Mass Transfer Notation

We will use a mass transfer notation that differs slightly from that presented in the handout from Cussler. Our notation is a bit more detailed and follows that used in the 10.302 text 'Fundamentals of Heat & Mass Transfer" by Incropera & Dewitt.

\( C_A = \) molar concentration of component A, (mol/L, mmol/L, etc.)

\( C = \) total molar concentration of all species = \[ \frac{1}{n} \sum_{i=1}^{n} C_i \]

\( x_A = \) mole fraction of component A

\( \rho_A = \) mass concentration of component A (gm/L, mg/mL, etc.) = \( M_A C_A \)

\( \rho = \) total mass concentration of all species = \[ \frac{1}{n} \sum_{i=1}^{n} \rho_i \]

\( m_A = \) mass fraction of component A

\( D_{AB} = \) diffusion coefficient of A in B (cm\(^2\)/s, m\(^2\)/s, etc.)

**Fick's Law (Diffusive Flux of Component A)**

Fick's law gives the diffusive flux relative to the average velocity of the fluid mixture. In the absence of any convective transport (velocity = 0), the diffusive flux and total flux are the same. Flux = rate of mass transfer per unit surface area normal to the direction of transport; units are mol/cm\(^2\)-s or gm/cm\(^2\)-s.

In terms of molar flux (mol/cm\(^2\)-s):

\[
J_{A}^* = -CD_{AB} \nabla x_A
\]

if \( C \) is constant:

\[
J_{A}^* = -D_{AB} \nabla c_A
\]

for one-dimensional diffusion:

\[
J_{A}^* = -D_{AB} \frac{dc_A}{dx}
\]

In terms of mass flux (gm/cm\(^2\)-s):

\[
j_A = -\rho D_{AB} \nabla m_A
\]

if \( \rho \) is constant:

\[
j_A = -D_{AB} \nabla \rho_A
\]

for one-dimensional diffusion:

\[
j_A = -D_{AB} \frac{d\rho_A}{dx}
\]
**MASS FLUX RELATIVE TO STATIONARY COORDINATES**

Fick's law applies to diffusion relative to the average velocity of the fluid. Diffusion relative to stationary coordinates is as follows:

<table>
<thead>
<tr>
<th>In terms of molar flux (mol/cm²-s):</th>
<th>In terms of mass flux (gm/cm²-s):</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{N}<em>A^* = J_A^* + C_A v^* = -CD</em>{AB} \nabla x_A + C_A v^* )</td>
<td>( \dot{n}<em>A^* = j_A + \rho_A v = -\rho D</em>{AB} \nabla m_A + \rho_A v )</td>
</tr>
<tr>
<td>where ( v^* ) = molar average velocity of fluid: ( v^* = x_A v_A + x_B v_B )</td>
<td>where ( v ) = mass average velocity of fluid: ( v = m_A v_A + m_B v_B )</td>
</tr>
<tr>
<td>total molar flux relative to stationary coordinates: ( \dot{N}^* = \dot{N}_A^* + \dot{N}_B^* = Cv^* = C_A v_A + C_B v_B )</td>
<td>total mass flux relative to stationary coordinates: ( \dot{n}^* = \dot{n}_A^* + \dot{n}_B^* = \rho v = \rho_A v_A + \rho_B v_B )</td>
</tr>
</tbody>
</table>

**DIFFERENCES BETWEEN THIS NOTATION AND THAT OF CUSSLER**

Cussler uses the following notation, which I find a bit more vague

\( J_1 \) = diffusive molar mass transfer rate, mol/s (NOT a flux! even though it says so in the book). The "1" subscript denotes it is for the 1-dimensional case (the "1" does not help us figure out which direction"

\( j_1 \) = diffusive molar mass flux, mol/cm²-s (Note -- a flux is a rate per unit area, so the statement in Cussler that \( j_1 \) is a "flux per unit area" is semantically flawed.)

\( n_1 \) = total molar or mass transfer rate relative to stationary coordinates

\( v \) = mass average velocity of fluid

\( v^* \) = molar average velocity of fluid

**CORRECTION OF LIQUID DIFFUSION COEFFICIENTS FOR TEMPERATURE**

\[ D_{AB,Temp_2} = D_{AB,Temp_1} \frac{T_2 \mu_1}{T_1 \mu_2} \] where \( \mu_1 \) and \( \mu_2 \) are the solvent viscosities at temperatures \( T_1 \) and \( T_2 \).