Financial Derivatives

18.095: IAP Mathematics Lecture Series 2000
Homework Problems for Lecture 5

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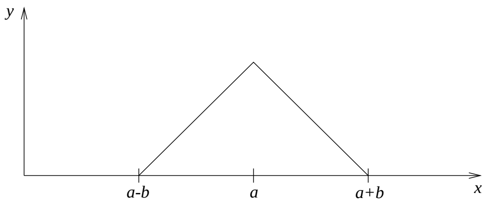

Figure 1: A butterfly spread payoff.

1. Explain how the "butterfly spread" option in Fig. 1 is equivalent to a certain linear combination of put and call options.

2. Consider a derivative with the general linear payoff function $y(x) = ax + b$, where a and b are constants. Construct a static hedged position $u_o = w - \phi x_o$ (by choosing u_o and ϕ) that is riskless (i.e. nonrandom at maturity), and thus determine the fair price of w. Verify explicitly that your value of ϕ agrees with the optimal hedge ratio ϕ^* formula for minimal quadratic risk.

3. Consider the general binomial model with outcomes x_1 and x_2 and probabilities p_1 and $p_2 = 1 - p_1$. Verify explicitly that the quadratic residual risk R^* vanishes. Find an effective probability p_1^* of outcome x_1 such that $\langle x \rangle = x_o$, and show that the risk-neutral valuation formula $w = p_1^* y_1 + p_2^* y_2$ gives the fair price.

4. Assume the following uniform transition probability

$$p(x|x_o) = \begin{cases} \frac{1}{2a} & \text{if } x_o - a \leq x \leq x_o + a \\ 0 & \text{otherwise} \end{cases}$$

for the underlying asset. Find the fair price(s) $w(x_o)$ for a call option struck at K that minimize either

1. the "quadratic risk" R, or
2. the "absolute-value risk" R'.

The first case can be done with the formulae in the notes, but the second will require some *geometric* thinking! Are the prices the same? Are the residual risks R^* and R'^* the same?