Capillary Stress and Structural Relaxation in Moist Granular Materials

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Supporting Information

ABSTRACT: A numerical and theoretical framework to address the poromechanical effect of capillary stress in complex mesoporous materials is proposed and exemplified for water sorption in cement. We first predict the capillary condensation/evaporation isotherm using lattice-gas simulations in a realistic nanogranular cement model. A phase-field model to calculate moisture-induced capillary stress is then introduced and applied to cement at different water contents. We show that capillary stress is an effective mechanism for eigenstress relaxation in granular heterogeneous porous media, which contributes to the durability of cement.

INTRODUCTION

Capillary condensation is a ubiquitous process of vapor–liquid phase transition in porous media, such as sand piles, plaster, paints, silica gels, and cementitious materials, which has an important yet poorly understood effect on mechanical behavior. The confined fluid can generate enormous local stresses, as observed in granular material aging, wet film friction, nanotribology, multiphase immiscible flows, cement drying shrinkage, and in everyday life experiences such as hardening of a drying sponge or building a sand castle on the beach. Capillary condensation and evaporation potentially bring undesirable fracture processes, as in drying and cracking of colloidal films and paints, but capillary stress can also be exploited in nanomaterial fabrication by capillary force lithography, capillary rise infiltration, evaporation-driven assembly and self-organization, and composite imbition and even used to evaluate the atmosphere of planets.

Despite the broad importance of capillary forces, they remain challenging to predict in complex porous materials over the full range of liquid saturation, either in equilibrium or during a dynamical process of drainage/imbibition. For granular or colloidal materials, existing models addressing partial saturations are oversimplified and only apply either to low humidity (the so called pendular/funicular regimes) or to idealized geometries (slit/cylindrical independent pores or a single sphere against a wall). At higher saturation, models based on geometrical analysis of the Young–Laplace equation for smaller clusters are proposed but restricted to only equilibrium liquid distributions inside monodisperse packings. A full molecular treatment here is beyond the current computational capability because that will require considering hundreds of millions of water molecules for our system.

Structural changes due to adsorption/desorption in porous media are known as "sorption-induced deformation". We refer the reader to Gor et al. for a review on the topic. We also refer the readers to the recent work of Schappert and Pelster reporting the poromechanics of Vycor, a porous silica glass, upon argon sorption. In this work, an unexpected sharp contraction followed by re-expansion was observed upon desorption. Notice that here, we are concerned with capillary stress-induced deformation only, therefore surface effects (Bangham effect) that might occur at low relative humidity are beyond the scope of this work.

In this article, we propose a numerical and theoretical framework to quantitatively predict capillary condensation/evaporation and compute associated capillary forces and structural relaxation in a 3D realistic nanogranular cement paste model using lattice-gas simulations of adsorbed water.
parametrized from atomistic simulations. In particular, this allowed us to access the adsorption/desorption mechanism and assess the role of (metastable) cavitation. In addition, we present for the first time to our knowledge, a phase-field model of the liquid—vapor mixture spatial distribution, whose inhomogeneous stress tensor is integrated over Voronoi polyhedra in order to calculate forces between each pair of neighboring grains. The capillary forces applied to the cement hydrate the nanograins in molecular dynamics (MD) simulations, and together with the cohesive interactions between these nanograins predict the overall stress relaxation. As an important application, we calculate drying shrinkage of cement paste, using a discrete lattice gas density functional theory (DFT) with interaction parameters imported from water/cement atomistic simulations. This DFT approach was first derived by Kiefl et al. for adsorption/desorption of a fluid in a quenched random porous solid. This method has been further applied to Vycor, controlled porous silica glasses and aerogels, to infer qualitatively adsorption/desorption isotherms from the minimization of the grand potential

\[ \Omega = -w_i \sum_{i,j} \rho_i \rho_j - w_i \sum_{i,f} \rho_i \eta_i - \mu \sum_i \rho_i + k_B T \sum_i \left( \rho_i \ln \rho_i + (1 - \rho_i) \ln(1 - \rho_i) \right) \]

where \( \rho_i \) is the normalized density of fluid on site \( i \), which can continuously vary from 0 to 1, \( \eta_i = 0 \) or \( \eta_i = 1 \), indicating if site \( i \) is occupied by solid or not. \( w_i \) and \( w_d \) are the fluid—fluid interactions and fluid—solid interactions, respectively, that are imported from atomistic simulation data. The cement paste porous structure used here comes from the out-of-equilibrium precipitation of cohesive interactions of calcium silicate hydrate (C–S–H) nanograins and has a realistic pore size distribution (gel pores and capillary pores below and above 3 nm, respectively) and connectivity (see the Supporting Information). They have volume fraction of 0.52 that corresponds to cement paste made with water to cement ratio (w/c) of 0.45. The lattice spacing, \( a \), of our DFT simulation is estimated from the surface tension that is energy per area \( \gamma \approx w_i / 2a^2 \) for nitrogen at \( T = 77 \) K, \( \gamma \approx 8.94 \) mN/m, which gives \( a \approx 0.345 \) nm; for water at \( T = 300 \) K, \( \gamma \approx 72 \) mN/m, which gives \( a \approx 0.24 \) nm. On the basis of these estimates, we choose a fine-grid cell size of \( a = 0.3 \) nm that is close to the molecular size. The fluid—fluid interaction, \( w_{ff} \), is determined by the bulk critical point \( k_BT_c = -\nu w_{ff}/2 \), where \( \nu \) is the number of nearest neighbors on the cubic lattice. The fluid—solid interaction, \( w_{sf} \), is estimated from molecular simulations of the isosteric heat of adsorption in the limit of zero coverage for water in the cement paste. Adopting the relation between chemical potential and relative saturating pressure, \( h \) (or relative humidity in the case of water) \( \mu = k_B T \ln(h) \), we show in Figure 1a that the lattice DFT method with appropriate grid fineness is able to quantitatively predict the room temperature hysteretic water adsorption/desorption isotherm in the cement paste in agreement with experiments with no adjustable parameter. In particular, the closure point of the hysteresis loop (\( h \approx 30\% \)) in the simulated isotherms is one that is observed in many water adsorption experiments at room temperature in disordered mesoporous materials.

Figure 1. (a) Simulated water adsorption/desorption isotherm of a mesoscale cement paste model of volume fraction, \( \eta = 0.52 \), compared to experimental data. The isotherm is renormalized with respect to the closure point. (b) Cross-section of the pore network showing water upon adsorption/desorption. Black—colloidal nanoparticles of CSH. Blue—water. Grey—vapor. Ink-bottleneck effect is exemplified by the red-circled areas.

There are many studies on water sorption in hardened cement paste reporting low pressure hysteresis loops in the adsorption/desorption isotherm.

There has been significant interest in the exchange of water molecules from the interlayer void inside the cement hydrates nanograins and gel pores similar to what is happening in clay that also exhibits low pressure hysteresis. Specifically, calcium clay exhibits smaller residual swelling than sodium clay because of the existence of the low pressure water adsorption/desorption isotherm at room temperature. More recently, Saiedpour and Wadsö using dynamic vapor sorption have shown the presence of a low pressure hysteresis. Along the same line, Baroghel-Bouny showed that water adsorption/desorption at 44 °C does not exhibit low pressure hysteresis, indicating that this is likely a kinetic issue due to the very large attractive electrostatic ICFs that probably do not allow the exchange of water molecules from the interlayer void inside the cement hydrates nanograins and gel pores that is said to occur at relative humidity lower than 30%. We assume no volume change of the cement hydrates nanograins.

The reversible part of the simulated adsorption/desorption isotherm curve shown in Figure 1a corresponds to the buildup of molecular nanometric films of various thicknesses depending on the local surface curvature. We found that metastable ink-bottleneck (cavitation) states are at the origin of sorption hysteresis in cement paste as shown in Figure 1b. The high connectivity of the cement paste pore network and the resulting percolating liquid distribution is

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Figure 2. Capillary stress—(a) simulation of local stresses in the capillary bridge between two spherical colloidal particles of diameter $D = 6 \text{ nm}$ at $h = 30\%$. (b) Total capillary force, $F$, on each particle (positive sign for pointing toward the other particle) vs humidity, $h$, compared with the two analytical solutions of the Kelvin–Laplace equation (because of the 3D geometry, see the Supporting Information eq 13) for sphere-center separation, $d = 0.9D$ (spherical particle diameter $D = 6 \text{ nm}$), including the surface adsorbed layer. (c) Cross-sectional view of the liquid density, $\rho$, filling the region between the three touching spherical particles, simulated at $h = 35\%$.

naturally obtained in our numerical framework without any ad hoc assumption on water distribution in contrast to the aforementioned pendular/unicular ring models.

**Capillary stress.** Next, we analyze the fluid distribution in the pore network (Figure 1b), and calculate the mechanical effect of capillary stress in such a complex porous structure. To calculate the capillary stress at a given $\mu$ (or $h$) value, the mean-field lattice gas DFT is written in its continuum limit, which is equivalent to the Cahn–Hilliard phase-field model (see e.g., eq 3.18 from Cahn and Hilliard and eq 59 from Bazant)\(^{58,59}\)

$$\Omega = \int (k_B T \mu \ln(\rho) + (1 - \rho) \ln(1 - \rho) - \mu \rho) dV + \int \frac{w_g}{2} \alpha^2 (\nabla \rho)^2 - \frac{w_s}{2} \rho \rho^3 dV + \int_{\partial V} \nabla \bar{S} \left( w_g \rho \nabla - \frac{w_s}{2} \rho \nabla \rho \right)$$

(2)

where the order parameter, $0 < \rho < 1$, here is the normalized liquid density, $\partial V$ represents the liquid–solid boundary, and all symbols are as explained after eq 1.

Once we have a suitable free energy, we define the capillary stress tensor, first derived by Korteweg\(^{60,62}\) (see eq 4 from Anderson et al.\(^{61}\))

$$\bar{\sigma} = \left( p_s (\rho) + \frac{\alpha^2 w_g}{2} (\nabla \rho)^2 \right) \mathbf{I} + \alpha^2 w_s \rho \nabla \otimes \nabla \rho + \bar{\sigma}_0$$

(3)

and use it to express the stress in terms of only the density profile from the DFT simulations in the porous structure. Here, $\mathbf{I}$ is the identity tensor, $\bar{\sigma}_0$ is an arbitrary tensor constant, $p_s (\rho) = \mu \rho + \frac{\alpha^2 w_s}{2} \rho^3 - k_B T \mu \ln(\rho) + (1 - \rho) \ln(1 - \rho)$ is the asymptotic bulk value of the hydrostatic pressure, and all symbols are as in eq 2 (see derivation for eq 3 in the Supporting Information).

In principle, surface forces can be calculated by integrating the normal stress over a space-filling tessellation of the microstructure. In this way, equal and opposite forces are applied at each face of the tessellation, perfectly satisfying Newton’s third law in the fluid, despite adsorbed fluid density fluctuations on the complex surface geometry. For colloidal or granular systems of convex particles, the most natural choice is the Voronoi tessellation, for which fast algorithms are available, such as the package Voronoi\(^{55}\) used below.

Using the lattice-gas DFT model for water, we first apply the method to the simplest case of two nanograins at short distance. Figure 2a shows a stable capillary bridge at appropriate relative humidity. Analytical results are available to describe the capillary force,\(^{21,57,64}\) and most models for wet granular materials rely on this picture of a capillary bridge.\(^{65–70}\)

The simulated capillary force versus humidity is shown in Figure 2b, in agreement with the solution of the Kelvin–Laplace equation,\(^{71–73}\) assuming bulk water surface tension. The forces are simulated on the adsorption branch and thus compared with the analytical solution that has the smaller Kelvin radius (in general the Kelvin equation for a capillary bridge between two spheres admits two solutions, see the Supporting Information), augmented by a wetting layer of thickness 0.25 nm.

Figure 2c shows three spheres nearly in contact to illustrate the challenges posed by any other geometry. At low humidity, the capillary bridge theory still holds and accurately predicts simulation results. However, at high humidity, these bridges coalesce to fill the region between the particles and drastically alter the forces, in a way that only a molecular-based approach can capture such as the DFT simulation used here. This is due in part to the liquid–vapor interface that takes a nonconvex shape in three dimensions.

Figure 3 shows the Voronoi tessellation\(^{85}\) on the 3D heterogeneous porous packing of polydisperse cement nanograins\(^{35}\) from which stresses can be efficiently and accurately computed. This example demonstrates the capability to predict the stress and deformation of porous materials for any spatial distribution of confined fluid as in the case of equilibrium capillary condensation during drying and wetting processes or in out-of-equilibrium multiphase flow.

**RESULTS**

Applying the capillary force on each nanograin in structural relaxation MD simulations, we compute the driving shrinkage of cement paste in quantitative agreement with the experimentally observed volume shrinkage (see Figure 4). Our findings indicate that capillary forces facilitate macro-
scopistic stress relaxation in granular or colloidal cohesive media with rough and glassy potential energy landscapes and having an abundance of metastable states.

Reactive heterogeneous materials such as cement contain residual stresses due to the out-of-equilibrium solidification process. The cement paste structure analyzed here has accumulated tensile eigenstresses of −47 MPa because of the out-of-equilibrium precipitation of nanogranod. We refer to this structure as "hardened" cement paste. Then, the eigenstress was relaxed (less than 10 kPa) using energy minimization under NPT conditions (see the Supporting Information). The relaxed cement paste structure is named "aged".

Figure 4a shows how capillary forces influence the shrinkage strain of "hardened" and "aged" cement paste. In a series of MD structural relaxation simulations, capillary forces calculated at corresponding relative humidities, $h$, were applied on the nanogranod as force vectors in addition to the cohesive interactions of cement hydrates nanogranod (see the Supporting Information). The "aged" model exhibits shrinkage only because of the capillary effect, whereas the "hardened" sample shrinks under the combined action of tensile eigenstresses and capillary forces. In the "aged" sample, the pure capillary effect decreases with increasing humidity, and the predicted strain is in agreement with the experiment$^{59}$ for $h > 30\%$ on the first drying cycle. In contrast, the "hardened" sample shrinks more than the "aged" one and with the opposite trend: the amount of shrinkage strain increases upon increasing humidity. This counter-intuitive behavior was also observed in short-term cycling $h$ cement experiments after long-term curing of the cement paste samples under a series of constant relative humidities.$^{42}$ Our relaxation scheme with capillary forces at constant $h$ coupled to nanogranod cohesive interactions would correspond to the long-term curing conditions of the experiments in ref 42. It is interesting to note that our relative strain results are in surprisingly good agreement with the total strain measurement of Figure 4 in ref 42.

In Figure 4b, we investigated stress relaxation under constant volume constraint. Minimal residual stresses are observed in the "aged" paste for all humidities. Residual stresses in the "hardened" paste were relaxed to about 30 MPa at $h = 30\%$ (closure point of the hysteresis loop), which is significantly different than that obtained under dry condition (−44 MPa). Overall, capillary forces help in relaxing eigenstresses with the lowest volume shrinkage. It is remarkable to notice the synergistic effect of capillaries and eigenstresses, reducing the shrinkage compared to dry or fully saturated conditions.

These results demonstrate that capillary stress at intermediate humidity is an effective mechanism of eigenstress relaxation that is developed during setting and, therefore, beneficial to cement durability. Although the magnitude of capillary forces on each nanograin is small (∼1 nN) compared to the grain−grain interactions (∼10 nN), these forces still have an observable effect at the mesoscale. For materials that are "softer" than cement paste (e.g., wood$^{78}$, biopolymers$^{79}$), capillary forces are expected to have even larger aging effects.

**CONCLUSIONS**

In conclusion, we have demonstrated that a mean-field lattice DFT approach with suitable molecular scale energy parameters and lattice discretization fineness is able to predict the hysteretic adsorption/desorption isotherm of water in a realistic mesoscale cement paste model, further validating this colloidal description of cement hydrates. We further extended this DFT approach to calculate capillary stress from the confined fluid distribution and formulated a general framework to calculate capillary stress in complex porous media, without any assumptions on the pore morphology or topology, across the full range of liquid saturation. Interestingly, enough, we found that when combined with the cohesive interactions of the nanogranod, the capillary stress at $h$ 30% actually helps in removing the eigenstress of the cement paste that results from the out-of-equilibrium setting conditions. Our approach is to predict for the first time capillary stress and structural relaxation in cement paste based on statistical physics combined with a realistic mesoscale texture, taking into account fluid−solid coupling explicitly. This is of both theoretical and practical importance because of cement’s multiscale challenging texture and its wide usage in everyday life.

**ASSOCIATED CONTENT**

* Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.langmuir.8b03400.

Phase-field formulation and capillary stress calculation, numerical simulations, microstructure of cement paste, simulation of adsorption/desorption isotherms, and structural relaxation with capillary stress. (PDF)
The authors declare no competing financial interest.

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