Brad Skow October 15, 2007

Philosophy 24.111: Philosophy of Quantum Mechanics, Fall 2007
Handout 2: An Outline of Bell's Theorem

1. Preliminaries

Bell's Locality Principle: Let P_{HV} be the probability function determined by a hidden variables theory. Then the theory is a local theory if

$$P_{HV}(A, B|a, b, \lambda) = P_{HV}(A|a, \lambda)P_{HV}(B|b, \lambda).$$

Expectation Values: Let $A_1, \ldots, A_n$ be a range of outcomes of some process, where exactly one of those outcomes must occur. Assign numerical values $V(A_1), \ldots, V(A_n)$ to each outcome. Then the expected outcome (or expectation value) relative to this range is $\sum_i (V(A_i)\text{Prob}(A_i))$.

We consider a range of outcomes of join spin experiments on particles in the singlet state: {The particles go the same way; the particles go different ways}. Assign value $+1$ to the first, -1 to the second.

We also consider a range of outcomes of spin experiments on a single particle. The range is {The particle goes up; the particle goes down}. Assign value $+1$ to the first, -1 to the second.

Let

- $E_{HV}(a, b, \lambda)$ be the expectation value of the joint spin measurement when the magnets are set at a and b and the hidden state is λ.
- $E^1_{HV}(a, \lambda)$ be the expectation value of the spin measurement on the left particle when its magnet is set at a and the hiden state of the pair is λ.
- $E^2_{HV}(b, \lambda)$ be the expectation value of the spin measurement on the right particle...

Note that if P_{HV} satisfies Bell's locality principle, then

$$E_{HV}(a, b, \lambda) = E^1_{HV}(a, \lambda)E^2_{HV}(b, \lambda).$$

2. Proof of Bell's Theorem.

Statement of the theorem: If P_{HV} satisfies Bell's locality principle, then $P_{HV} \neq P_{QM}$.

proof (outline): First we prove a

Lemma: if $x, y, x', y' \in [-1, 1]$ then $S(x, y, x', y') \in [-2, 2]$, where $S(x, y, x', y') = xy + xy' + x'y - x'y'$.

Then we use the Lemma to show that if P_{HV} satisfies the locality principle,

$$|E_{HV}(a, b) + E_{HV}(a, b') + E_{HV}(a'b) - E_{HV}(a'b')| \leq 2.$$

Then we pick particlar angles a, b, a', b' and show that for those angles

$$|E_{QM}(a, b) + E_{QM}(a, b') + E_{QM}(a'b) - E_{QM}(a'b')| > 2.$$