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Philosophy 593S: Philosophy of Space and Time, Fall 2005
Handout 5: Relationalist Physics

1. What must a relationalist physics do?

One thing physical theories do is tell us which possible worlds are physically possible. A relationalist physics must at least do that. Must it do more?

Suppose that substantivalism is true and the world is Newtonian. Then a complete description of a possible world specifies three things:

(1) How many material bodies there are.

(2) The mass (and other intrinsic properties, like charge) of each material body.

(3) The location of each body in space, at each time.

Now suppose relationalism about ontology is true. What must a complete description of a possible world specify? One answer:

(1) How many material bodies there are.

(2) The mass (and other intrinsic properties, like charge) of each material body.

(3) For each pair of bodies, the distance between them, at each time.

But: note reference to time. To remove it, suppose that (1) and (2), together with a specification of a distance between each pair of bodies is a description of an *instantaneous state* of a possible world. Then a relationalist might say that a complete description of a possible world specifies

(1) Which instantaneous states the world passes through, and in what order.

(2) The temporal distance between each pair of instantaneous states.

Or, a relationalist might say that a complete description of a possible world specifies only

(1) Which instantaneous states the world passes through, and in what order.

Does one of these fit better with the arguments relationalists give?

2. Relationalist Theory: Version One

Those histories of interparticle distances are physically possible which can be embedded in space and time so that Newton's laws are true.

3. Determinism

Determinism, the general definition: a physical theory is deterministic just in case any two physically possible worlds that agree at a time agree at all times.

Suppose that Newtonian mechanics is true. Under what conditions do possible worlds agree at a time? At all times?

Is Relationalist Theory One deterministic?

4. **Relationalist Theory: Version Two**

Those histories of interparticle distances are physically possible which can be embedded in space and time so that Newton's laws are true, the total energy of the universe is zero, and the total angular momentum of the universe is zero.

5. **The Lagrangian formulation of Newtonian Mechanics**

If substantivalism is true, then a description of an instantaneous state of the world specifies

(1) How many material bodies there are.

(2) The mass (and other intrinsic properties, like charge) of each material body.

(3) The location in space of each material body.

Pick a coordinate system for space and answers to (1) and (2). We can set up a correspondence between descriptions of instantaneous states of the world and points of an abstract space. This set is called "Configuration Space." (A "configuration" is an instantaneous state of the world.)

Example: suppose there is just one particle. Then $\mathbb{R}^3$ is configuration space. The point $(0,0,1)$ represents an instantaneous state in which the one particle is located at the point in space with coordinate $(0,0,1)$.

Example: suppose there are two particles. Then $\mathbb{R}^6$ is configuration space. The point $(0,0,0,0,0,1)$ represents an instantaneous state in which the first particle is located at the point in space with coordinate $(0,0,0)$ and the second is located at the point with coordinate $(0,0,1)$.

In general: if there are n particles then configuration space is $\mathbb{R}^{3n}$. The $3i-2, 3i-1, 3i$-eth coordinates on configuration space represent the position in space of the i-th particle.

Definition: a path in configuration space is a function $f : \mathbb{R} \to \mathbb{R}^{3n}$. Each function corresponds to a possible world: if we use $\mathbb{R}$ to represent time, then the function tells you what, at each time, the instantaneous state of the world is.

Note: As defined a path through configuration space is *not* just a set of points in configuration space. There are distinct paths that run through the same points in configuration space. They differ over "how fast" they run through them.

The Lagrangian L is a function on configuration space.[1] (Usually it is the kinetic energy minus the potential energy of that configuration.)

Some paths have this feature: the value of $\int L\,dt$ along that path is less than its value along all "nearby" paths (or, alternatively, greater than its value along all nearby paths). Such paths are "extremal."

The law of Newtonian mechanics are then equivalent to the following law: those paths through configuration space for which $\int L\,dt$ is extremal are physically possible.

6. Barbour's Theory

Barbour's epistemic motivations.

Barbour wants to write down laws with two features.

First, it should not use the same configuration space that we used above. Instead it should use "relative" configuration space: each point should represent instantaneous interparticle distances, not positions in absolute space. (Barbour calls relative configuration space "Platonia," or (in the three particle case) "Triangle land.")

Example: if there are just two particles, then configuration space is $\mathbb{R}$ (not $\mathbb{R}^6$). The point 4 represents an instantaneous state in which the two particles are 4 meters apart.

Second: it should have no temporal metric. That is: if two paths through configuration space agree on which states the world passes through, then either they are both physically possible, or neither is physically possible. "How fast" the universe is moving through the states should play no role in the dynamics.

The problem: (i) for any given path in relative configuration space there are many paths in "substantivalist" configuration space; (ii) in many cases, some but not all of those paths in substantivalist configuration space correspond to physically possible worlds (according to Newtonian mechanics).

Barbour's response: in his theory, some paths through substantivalist configuration space that Newton says are physically possible are *not* physically possible. (That is: they are not physically possible, even when interpreted relationalistically. They do not even describe physically possible sequences of interparticle distances.)

The details (very sketchy): Barbour writes down a metric on relative configuration space. It allows us to compute the length of any path through configuration space.

[1] Congnoscenti: actually it is a function on the tangent bundle over configuration space.

Shortest paths through configuration space are called "geodesics." The law is: the physically possible paths through configuration space are the geodesics.

(In the book Barbour discusses how to define a "distance" between points in relative configuration space without invoking distance in space itself. The metric on configuration space is derived from this function.)

Consequences:

(1) The sequences of interparticle distances that are physically possible in Barbour's theory are the sequences of interparticle distances that are physically posssible in Newtonian mechanics, and in which the angular momentum of the universe and the total energy of the universe are zero.

(2) If a path through relative configuration space is physically possible, then every other path that runs through the same points is also physically possible. It does not matter if they run through the points "at a different speed." ("Proof": the length of a path does not depend on how it is parameterized.)

7. Assessing Barbour's Theory

According to Barbour's theory, it is a law of physics that the angular momentum of the universe is zero. Not so according to Newtonian mechanics. As far as we can tell, the angular momentum of the universe *is* zero. Does this favor one theory over the other?

Systems that are far away from other systems are relatively isolated. Relatively isolated systems can have non-zero angular momentum. If Newtonian mechanics is true, we can apply the laws of the theory not just to the universe as a whole, but also to relatively isolated subsystems. Not so with Barbour's theory. Does this favor one theory over the other?

According to Barbour's theory, it is a law of physics that the total energy of the universe is zero. Not so according to Newtonian mechanics. Does this favor one theory over the other?