24.111: Philosophy of Quantum Mechanics, Spring 2016
Homework 6: GRW

1. Suppose that before a GRW collapse happens the wavefunction of a two-particle system is as below, where, for example, $|x\rangle_1$ is a state vector for particle 1 in which it is definitely located at point of space x. Suppose then that a collapse does happen. (i) Using the easy version of GRW, what is the probability that the collapse is associated with particle 1? (ii) Using the easy version of GRW, what is the probability that, **if the collapse is associated with particle 2**, then after the collapse particle 1 has a definite position?

$$\frac{1}{\sqrt{3}}\,|x\rangle_1\,|y\rangle_2 + \frac{1}{\sqrt{3}}\,|x\rangle_1\,|z\rangle_2 + \frac{1}{\sqrt{3}}\,|y\rangle_1\,|z\rangle_2\,.$$

Solution: (i) 1/2. (ii) the official way to figure this is to rewrite the wavefunction so that, in each term, particle 2 has a definite position, and in no two terms does it have the same definite position. Do that and you get

$$\frac{1}{\sqrt{3}}\,|x\rangle_1\,|y\rangle_2+\frac{1}{\sqrt{3}}(|x\rangle_1+|y\rangle_1)\,|z\rangle_2 = \frac{1}{\sqrt{3}}\,|x\rangle_1\,|y\rangle_2+\sqrt{2}\frac{1}{\sqrt{3}}(\frac{1}{\sqrt{2}}(|x\rangle_1+|y\rangle_1))\,|z\rangle_2\,.$$

Particle 1 has a definite position only if the collapse puts particle 2 into the state $|y\rangle$, thereby collapsing this whole wavefunction onto its first term.

2. The statistical algorithm is a **procedure for predicting the outcomes of experiments**. It's not a **theory of what the world is like** (at least, not of what it is like when no experiments are going on). An **interpretation of quantum mechanics** (and interpretations of QM are supposed to be theories of what the world is like) is worth taking seriously only if it has this feature: that interpretation can explain why, if it is true, the statistical algorithm is a good procedure for predicting outcomes (for this question, we can take "good" to mean: the probabilities the algorithm assigns to outcomes are equal to, or extremely close to, the "true" probabilities of those outcomes). The question: how does GRW explain why the statistical algorithm is good?

To make the question concrete: if I set out to measure the spin at 0 degrees of an electron in state $|90\uparrow\rangle$, then the statistical algorithm says that the up outcome has a probability of .5. In your answer I want you to explain why, according to (the easy version of) GRW, this is the correct probability. (Or at least: explain what the proponents of GRW hope is the correct answer to this question.)

(That's a pretty "open ended" question. Here are a few hints: you'll want to say something what must be going on, physically, when I set out to measure the electron's spin. For full credit you're going to have to write out a wavefunction explicitly, maybe more than one. You're going to have to go through what GRW says about wavefunction collapse.)

Solution: When you measure this electron's spin, it is going to interact with some measuring device made up of a ton of particles. Writing $|r\rangle_n, |u\rangle_n, |d\rangle_n$ for the nth particle being located where it needs to be for the device's pointer to indicate 'ready,' 'up,' or 'down,' respectively, the wave function starts as $|90\uparrow\rangle |r\rangle_1 \ldots |r\rangle_n$, then changes to $\frac{1}{\sqrt{2}}|0\uparrow\rangle |u\rangle_1 \ldots |u\rangle_n + \frac{1}{\sqrt{2}}|0\downarrow\rangle |d\rangle_1 \ldots |d\rangle_n$. Since n is so large, almost immediately there will be a collapse. The collapse law says that there's a $1/\text{n}$ chance for each particle to be associated with the collapse (I am ignoring the position of the electron). Suppose it's associated with the kth particle. The above wavefunction is already written in the right form for figuring probabilities of post-collapse wavefunction; the rule says that it's a 50-50 chance for each term. But the first term is one in which the outcome of the measurement was 'up,' and the second is one in which the outcome was down.

3. Many philosophers endorse the following thesis about the connection between mental properties (like the property of believing that it is raining, or the property of feeling a pain in one's knee, or the property of believing that electron e is a spin-up-at-0-degrees electron) and physical properties (like the property of having one's neurons firing at a certain rate, or the property of having exactly three million electrons inside one's head): a person's physical properties "determine" their mental properties. More specifically,

(X) Two people can't have different mental properties unless they there is at least one physical property that one them has that the other does not.

Now think about the John argument. The first premise was: John is a possible person/thinker. Now a defender of GRW might respond to the John argument by saying that this premise is false. But what could be his reason for rejecting it? He could say: there is some necessary truth about thinkers that John would violate, if he existed. (X) is a candidate truth of this sort; but it cannot be used for this purpose. John, if he existed, would not be in conflict with (X). Your task for this question is: explain why.

Solution: Suppose John, and someone else, have different mental properties. Say, John believes that the electron he just sent through his head is an up at 0 electron, and Joan believes that the electron she just sent through her head is a down electron. This scenario doesn't seem like a threat to (X). For either Joan's brain is hooked up just like John's, or not. If it is hooked up just like John's, then the particle inside her head is 'down,' while John's is 'up.' The particles are in different positions. And the positions of the particles that make up your brain is a physical property of a person. So John and Joan have different physical properties. Now if Joan's brain is hooked up differently from John's, say it records the spin of the electron she passed through her head otherwise than in the position of a particle (maybe it is recorded in the position of a big pointer instead), then automatically they always have different physical properties.