24.111: Philosophy of Quantum Mechanics, Spring 2016
Homework 2 Solutions

1. (2pts) Write the state $|0^\circ \downarrow\rangle$ as a linear combination of $|60^\circ \uparrow\rangle$ and $|60^\circ \downarrow\rangle$.

Solution: $|0 \uparrow\rangle = 1/2\,|60 \uparrow\rangle + \sqrt{3}/2\,|60 \downarrow\rangle$.

2. (2 pts) What (according to the statistical algorithm) is the probability that an electron in the state $|40^\circ \downarrow\rangle$ will be deflected up through magnets oriented at 15°?

Solution: one way to compute the answer is to compute $\langle 40 \downarrow | 15 \uparrow\rangle^2 = (\sin(25/2))^2 \approx .047$.

3. (4 pts) Show that Born's rule, in the statistical algorithm, entails the $\cos^2$ law describing the observed behavior of electrons in spin experiments.

(For purposes of this question, take the law to be this: if an electron was just deflected up through magnets oriented at θ_1, and has not passed any S-G magnets since, then the probability that that electron will be deflected up through magnets oriented at θ_2 is $\cos^2((\theta_1 - \theta_2)/2)$.[1] You will need to remember / look up a trig identity.)

Solution: By Born's rule, the probability is

$$\begin{aligned}
\langle \theta_1 \uparrow | \theta_2 \uparrow\rangle^2 &= [\big(\cos(\theta_1/2), \sin(\theta_1/2)\big) \cdot \big(\cos(\theta_2/2), \sin(\theta_2/2)\big)]^2 \\
&= [\cos(\theta_1/2)\cos(\theta_2/2) + \sin(\theta_1/2)\sin(\theta_2/2)]^2 \\
&= [\cos(\theta_1/2 - \theta_2/2)]^2.
\end{aligned}$$

where the last line comes from a standard trig identity.

[1] The law as I wrote it on the slide spoke of what proportion of a large number of electrons will be deflected up, not what the probability is that an individual electron will be deflected up. The differences between these are (philosophically) important, but too distracting to worry about at this point in the course.

4. (4 pts) Let v be a unit-length vector in $\mathbb{R}^2$ that we have associated with some electron for the purposes of making predictions about experiments on that electron. Explain why, as far as the statistical algorithm is concerned, it make no difference whether we associate v or $-v$ with the electron.

The statistical algorithm uses v to assign probabilities to outcomes of experiments. It says that the probability of the outcome associated with a vector w (eg $|0\uparrow\rangle$ is associated with the 'up' outcome of a spin-at-0 measurement) is equal to $(v \cdot w)^2$. But because the dot product is linear, $((-v) \cdot w)^2 = ((-1)(v \cdot w))^2 = (v \cdot w)^2$. So v and $-v$ give the same probabilities to all outcomes.

5. (4 pts) Suppose we had a (fictional) quantum-mechanical system. We find that when we perform a certain experiment (measurement) on the system, we always get one of three outcomes: green, red, or blue. (i) What vector space would you choose to represent states of the system, for the purposes of predicting outcomes of this kind of experiment? (ii) Choose vectors to represent the three outcomes of the experiment (and say what they are; note that QM does not allow just any three vectors to represent the outcomes, there are rules). (iii) Now choose a "state" vector for your system other than one of these three (and say what it is). (iv) Compute the probability that an experiment on a system in the state you choose will have either a red or a green outcome.

Solution (for example): use $\mathbb{R}^3$. Let $(1, 0, 0)$ be green, $(0, 1, 0)$ be red, and $(0, 0, 1)$ be blue. Then $(1\sqrt{2}, 1\sqrt{2}, 0)$ (or any other unit length vector) is a possible state. The probability of getting either red or green in this example is 1.