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MATHEMATICAL EXPLANATION AND INDISPENSABILITY ARGUMENTS

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We defend Joseph Melia's thesis that the role of mathematics in scientific theory is to 'index' quantities, and that even if mathematics is indispensable to scientific explanations of concrete phenomena, it does not explain any of those phenomena. This thesis is defended against objections by Mark Colyvan and Alan Baker.

I. INTRODUCTION

Is the role of mathematics in scientific theories genuinely explanatory? Much recent discussion of this issue has taken place in the context of discussions of indispensability arguments for mathematical Platonism ('Platonism' for short), the view that mind-independent mathematical entities exist. The common pattern of these arguments runs as follows. First-order regimentations of the best scientific theories unavoidably quantify over mathematical entities such as sets (the indispensability thesis).[1] Moreover, scientists are ontologically committed to all those entities that are indispensable to the best current scientific theories. It follows that they are ontologically committed to all the mathematical entities quantified over in the best scientific theories.[2]

Joseph Melia has provided a powerful strategy for dealing with the indispensability argument for Platonism.[3] Briefly, he has argued that to establish Platonism it is not enough to show that mathematics is indispensable to best science. What must be shown is that the entities quantified over in

[1] This premise can be and has been contested, but we shall not engage with arguments on this issue here. See H. Field, *Science without Numbers* (Oxford: Blackwell, 1980), for an attempt to dispense with mathematics.

[2] See W.V. Quine, 'Two Dogmas of Empiricism', *Philosophical Review*, 60 (1951), pp. 20–43, §VI, and H. Putnam, *Philosophy of Logic* (New York: Harper & Row, 1971), §VIII, for early statements of this style of argument.

[3] J. Melia, 'Weaseling Away the Indispensability Argument', *Mind*, 109 (2000), pp. 455–79.

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mathematics have an indispensable and *genuinely explanatory* role to play in best science. Otherwise it can be argued that mathematics is indispensable merely in *indexing* physical facts, and that the mathematics itself is not explanatory. Melia claims that if the role of mathematics is one of indexing, not explaining, there is no good reason to believe that there are mathematical entities. We shall call this 'Melia's indexing argument'.[4] We consider the argument in more detail in §II.

Mark Colyvan and Alan Baker have sought to defend Platonism by arguing, with the use of specific examples, that mathematics *does* play an indispensable explanatory role in science.[5] We consider their defences of Platonism in turn in §§III and IV. We shall argue that their defences are unsuccessful, and that they fail because ultimately they do not provide compelling reasons to think that the role of mathematics in the examples they describe is anything other than to index physical facts.

II. MELIA AND THEORETICAL INDISPENSABILITY

Some philosophers (notably Quine) claim that the postulation of unobservable concrete entities, such as atoms, is justified by their theoretical indispensability. They also think that these considerations lend the same kind of support to the postulation of abstract objects, specifically of mathematical objects. Melia thinks that Quine and company are mistaken about the latter claim. He offers the following account of how the postulation of concrete unobservables brings explanatory benefits. By postulating comparatively few basic kinds of unobservables, and comparatively few fundamental ways of arranging them, science is able to explain the behaviour of a wide range of diverse observable objects economically and elegantly. Theories which shun unobservables are left with a motley collection of observables which are not explicable in such simple terms. The world, it seems, is a simpler place if there are unobservables.

Melia provides an example to illustrate why these considerations are of no use to the Platonist. Physical theories describe spatiotemporal relations

[4] Similar arguments have since been offered by M. Leng, 'What's Wrong with Indispensability?', *Synthese*, 131 (2002), pp. 395–417, §§5–6, who speaks of mathematics as merely *modelling* concrete facts, and C. Pincock, 'A Revealing Flaw in Colyvan's Indispensability Argument', *Philosophy of Science*, 71 (2004), pp. 61–79, §3, who speaks of mathematics as merely *mapping onto* concrete facts.

[5] M. Colyvan, *The Indispensability of Mathematics*, henceforth *IM* (Oxford UP, 2001), and 'Mathematics and Aesthetic Considerations in Science', henceforth MACS, *Mind*, 111 (2002), pp. 69–74; A. Baker, 'Are There Genuine Mathematical Explanations of Physical Facts?', *Mind*, 114 (2005), pp. 223–38.

between objects. This requires the theories to have predicates ascribing distances between objects. Theories may do this in different ways. Suppose theory T_1 takes the 2-place predicates '*x* is 3 metres from *y*', '*x* is $^7/_{11}$ metres from *y*', and infinitely many others, as primitive. Suppose theory T_2 includes the single 3-place predicate '*x* is *r* metres from *y*', where the variable '*r*' ranges over real numbers, plus enough names for numbers for T_2 to be able to ascribe the same distance relations to objects as T_1. T_1 has a simple ontology, but has to take infinitely many predicates as primitive. T_2 has fewer primitive predicates than T_1, but has a less simple ontology. Which is the overall simpler theory?

T_2 can express an infinite number of distance relations in a simple way. It uses fewer primitive predicates than T_1. It is capable of generating many distance predicates using only a few primitives. Nevertheless, Melia claims (p. 473), it does not follow that 'the distance relations expressed by these predicates are themselves not primitive, irreducible relations'.

This can be made clear by an example. Suppose T_2 expresses the fact that

(i) *a* is $^7/_{11}$ metres from *b*

by using a 3-place predicate relating *a* and *b* to the number $^7/_{11}$. (i), however, does not hold in virtue of the following fact:

(ii) A 3-place relation holds between *a*, *b* and the number $^7/_{11}$.

Melia is not to be taken as denying that (ii) is a fact. He is denying that (i) obtains *because* (ii) does. We suggest that he might give the following reason for this. It seems doubtful that

(iii) *a* is 63.63 centimetres from *b*

obtains in virtue of the fact that a 3-place relation holds between *a*, *b* and the number $^7/_{11}$. If anything, if (iii) obtains in virtue of a 3-place relation between *a*, *b* and a number, that number would be the number 63.63, not the number $^7/_{11}$. That is, it seems doubtful that (iii) obtains in virtue of (ii). But (i) is the same fact as (iii). (A theory such as T_2 can use different 3-place predicates ('is $^7/_{11}$ metres from', 'is 63.63 centimetres from') to describe one and the same fact about the distance between *a* and *b*.) But then it should be doubtful that (i) obtains in virtue of (ii).

Melia's preferred explanation of (i) is given (p. 473) by

(iv) The various numbers are used merely to index different distance relations, each real number corresponding to a different distance relation.

Although Melia does not provide further explanation of what he means by numbers indexing distance relations, we take the guiding idea to be familiar from the work of other philosophers. For instance, Swoyer presents the idea in the following way:

> When we measure the lengths of physical objects in metres, we pair the objects with numbers in such a way that the two exhibit a common pattern. We can view our measurement scale as correlating each physical object with a unique numerical surrogate or representative in the set of positive real numbers. For example, a metre bar is paired with 1 and a twelve-inch ruler with 0.3048. Furthermore, properties and relations of the physical objects are paired with numerical properties and relations. For example, the relation that two objects stand in whenever one is longer than the other is represented by the greater-than relation on the real numbers, so that one object is longer than a second [if and only if] the number representing it is larger than the number representing the second. [More generally] the *pattern* of relations among the constituents of the represented phenomenon is mirrored by the pattern of relations among the constituents of the representation itself.[6]

If (iv) is correct, then it is an open question whether distance relations are fundamental. In that case, it is an open question whether T_2 'posits no fewer fundamental distance relations than T_1' (Melia, p. 473). Melia concludes that there is no reason to suppose that T_2 takes the world to be a simpler place than T_1 does.

The contrast with positing concrete unobservables is clear. Positing concrete unobservables allows the behaviour of observable facts to be explained in a simple, elegant way. According to Melia, positing mathematical objects does not provide a simple, elegant explanation of distance relations. Rather, it leaves the complexity of the concrete facts exactly where it is. The role of mathematical objects is not to *explain* concrete facts in this example, but merely to *index* those facts.

This line of argument, which we call Melia's indexing argument, poses a serious challenge to Platonists. If they wish to model the indispensability argument for mathematical objects on the indispensability argument for concrete unobservables, Platonists must show that mathematics has a genuinely explanatory role to play in science. It is not enough to show that mathematics is indispensable to the formulation of scientific theories. (Platonists might contend that indispensability is enough, but we shall not pursue this issue here.) In the next two sections we shall consider attempts by Colyvan and by Baker to show that mathematics does have a genuinely explanatory role in science.

[6] C. Swoyer, 'Structural Representation and Surrogative Reasoning', *Synthese*, 87 (1991), pp. 449–508, at pp. 451–2.

III. COLYVAN AND THE EXPLANATORY POWER OF MATHEMATICS

In his response to Melia (MACS, p. 72), Colyvan refers to his book *The Indispensability of Mathematics*, where he gives three scientific examples to illustrate the explanatory power of mathematical claims and their associated entities. The examples he gives are the explanation of the bending of light by massive bodies (*IM*, pp. 47–9), antipodal weather patterns (pp. 49–50), and FitzGerald–Lorentz contraction (pp. 50–1).

Here are some questions and some suggested answers to them. (A) Why does light bend around massive objects? Because light travels along space-time geodesics, and the curvature of space-time is greater around massive objects. (B) At any given moment, why do there exist two antipodes with the same pressure and temperature? Because of the causal history of weather patterns and the proof of a theorem of algebraic topology according to which, for any time, there are antipodal points on the Earth's surface which match in temperature and pressure. (C) Relative to a given inertial reference frame, why does a body in motion contract in the direction of motion? Because (in barest outline) of the space-time manifold and its geometric properties.

Examples (A) and (C) can be dealt with together. Colyvan's discussion of these examples has a common pattern. First, he argues against a causal explanation of the *explananda*. Then he argues that the *explananda* have a non-causal explanation in which mathematics figures indispensably. He concludes that the *explananda* have mathematical explanations. Our worry is with this last step. (Some philosophers argue that because the geometric structure of space-time makes a dynamic difference, it is a causally relevant feature of space-time, and so they claim that the explanation given in (A) *is* a causal explanation.[7] We do not take sides on this matter.) In (A) and (C), the explanations in question make use of geometric features of space-time. Even granted that the formulation of these explanations involves indispensable mention of mathematical entities, it does not follow that these entities play a part in the explanations. Melia's indexing argument shows that the part which mathematical entities play may only be to pick out the features of space-time which provide the whole of the geometric explanations. (This is indeed the line which Melia takes to example (A) in his reply to Colyvan.[8])

[7] See, for example, C. Mortensen, 'Arguing for Universals', *Revue Internationale de Philosophie*, 160 (1987), pp. 97–111, at p. 105.

[8] Melia, 'Response to Colyvan', *Mind*, 111 (2002), pp. 75–9, at p. 76.

Colyvan provides no reason to think that mathematics has more than an indexing role to play in this example. Without providing such a reason, he cannot claim to have shown that the role of mathematics is genuinely explanatory.

What is in debate here is whether there is a genuine explanatory role for mathematics in science, or whether mathematics has a merely indexing role. There is a general issue about what each party needs to establish. Is there a natural default position? Should indexing be assumed unless there is a knock-down argument for genuine explanatoriness? Or should explanatoriness be presumed unless it is shown why this leads to incoherence or implausibility?

Our answer to each of these questions is 'No'. We see the terms of the debate as follows. The anti-Platonist, or nominalist, needs to show that the application of mathematics to the concrete world can reasonably be taken to have only an indexing role. If he can show this, then he is not unreasonable in taking mathematics to lack an explanatory role. That would take him a step closer to his strategic goal, which we do not attempt to assess in this paper, of showing that he is not unreasonable in disbelieving in the existence of mathematical objects. The Platonist, on the other hand, needs to show that the application of mathematics to the concrete world has more than an indexing role. (Colyvan presents his case by saying that 'there are applications of mathematics in physical science that prove problematic for Melia': MACS, p. 69.) If the Platonist can show this, then he is not unreasonable in taking mathematics to have an explanatory role. That would take him a step closer to *his* strategic goal, which again we do not attempt to assess in this paper, of showing that he is not unreasonable in believing in the existence of mathematical objects. Both parties agree that mathematics has an indexing role. The key question is whether the application of mathematics to the concrete world can reasonably be taken to have only an indexing role. Neither party need have knock-down arguments for their own respective answers. As in any other philosophical debate, one party may present a stronger case than the other without having a conclusive case, and it may not be an easy matter to reach an overall and widely shared judgement about which party has the stronger case and just how strong it is.

This is a good point at which to consider an example which Colyvan gives (*IM*, p. 81) of how mathematics contributes to 'the unification and boldness of ... physical theory'. He claims that the introduction of complex numbers increased unity within pure mathematics and also within the theory of differential equations. This is significant because the degree of unification of a theory is often held to be, and is held by Colyvan to be, a measure of its explanatory power. Colyvan says that

> [The single method for solving differential equations] arises out of deep structural similarities between the systems portrayed by these equations. For example, if two different physical systems are governed by the same differential equation, it's clear that there is some similarity between these systems, no matter how disparate the systems may seem.... It seems plausible, at least, that this similarity is structural and is captured by the relevant differential equation. Even when the systems are governed by different differential equations, structural similarities may still be revealed in the mathematics.[9]

Unfortunately, Colyvan does not explain what he means by a 'structural similarity'. One might reasonably wonder there is anything more to systems sharing a structural similarity than their being describable by similar differential equations. It would then be trivial that 'even when the systems are governed by different differential equations, structural similarities may still be revealed in the mathematics'. At any rate, nothing Colyvan says blocks such a debunking treatment of the above passage. Alternatively, one might take a structural similarity to be a mathematical object of some kind (or at least an abstract object of some kind). For although talk of structures is often found in the philosophies of science and mathematics, different philosophers seem to have quite different notions of structure in mind. For example, in one sense, two systems share a structural similarity if there is an isomorphism between the entities in their respective domains, whereas in another sense there is a structural similarity between two systems if those systems attribute the same relational properties to the same entities. We think that if Colyvan has in mind a notion of structure in either of these senses, then his current example fails to tell against Melia. There is a distinction between (1) something's being a heuristic device for detecting a feature F, and (2) something's being responsible for the possession of feature F. A special case of this is the distinction between (1*) something's being a heuristic device for detecting unifying features of physical systems, and (2*) something's being responsible for the unification. Of these two factors, it is only (2*) – that which is responsible for unification – which has explanatory power. What is responsible for unification is an ontological reduction in the number of independent kinds of phenomena that a given system needs to posit. Simply finding that different systems fall under similar differential equations does not achieve ontological reduction. So the role of complex numbers in physics does not itself achieve theoretical unification, and the mathematics is not itself explanatory.

In the case of example (B), we should distinguish the theorem from the proof of the theorem. To take these in turn, the mathematics of the theorem

[9] Colyvan, *IM*, p. 83. Colyvan's example is also discussed by Pincock, §5.

– its talk of continuous functions and spheres – serves a role similar to the mathematics in (A) and (C). In this example, the mathematics helps to pick out various topological features of the Earth's weather systems with their temperature and pressure changes. It is common ground between Colyvan and his opponent that the Earth's weather system consists, at any given time, of various spatial regions, and that each region has certain temperature and pressure gradients. It is also common ground between Colyvan and his opponent that these spatial regions, and the features ascribed to them above, have explanatory power. It is because certain regions of the Earth's weather system have the temperature and pressure gradients they have that certain of its other regions have the temperature and pressure gradients they have. These regions and the features in question are not mathematical entities, although mathematics can be used to index the regions and features. But the fact that mathematics can be used to pick out entities which are both non-mathematical and explanatory does not entail that the mathematics itself is (also) explanatory. As to the proof of the theorem, arguably it plays only a justificatory role. What we mean by this is that the proof shows that the theorem is true, and why it is true. Given that the proof justifies the theorem, we are then entitled to make use of the theorem, e.g., in applications to physical facts. Colyvan's discussion of this example does nothing to rule out this line of thought, and so his discussion at this point fails to establish the explanatory role of mathematics. What we offer here, then, is a 'divide-and-conquer' strategy. If the indexing tactic is successful in accounting for any case of the application of mathematics to the concrete world, the role of the theorem of algebraic topology can be handled by the indexing tactic. The role of the proof of that theorem is to justify the acceptance of that theorem. In neither case is mathematics being taken to explain facts in the concrete world.

The common problem facing Colyvan's arguments is that they merely highlight the fact that mathematics is very useful. His opponent can grant this, but can add that it provides no reason to deny the claim that mathematics is useful simply because it indexes or models the concrete structures that do the real explaining.

It may be felt that in questioning whether Colyvan's examples involve mathematical explanation we have been setting the bar too high. Or, perhaps worse still, we have set the bar higher for mathematics than for other parts of science. We do not think we have, because, independently of this debate in the philosophy of mathematics, various philosophers have suggested that one may cite certain entities with a view to giving an explanation of an event, without thereby claiming that those entities provide part of the explanation. We particularly have in mind here the idea of 'programme explanation'.[10] According to this idea, in explaining something we often cite a certain feature of an object or event so as to point to some other feature of it, where it is the latter feature which explains the *explanandum*. Two brief illustrations: why does opium put people to sleep? Because it has a dormitive power. Citing a dormitive power alludes to some chemical property of opium, the genuinely explanatory factor, which produces sleep in opium smokers. Why did a certain string of letters appear on your computer screen? Because you typed a series of key strokes. Citing the key strokes alludes to some properties of the electronics, genuinely explanatory factors, connecting keystrokes with screen letters. Whether or not this idea of programme explanation is ultimately acceptable, we hope at least to have indicated why we are not guilty of setting the bar higher for mathematics than for other sciences on the issue of explanation-giving. There seems to be a kind of phenomenon endemic in other parts of scientific practice and in everyday explanation-giving which occupies broadly the same role as Melia thinks mathematics plays in scientific explanation, namely, that of indexing the properties which are genuinely explanatory. Using the indexing argument against Colyvan does not, therefore, amount to special pleading against the idea of mathematical explanation.

IV. BAKER AND THE EXPLANATORY POWER OF MATHEMATICS

Alan Baker, in his 'Are there Genuine Mathematical Explanations?', responds to Melia's claims that the role of mathematics in best science is merely to index physical facts, and that it does not provide genuine explanations. Baker makes the following two key claims. (1) He provides an example of applied mathematics which he claims is not subject to one reading of Melia's argument. (2) Though he admits the example is subject to the other reading of Melia's argument, he claims that the example can be defended against this reading, and that it provides a case of genuine mathematical explanation. We believe Baker's example does not sidestep either of his readings of Melia's argument, and that his defence of his example fails to show that it involves genuine mathematical explanation. We shall examine the two readings of Melia's argument which Baker offers.

Baker cites the following passages from Melia:

> [When] we come to explain [physical phenomenon] *F*, our best theory may offer as an explanation, '*F* occurs because *P* is $\sqrt{2}$ metres long'. But ... though the number $\sqrt{2}$

[10] See F. Jackson and P. Pettit, 'Functionalism and Broad Content', *Mind*, 107 (1988), pp. 381–400, at pp. 391–7.

> is cited in our explanation, it is the *length* of *P* that is responsible for *F*, not the fact that the length is picked out by a real number ('Response to Colyvan', p. 76).
>
> [Theory] T_2 expresses the fact that *a* is $^7/_{11}$ metres from *b* by using a three-place predicate relating *a* and *b* to the number $^7/_{11}$.... [But] the various numbers are used merely to index different distance relations, each real number corresponding to a different distance relation ('Weaseling Away', p. 473).

Baker offers two interpretations of these passages. One is that 'reference to $\sqrt{2}$ and $^7/_{11}$ is not explanatory in the above examples because these numbers are *acausal*' (p. 228). The other is 'that the mathematical apparatus in these cases is not genuinely explanatory because the role of the numbers $\sqrt{2}$ and $^7/_{11}$ is *arbitrary*' (p. 227).

We agree with Baker that it would be a poor argument to urge that reference to $\sqrt{2}$ and $^7/_{11}$ is not explanatory in the above examples because numbers are acausal. This does not, however, seem to be a line of argument which Melia takes. The suggestion that the role of numbers in explanations is arbitrary is more promising. One might be persuaded that, say, $^7/_{11}$ is not explanatory in the above example on the ground that reference to this number depends on an arbitrary choice of measuring unit. However, what is essential to Melia's argument against Platonism is that the role of number talk is only to index physical facts. He would be ill advised to make Baker's arbitrariness claim pivotal to that argument. Given the distance between *a* and *b*, and given that we are using the metre as our unit of length, it need not be arbitrary – it need not be a matter of choice on our part – that the distance between *a* and *b* is $^7/_{11}$ metres. Of course, we could have used a different unit of measurement, and in terms of this other unit of measurement the distance relation between *a* and *b* might not be indexed with the number $^7/_{11}$. In this sense, the use of $^7/_{11}$ *is* arbitrary. So in one respect it is not arbitrary that a certain number is used to index a given distance relation, whereas in another respect it is. The issue is now in danger of stalemate: Melia would have to show that although the role of numbers is not arbitrary in the first sense, the fact that it is arbitrary in the second shows their role is unexplanatory. In any case, the issue of arbitrariness can be eliminated altogether with a change of example. Instead of describing the distance relation between *a* and *b*, we could instead describe their relative lengths. Suppose *a* is $^7/_{11}$ times as long as *b*. The role of $^7/_{11}$ in this description is not arbitrary in any sense, and so it cannot be unexplanatory because it is arbitrary. Melia can, and should, stick to the claim that the role of number talk, whether arbitrary or not, is solely to index physical facts, not to explain their behaviour.

To sum up, Baker offers two interpretations of the above passages from Melia. Both interpretations should be rejected as potential grounds for

Melia's objection to Platonism. We also think that Melia's argument is untouched by the philosophical deficiencies in Baker's interpretations of it.

Baker offers an example of applied mathematics of his own (§§2 and 3). Here is a sketch of the example, which concerns the life-cycle of the 'periodical' cicada. Certain species of cicada are born in soil and remain there for either 13 or 17 years until they emerge as adults. Why are the life-cycle periods prime? There are two evolutionary explanations currently available which answer this question. One is that it is evolutionarily advantageous for the cicada species to intersect as rarely as possible with predator species. The other is that it is evolutionarily advantageous for the cicadas to have sufficient mating opportunities during their adult stage, while not mating with subspecies which have a life-cycle period different from their own, since that would decrease the mating opportunities of their offspring. These evolutionary theories have a common mathematical basis in number theory. There is a mathematical link between being prime and minimizing the intersection of periods. The mathematics shows that prime periods minimize intersection, at least as compared with non-prime periods. Evolutionary theory then claims that cicadas with periodic life-cycles are likely to evolve periods that are prime. Mathematically-informed evolutionary theory can predict that, given that cicadas in a certain type of ecosystem are limited by biological constraints to periods from 14 to 18 years, those cicadas are likely to evolve 17-year cycles.

In effect, Baker then poses the following three questions: does the cicada example involve reference to mathematical objects? Is the cicada example a case of genuine explanation? Is the cicada example a case of genuine mathematical explanation? All parties are agreed that the cicada example involves reference to numbers. We also grant that the example is a case of genuine explanation. As Baker notes (p. 234), since an explanation involving mathematics need not be a mathematical explanation, affirmative answers to the first two questions do not guarantee that the answer to the third is also affirmative. The crux, then, is how to answer the third question.

If the cicada example is a case of genuinely mathematical explanation, Baker needs to show that it is immune to Melia's argument that number talk has no explanatory role. At least on the face of it, the points Melia makes against examples of applied mathematics in the passages cited above will carry over to this example. Just as Melia urges that the role of numbers in those examples is merely to index distance relations, so too he could urge that the role of 17, or of any other prime number, in the cicada example is merely to pick out a given duration, namely, the life-cycle of the cicada.

Baker's first key claim was that the cicada example avoids Melia's argument altogether on one of his (Baker's) readings of it. Baker claims (p. 236) that the role of primes in the cicada example is not arbitrary in any sense, unlike that of the role of numbers in Melia's examples of applied mathematics. As we have shown, it is questionable whether Melia's case against assigning mathematics an explanatory role turns on a charge of arbitrariness. So responding to him by finding an example that simply avoids arbitrariness may miss the point. But in any case Baker fails to show that the role of primes in the cicada example is *not* arbitrary. In Melia's examples, the use of $\sqrt{2}$ or $^{7}/_{11}$ to index certain distances is arbitrary because it involves a conventional choice of a unit of measurement. A similar point applies to Baker's example. It is vital to distinguish between a magnitude (a distance, a duration, etc.) and the unit used to measure degrees of that magnitude. It is not a matter of choice what magnitude something has. *A fortiori*, it is not an arbitrary matter what magnitude something has. It is, however, a matter of convention what unit is used to measure that magnitude. Since different, and equally useful, units of measurement are available, it is arbitrary which unit of measurement is used to measure the magnitude in question. Prime numbers appear in the description of the life cycle of cicadas because, and only because, we measure that life cycle *in years*. But we could choose to measure the duration of that cycle differently. Instead of measuring it as 17 years, we could measure it as 68 seasons, for instance, or as 204 months. Since neither 68 nor 204 is a prime number, it seems that the role of primes in the life cycle of cicadas is arbitrary in just the same way as $\sqrt{2}$ or $^{7}/_{11}$ is in Melia's example.

Baker does not spell out why he thinks the role of primes is not arbitrary, and does not address the objection. But it might be thought that his claim could be defended on the ground that the choice of measuring unit in his example is *not* arbitrary: a year is a biologically significant period of time. It would be no accident, then, that life cycles are measured in years. Further, it may be urged that what is remarkable about the example is that given that the year is the appropriate measuring unit, cicada life cycles turn out to be prime numbers of years. Explaining this remarkable fact will involve an appeal to the properties of primes; hence primes have explanatory value.

We do not find this defence of Baker's claim persuasive. It is not obvious that the year is biologically a more significant period of time than the season or the month. What makes the year a biologically significant period of time is the fact that each year consists of the four seasons, and the seasons are evidently biologically significant periods of time. Seasonal changes trigger such biological phenomena as germination, migration and hibernation. At any rate, insisting on the year, *as opposed to the seasons*, as the appropriate measurement unit seems unwarranted. The seasons have at least as good a case as the year for being an appropriate measurement unit.

Baker might reply as follows: 'Certainly one can find biological phenomena for which seasons or months are a more significant unit of time. But surely the point at issue is what the most salient unit of time is when discussing periodical cicadas. The simple fact is that all discussions and putative explanations of the life-cycle of periodical cicadas by biologists use the basic unit of the year. Does this not undercut your argument?'[11]

This reply consists of a datum and a contention. The datum is that biologists standardly use a certain unit, the year, when measuring the cicada cycle. The contention is that this measurement unit is the most salient unit. The effectiveness of this reply turns on what the appeal to salience amounts to. If the contention that something is the most salient unit just amounts to saying that it is the unit uniformly used by biologists in this field, then the contention does not go beyond the datum. The datum is itself compatible with our liberal view that each of many units of measurement is admissible in measuring cicada cycles. If the contention is that the most salient unit is the uniquely applicable unit, the contention would be false. Other measurement units are just as applicable, even if they would not be as convenient. Last, if the contention is that the unit of the year is explanatorily privileged, it is difficult to see how the datum would warrant it, and how the privilege would be compatible with the equivalence of a year with four seasons, or again with twelve months. To take a parallel case, galactic distances are standardly measured in terms of the light year. They could as well be measured in terms of the kilometre, since 1 light year = 9.461×10^{15} km. This practice would be non-standard because it would be unnecessarily cumbersome, but it would not be because the unit of a light year is uniquely correct or explanatorily privileged.

To resume, it cannot be that the properties of the primes are of crucial explanatory importance if we could just as well measure the cicada cycle in seasons – periods to which the primes do not bear any interesting relation. We need to distinguish between a duration and the unit of time used to measure that duration. What is arbitrary is the unit of time used to measure the duration. One thing we might want to explain is why the life cycle of cicadas has the duration it does. This can be explained without taking the year as the unit of measurement, and so it can be explained without appealing to prime numbers. Another thing we might want to explain is why the duration can be measured as a prime number of revolutions of the earth around the sun. But here we can reuse the previous explanation. Suppose the explanation took the season as its unit of measurement, and took the duration in question to be *m* seasons long. We can then point out that a

[11] We are grateful to a referee for this suggested reply.

duration of m seasons is equivalent to a duration of n years, where n is some prime number. We then have a *bona fide* explanation of why the duration can be measured as a prime number of revolutions of the earth around the sun.

Perhaps a way forwards for Baker would be to focus on the duration of the cicada life cycle *relative* to that of other organisms. Relative duration (x's being twice as long as y, etc.) does not depend on the choice of measuring unit at all and so will avoid the arbitrariness issue. Given that the cicada life cycle is n times as long as those of various organisms, the chances of adult cicadas encountering predators will be less, and so the chances of the survival of the cicada species is increased. This approach certainly yields a number that is not arbitrary in the sense in which the numbers used in Melia's examples are (though in this example it need not be a prime). But even this approach will not help Baker's response to Melia's argument if, as we have argued, Melia's argument does not rest on worries about arbitrariness.

In making his second key claim Baker considers the cicada example in the light of each of three accounts of explanation: the causal, the deductive–nomological, and the pragmatic (pp. 234–6). His intention is to show (p. 234) that the example is a case of genuine mathematical explanation, where this means showing

> that the mathematical component is explanatory in its own right, rather than functioning as a descriptive or calculational framework for the overall explanation.

Taking the three accounts in turn, the causal account of explanation says that to explain something is to give information about its causal history. In considering this account Baker is effectively responding to the first of his readings of Melia's argument against Platonism, according to which the role of numbers is not explanatory because numbers are acausal. Baker admits that since numbers are acausal, the causal account cannot show that the example is a case of mathematical explanation. So the causal account will not serve Baker's purposes.

The deductive–nomological account says that to explain a phenomenon is to provide a sound argument whose premise-set essentially includes a law statement, and whose conclusion is a statement describing the *explanandum*. Baker claims (p. 235) that

> A broadening of the category of laws of nature to include mathematical theorems and principles, which share commonly cited features such as universality and necessity, would count the mathematical theorem [that prime periods minimize intersection, compared with non-prime periods] as explanatory on the same ground as the biological law [that having a life-cycle period which minimises intersection with other (nearby/lower) periods is evolutionarily advantageous].

Here Baker notes certain features which law statements share with statements in pure mathematics, and he advocates broadening the category of a law of nature so that statements in pure mathematics count as law statements. He is, of course, free to understand the latter category more broadly if he so wishes. As he remarks, law statements and mathematical theorems share certain features. But what is precisely at issue is whether mathematical theorems are like law statements *in that they explain events*. When the category of a law of nature is understood narrowly, all parties can agree that all law statements explain. But when that category is broadened to include mathematical theorems, it can no longer be assumed that all law statements explain. This is what has to be shown. Baker's recommendation either begs the question or is useless.

His recommendation is question-begging if it assumes that mathematical theorems are like law statements in having explanatory power. That this is question-begging might be denied; if the question of whether mathematics explains is to be discussed at all, it might be said, the parties to the debate need to agree to some way of making mathematical explanations possible in the framework in question, so Baker is modifying the framework in a minimal way in order to get the debate off the ground. The charge of question-begging is very often a delicate one, but we think that this reply underestimates the depth of the dispute between Baker and his opponents. Opponents would doubt not only whether there are any mathematical explanations, but whether such things are so much as possible (just as Leibniz rightly doubted whether a decahedron, a solid whose sides are ten regular polygons of identical size and shape, is possible). So the disputing parties cannot, as the reply urges, 'agree to some way of making mathematical explanations possible in the framework in question', and then go on from there. What is at issue is precisely whether a mathematical explanation is something which is even possible. 'Modifying the framework' in any way that makes mathematical explanation possible is not 'getting the debate off the ground'. On the contrary, it is immediately settling the debate in Baker's favour. For what is at dispute is whether mathematical principles should be classified as forming a subclass of the explanatory principles properly called laws of nature.

Here is another way to make this point. Our topic is mathematical explanation. Baker thinks that there are mathematical explanations. Is the opposing nominalist's view simply that there are no mathematical explanations, but that there could be, and that it is just an unfortunate coincidence that to date no mathematical explanations have been produced? We suggest that the nominalist's view should be that there could not be mathematical explanations. In this paper we have tried to show the resilience of

Melia's indexing strategy. If his strategy works against some cases of putative mathematical explanation, it works against all possible putative mathematical explanations. Indeed, much of the interest of his strategy lies precisely in its potentially sweeping range of application. Certainly it would be dogmatic simply to deny the possibility of mathematical explanation. But that should not be the nominalist's position. Thanks to Melia's indexing strategy, he has an argument which calls into question the possibility of mathematical explanation. It is this argument which Baker addresses, and which we in turn think that he has failed to defeat.

Baker's recommendation is useless if it simply groups into one category both law statements as originally so-called, *viz* statements with acknowledged explanatory power, and mathematical principles. That leaves open the question whether mathematical principles have explanatory power. After all, logical truths (e.g., truths of the form 'all Fs are Fs') have many of the features of law statements – they are universal and necessary – but it does not follow that they have the added feature of explanatory power. Simply lumping logical truths together with laws of nature will not confer explanatory power on the former.

Last, Baker considers the pragmatic account of explanation. As he understands this account, 'explaining a phenomenon involves providing an answer to a "why"-question which shows how the phenomenon is more likely than its alternatives' (p. 235). He continues

> ... genuine explanatory applications of mathematics ought to be reconfigurable as answers to questions about why a certain physical phenomenon occurred.... Why do periodical cicadas have prime periods? Because prime numbers minimize their frequency of intersection with other period lengths.

This strategy fails because it does not provide any good reason to think that, in Baker's words, 'the mathematical component is explanatory in its own right, rather than functioning as a descriptive or calculational framework for the overall explanation'. That is, the strategy fails to answer Melia's objection that the mathematical component merely indexes the physical facts which provide the real explanation. A nominalist might approach the example in the following way. The first question to ask is 'Why, in the case of any particular species of cicada, is their periodic life-cycle of *this* duration rather than any other?'. This question focuses on the physical phenomenon of duration rather than on a mathematical theory which might be used only to index durations. The answer, supplied by evolutionary theory, will be along the following lines: given that certain relevant creatures also present in the cicada habitat have periodic life-cycles of *some other* duration, it is advantageous for the cicada life-cycle to be of the particular duration it is, for this minimizes the encounters between the organisms.

Turning now to Baker's question 'Why do the periodic cicada life-cycles have prime periods?', the nominalist may answer as follows. First, whether they have prime periods or not depends on the measuring unit chosen. But whatever measuring unit is chosen, the reason why the life-cycle of a given species of cicada is of the given particular number of units is that the duration which that number of units picks out minimizes the cicadas' contact with other predatory species that inhabit the cicadas' environment. This answer once again points back to the concrete facts – the durations of the life-cycles of the cicada and of the other relevant organisms in its environment, and the forces of evolution. The nominalist argues that the number of units those durations amount to, relative to a given measuring system, play no explanatory role at all. They merely index the durations measured, which, together with evolutionary pressures, provide the genuine explanation.[12]

As it happens, primes have properties mirrored by the cicada life-cycle durations. Suppose we begin with the number 0 and increase incrementally by thirteen each time. We then get a sequence of numbers that will intersect infrequently with sequences that result from similar processes that begin with a number close to 0 (1 or 2, say) and increase incrementally by some number *n* close to 13 (10, say). If we start from a given time and move forwards in time incrementally by the duration of the cicada life-cycle, we get a sequence of times that intersect infrequently with sequences of times that have a nearby starting time (at a distance of 1 or 2 years, say) and move forwards in time incrementally by the duration of predatory organisms' life-cycles, where these organisms also inhabit the cicadas' environment. It is this property of the cicadas' life-cycle duration – this periodic intersection with the life-cycles of certain predatory kinds of organism – that plays an explanatory role in why their life-cycle has the duration it has. Primes have similar properties, and so successfully index the duration. But then so also do various non-primes: measuring the life-cycle in seasons produces analogous patterns.

[12] A referee rightly pointed out that it is contentious at best to classify a duration as a concrete phenomenon. Many philosophers would count a duration as abstract. This raises the pertinent but vexed issue of how the abstract/concrete distinction is to be drawn. It also recalls a familiar criticism of Hartry Field's philosophy of mathematics. Although Field avowedly posits only concrete objects, he posits space-time points. Yet some philosophers would count a space-time point as abstract: cf. the discussion in M.D. Resnik, review of Field's *Science without Numbers*, *Noûs*, 17 (1983), pp. 514–19, at p. 516. Our working definition of 'concrete object' is that *x* is a concrete object if and only if *x* stands in a space-time relation to another object, and *x* has causal powers. By this definition, durations and space-time points arguably both count as concrete objects.

The point, then, is that the indexing argument can be effectively exploited against Baker's arguments. Nothing that Baker has said tells against a nominalistic reading of the example which assigns mathematics no explanatory role at all, and he has not made the case for the explanatory role of mathematics by basing it on the pragmatic account.[13]

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