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Philosophy 593S: Philosophy of Space and Time, Fall 2005
Handout 7: space and time in Minkowski spacetime

A is the worldline of an inertial observer in Minkowski spacetime. Any two points of spacetime have an apparent spatial distance, relative to A, and an apparent temporal distance, relative to A. If you know the spacetime interval between every pair of spacetime points, then you can compute apparent spatial and temporal distances as follows.

1. **Apparent temporal distance.** If the line through p and q is parallel to A's worldline, then the apparent temporal distance between p and q is $\sqrt{-I(p,q)}$ (where $I(p,q)$ is the spacetime interval between them).

2. Definition: Lines that make the same angle with the horizontal axis that A's worldline makes with the vertical axis, and which are "tilted toward A's worldline," are the hyperplanes of simultenaity, according to A. (A regards all points on any one of those lines as happening at the same time.)

3. If the line through p and q is not parallel to A's worldline, then:

   (a) Find the (unique) hyperplane of simultenaity (according to A) that q lies on.

   (b) Find the (unique) line parallel to A's worldline that p lies on.

   (c) Find the intersection of these two lines. Label the point of intersection "r."

   (d) By (1), the apparent temporal distance between p and r is $\sqrt{-I(p,r)}$.

   (e) The apparent temporal distance between p and q = the apparent temporal distance between p and r.

   (Why does this makes sense? A can reason: q and r happen at the same time. So if t seconds elapse between p and r, t seconds will also elapse between p and q.)

4. **Apparent spatial distance.** If p and q lie on one of A's hyperplanes of simultenaity, then the apparent spatial distance between them is $\sqrt{I(p,q)}$.

5. If p and q do not lie on one of A's hyperplanes of simultenaity, then:

   (a) Find the (unique) line parallel to A's worldline that q lies on.

   (b) Find the (unique) hyperplane of simultenaity (according to A) that p lies on.

   (c) Find the intersection of these two lines. Label the point of intersection "r."

   (d) By (4), the apparent spatial distance between p and r is $\sqrt{I(p,r)}$.

   (e) The apparent spatial distance between p and q = the apparent spatial distance between p and r.

   (Why does this make sense? A can reason: q and r happen in the same place. So if p and r are x meters apart, p and q are also x meters apart.)

"But why are those particular tilted lines the hyperplanes of simultenaity, relative to A? Why doesn't A think that the horizontal lines are the hyperplanes of simultenaity, or the lines that are at right angles (in the diagram) to his worldline?"

Answer 1: In Minkowski spacetime, all inertial observers agree that light moves at $3 \times 10^8\, m/s$. Only if A's hyperplanes of simultenaity are the ones I mentioned will he agree with other inertial observers about the speed of light.

Answer 2: Prove it to yourself geometrically:

We know this much: If p is a point on A's worldline, and A bounces a photon off of q; then A will regard p and q as simultaneous just in case the following is true:

(T) The apparent elapsed time t_1 between the spacetime point at which A emits the photon and $p =$ the apparent elapsed time t_2 between p and the spacetime point at which A receives the photon back.

("The photon took $t_1 + t_2$ seconds to get from me to q and back; since light travels the same speed in all directions, it reached q $\frac{1}{2}(t_1 + t_2) = t_1$ seconds after I emitted it; but that was just when I experienced point p on my worldline. So p and q are simultaneous.")

So pick a point p on A's worldline. Chose a point r on A's worldline that occurs after p and a point s on A's worldline that occurs before p, so that the apparent elapsed time between s and p is the same as the apparent elapsed time between p and r. (That means that $I(s, p) = I(p, r)$; visually, it means that they are both the same distance from p on the diagram.)

Draw the future light cone at s and the past light cone at r. These light cones intersect at two points, call them a and b.

The points p, a, and b all lie on a line. This is the hyperplane of simultenaity, relative to A, through point p. Notice the way it tilts.

You can verify that not just a and b, but every other point on this surface has property (T) above.

(There is another way to explain why hyperplanes of simultenaity are geometrically special. In Minkowski geometry, those tilted lines are at (something like) right angles to A's worldline.

They are not, of course, at Euclidean right angles *in the diagram.* But they are at (something like) right angles *in the spacetime the diagram represents.* This is a drawback of using a two-dimensional Euclidean plane to represent the non-Euclidean geometry of two-dimensional Minkowski spacetime.)