}{\partial c_{\ell}} + \frac{\partial}{\partial c_{\ell}} \sum_i \mu_i^\Theta c_i = \frac{\partial \bar{g}}{\partial c_{\ell}} + \mu_\ell^\Theta.$$  

(2)

Then, when we differentiate with respect to the gradient of $c_\ell$, we have

$$\frac{\partial}{\partial \nabla c_\ell} = \frac{1}{c_{\ell}^\Theta} \frac{\partial}{\partial \nabla \tilde{c}_\ell}.$$  

(3)

The only term in $g$ which contains gradients of $c$ (or $\tilde{c}$) is the final term, and when $\ell \neq i$, the derivative is zero, so the differentiation selects only the $\ell$th component. Then, what we will end up with (explanation of the factor of 2 follows below) is

$$\frac{\partial g}{\partial \nabla c_\ell} = \frac{1}{c_{\ell}^\Theta} \sum_k \kappa_{jk} \partial_k \tilde{c}_\ell$$

(4)

which is a rank-1 tensor indexed over directions by $j$. Then

$$-\nabla \cdot \frac{\partial g}{\partial \nabla c_\ell} = -\frac{1}{c_{\ell}^\Theta} \nabla \cdot \sum_k \kappa_{jk} \partial_k \tilde{c}_\ell$$

(5)

and replacing $\ell$ with $i$,

$$\mu_i - \mu_i^\Theta = \frac{\partial \bar{g}}{\partial c_i} - \nabla \cdot \sum_k \kappa_{jk} \partial_k \tilde{c}_i$$

(6)

$$= \frac{\partial \bar{g}}{\partial c_i} - \sum_j \sum_k \partial_j \kappa_{jk} \partial_k \tilde{c}_i$$

(7)

which is similar to what is in the paper, albeit with more clear summations distinguishing between species and directions.
Equation (57)

It would be reasonable to replace this with

$$\xi = \frac{c - c_A}{c_B - c_A},$$

and this might have been a more appropriate choice for the paper. Nevertheless, whether $\xi$ varies between 0 and 1 or 0 to -1 is not particularly important to the results. The primary goal of that section was to demonstrate the similarity between the developed model and the more familiar notation of an Allen-Cahn style equation. So yes, the negative of the term proposed might make more sense, but the results are identical.

Equation (60)

This seems to be a typesetting error that slipped by us. The arXiv print is correct on this one. The last term in the correct equation should be

$$-\frac{\kappa}{c_a} \nabla^2 \tilde{c},$$

not

$$-\frac{\kappa}{c_a} \tilde{\nabla}^2 \tilde{c},$$

Equation (75)

The second term on the right hand side is incorrect. The equation should be

$$c_0 k_B T \ln a_+ = W \tilde{c} + k_B T (\ln \tilde{\gamma}_+ + 1) - \frac{\partial \varepsilon_p}{\partial \tilde{c}_+} |\nabla \phi|^2.$$ (11)

Equation (85)

This should be the stress-free strain

$$\tilde{\mu} = \ln \frac{\tilde{c}}{1 - \tilde{c}} + \tilde{\Omega}(1 - 2\tilde{c}) - \tilde{\kappa} \tilde{\nabla}^2 \tilde{c} - \tilde{\sigma} : \tilde{\varepsilon}. $$ (12)