# MODELING OF MULTI-JOINT MOTOR SYSTEMS

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**Abstract.** *A model of elastic mechanical systems is presented in the form of a network of constraint equations expressing the geometrical relations among component elements and their steady-state mechanical behavior. The model takes into account the non-linear features of motor system geometry and is used to represent mechanical interactions with the environment as well as to derive appropriate patterns of control inputs given a wide variety of motor tasks. This approach can be applied to the analysis of biological motor systems.*

## Introduction

Several experimental investigations of biological systems [1,3,9,10] have focused on the role of muscle mechanical properties in motor control, suggesting that a muscle is mechanically analogous to a "tunable" spring: i.e. it is characterized by a set of integrable functions between length and tension at steady state. The neural input to a muscle selects a particular function (a length/tension curve) out of this set. The equilibrium position and the stiffness of a joint is then defined, for any given value of muscle activation, as the position at which the length-dependent forces of opposing muscles generate equal and opposite torques about the joint. This view of posture has been more recently extended to the analysis of movement and trajectory formation [2,5]. It has been proposed that arm movements are represented and generated by the central nervous system (CNS) as smooth transitions in posture along *virtual trajectories*. The virtual trajectory is given as a time sequence of equilibrium configurations defined by the muscle elastic properties. Experimental and simulation evidence supporting this view has been obtained for single-joint movements [2] as well as for multi-joint arm trajectories [4,8].

In this paper we present a framework for modeling the mechanical properties of springs and linkages. Using this system we can model the static interactive behavior of a manipulator acting within its environment. This system is then used to test the performance of an adaptive control update procedure in which changes in the control signals to the actuators of a redundant system are computed to achieve a desired endpoint state (position and effort). Finally, an example is given where the system is used to model the control of a human arm.

## Modeling Elastic Systems

We have taken a modular approach to modeling mechanical systems in which the characteristics of the manipulator are lumped in discrete elastic elements connected by non-energic junctions. This approach is analogous to modeling with bond graphs [6], but since we are interested primarily in the steady state elastic properties of the system, the variables of interest are the efforts and positions (e.g. force and length) of the elements, instead of the efforts and flows (velocities). Using this approach, the goal is to compute either a net output effort for the system at a given position, or compute the resultant position for a specified applied effort. In addition, we wish to compute the net stiffness ($k = de/dp$) and/or the net compliance ($c = dp/de$) at the endpoint.

**Primitive Elements:** The basic building blocks used in these model systems are described as tunable, generalized springs in which the *effort output* ($e$) of the element is an integrable function of its *position* ($p$), or conversely, the position of the element is an integrable function of the applied effort. Each element typically has a *control input* ($u$) which acts to modify this relationship between position and effort. Thus we have two types of elements, a stiffness element:

$$e = f(p,u); \qquad k = \frac{\partial f}{\partial p}; \qquad c = k^{-1}\ (k \neq 0).$$

or a compliance element:

$$p = g(e,u); \qquad c = \frac{\partial g}{\partial e}; \qquad k = c^{-1}\ (c \neq 0).$$

**Transformers:** Mechanical linkages may be used to transform the actions of an elastic element from one physical domain to another. For instance, modeling a set of muscles acting around the joints of an arm involves the transformation from joint angles to muscle lengths and muscle forces to joint torques. These structures can be modeled by a *transformer* element which, when connected to a component element having position $p$ and effort $e$, forms a compound element whose state is defined in a different coordinate system, with position $P$ and effort $E$. When the transformation from compound position $P$ to component position $p$ is well defined, as in the case of joint angles to muscle lengths, the transformer produces a stiffness element described by:

$$p = L(P); \quad E = J(P)^t e; \quad K = J^t(P)kJ(P) + \Gamma.$$

$$\text{where } J = \tfrac{\partial p}{\partial P} \text{ and } \Gamma_{ij} = \textstyle\sum \tfrac{\partial^2 p}{\partial P_i \partial P_j} e_k.$$

Conversely, given the transformation $p$ to $P$, a transformer produces a compliance element. The transformation from joint angles ($p$) to tip position ($P$) for a redundant, multi-link arm is described by such an element. The constraint equations for this transformer are:

$$P = L(p); \qquad e = J^t(p)E; \qquad C = J(p)(k-\gamma)^{-1}J^t(p).$$

$$\text{where } J = \tfrac{\partial P}{\partial p} \text{ and } \gamma_{ij} = \textstyle\sum \tfrac{\partial^2 P}{\partial p_i \partial p_j} E_k.$$

**Summing Junctions:** Two or more elements may be connected together to form a composite stiffness element by a *common position junction* in which the same position is imposed on each element. This corresponds to a set of springs connected in parallel. The constraint equations describing this type of junction are:

$$p_i = P; \qquad E = \sum e_i; \qquad K = \sum k_i.$$

Similarly, a *common effort junction* (springs in series) generates a compliance element via constraint equations:

$$e_i = E; \qquad P = \sum p_i; \qquad C = \sum c_i.$$

## Implementation

We have implemented a computer system for the simulation of elastic motor systems, based on the equations presented above.

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The system is implemented in LISP on a Symbolics 3600 series computer, using an object-oriented programming approach. Each element is represented by an instance of a flavor, with pointers to the objects to which it is connected.

**Numerical Methods:** A real physical system produces a mechanical response to perturbations in both position and effort from the environment, but the causality constraints of the model may prevent us from solving for this response directly. Numerical methods can be used to solve for the steady state response in these cases.

The simplest approach is to use the stiffness or compliance at the endpoint to search for the equilibrium response using the Newton-Raphson method. To drive a stiffness element to a desired output effort ($e_d$), the algorithm is:

$$\textit{while}\ (e \neq e_d)\ \textit{do}\ \{p_{n+1} = p_n + k_n(e_d - e_n)\}.$$

For a compliance element, it is necessary to compute the effort output for a desired position change:

$$\textit{while}\ (p \neq p_d)\ \textit{do}\ \{e_{n+1} = e_n + c_n(p_d - p_n)\}.$$

## Control Input Update

In order to control the behavior of a manipulator modeled in this way, it is necessary to solve the inverse control problem. That is, one must compute a time series of control input values which will produce the desired output states. In particular, it is necessary to compute a change in control input $u$, which causes the manipulator to change from one equilibrium state, $(p_0, e_0)$, to another $(p_1, e_1)$. This is typically an ill-posed problem for biological systems. A pseudo-inverse solution to the problem has been presented in which active changes in input values are computed in response to passive displacements of the system [7]. The implementation of this solution is described here:

Each element in the model is able to perform two complementary functions: compute a change in input which will produce a desired change in position with zero change in effort $(\partial u/\partial p)$ and compute a change in input that causes a desired change in effort without affecting the position$(\partial u/\partial p)$. These transformations are combined to produce arbitrary changes in position and effort at the endpoint. One of these two operations is presumed to be defined for each of the primitive elements. Transformers and junction elements achieve these operations by invoking the transformations of its component elements. In this way desired changes in the output of a system are propagated to produce changes in input at the level of the primitive elements.

Causality constraints may preclude the direct computation of $\partial u/\partial p$ or $\partial u/\partial e$. In these cases, the stiffness or compliance of the element can be used to transform from one computation to the other. For stiffness and compliance elements we have, respectively:

$$\frac{\partial u}{\partial e} = \frac{\partial u}{\partial p}c; \qquad \frac{\partial u}{\partial p} = \frac{\partial u}{\partial e}k.$$

Junction elements combine the appropriate transformations from each component element to compute a change in input for each element. For a common position junction:

$$\frac{\partial u_i}{\partial P} = \frac{\partial u_i}{\partial p_i}$$

Common effort junctions:

$$\frac{\partial u_i}{\partial E} = \frac{\partial u_i}{\partial e_i}$$

Transformer elements must compute the appropriate change in input for for the component element to produce the desired change at the endpoint. For changes in output effort:

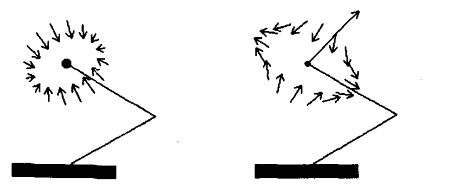
$$\frac{\partial u}{\partial E} = \frac{\partial u}{\partial e}J^t \quad \textit{with}\ J = \frac{\partial P}{\partial p}.$$

The dependence of the jacobian on the position of an element creates a complication when computing input changes for desired displacements of position. An element producing the same effort output at the new position may cause a different effort output at the endpoint. To correct for this we have:

$$\frac{\partial u}{\partial P} = \frac{\partial u}{\partial p}JC(J^t kJ) \quad \textit{with}\ J = \frac{\partial p}{\partial P}.$$

## Applications

Figure 1 shows the initial and final control values which produced the illustrated change in output force at the end of the two link arm. Note that the model predicts an unstable stiffness field at the endpoint. This leads to the prediction that human subjects may co-contract agonist/antagonist pairs of muscles in order to increase joint stiffnesses and stabilize a high force load.



a) Tip Position P = (0,1)
Output Force E = (0,0)
Inputs u = (.2,.2,.2,.2,.2,.2)

b) Tip Position P = (0,1)
Output Force E = (5,5)
Inputs u = (0,.526,0,.691,0,.452)

*Figure 1: Simulation of the postural characteristics for a two link planar arm with six muscles. Each muscle is modeled by a tunable spring with its stiffness and rest-length a linear function of input. The arrows surrounding the tip illustrate the stiffness field computed for small displacements at the endpoint. (a) Initial state of the limb. (b) Final state achieving a desired force output.*

## Conclusions

We have presented a formalism for modeling the static or quasi-static behavior of networks of elastic mechanical devices. These models can then be used to compute the control inputs required to produce a desired behavior by the system. Models such as these will provide valuable tools for the study of human motor performance.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