24.111: Philosophy of Quantum Mechanics, Spring 2016
Homework 3

1. Read the excerpt on the readings page from Feynmann's Lecture on Physics, on the two-slit experiment. This experiment illustrates the "mystery of superposition" in much the same way the two-path experiment (discussed in class) does. The goal of this question is for you to see how close the parallel is. Consider these three hypotheses:

(1) Every electron (in the two-slit experiment) either passes through the top slit or passes through the bottom slit (but not both).

(2) Some electrons pass through both slits.

(3) Some electrons pass through neither slit.

For each of these hypotheses, provide arguments (reasons to believe) that it is false. (Note! Do not spend lots of time looking in the Feynman for the arguments — though his description of the experiment is definitely relevant. It's not clear that Feynman is trying to provide the arguments I am asking for. It's better to think about the analogous arguments with the two-path experiment.)

2. Suppose an electron in state $|0\uparrow\rangle$, followed by an electron in state $|180\downarrow\rangle$, will pass through magnets oriented at 180°. Will they be deflected in the same direction? Different directions? Or are you unable to make any prediction with certainty? Justify your answer.

3. In the following questions we have two electrons, and we represent their joint spin state with vectors in the tensor product space $\mathbb{R}^2 \otimes \mathbb{R}^2$. So in product vectors $|x\rangle\,|y\rangle$ the "vector on the left" $|x\rangle$ represents the state of electron 1, the vector on the right, the state of electron 2.

Show, in painstakingly pedantic detail, that the product vector $|0\uparrow\rangle\,|0\downarrow\rangle$ is orthogonal to $|0\uparrow\rangle\,|0\uparrow\rangle$. (This is not hard, if you look up how the inner product of product vectors is defined in the notes.)

4. Write the electron-pair state $\frac{1}{\sqrt{2}}\,|0\uparrow\rangle\,|0\downarrow\rangle - \frac{1}{\sqrt{2}}\,|0\downarrow\rangle\,|0\uparrow\rangle$ as a linear combination of the basis vectors $|90\uparrow\rangle\,|90\uparrow\rangle\,,|90\uparrow\rangle\,|90\downarrow\rangle\,,|90\downarrow\rangle\,|90\uparrow\rangle\,,|90\downarrow\rangle\,|90\downarrow\rangle$. (Substitute $|0\uparrow\rangle = \frac{1}{\sqrt{2}}(|90\uparrow\rangle - |90\downarrow\rangle$, and a similar expression for $|0\downarrow\rangle$, into the above expression and use the linearity of the tensor product operation. For full credit you must show your work, not just write down the answer.)

5. Use the statistical algorithm to compute the probability that electron 1 will be deflected up through magnets at 0 degrees, when the state of the pair is $|90\uparrow\rangle\,|62\downarrow\rangle$.

6. Similarly, compute the probability that electron 1 will be deflected up through magnets at 0 degrees, when the state of the pair is $1/\sqrt{2}\,|0\uparrow\rangle\,|90\uparrow\rangle + 1/\sqrt{2}\,|0\uparrow\rangle\,|90\downarrow\rangle$.