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A SPECIAL CASE OF DYNAMIC PRICING POLICY*

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This article studies the implications of experience curves and brand loyalty for optimal dynamic pricing policy. In a continuous time model, we synthesize several results from the literature on open loop equilibria. Specifically, we show that prices should decrease over time for high discount rates and steeper exogenous declines in variable costs. Conversely, the prices should increase over time if experience curves affect fixed costs and if consumers are brand loyal.
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1. Introduction

The literature on the implications of experience curves has offered several different conclusions about optimal dynamic pricing policy. Spence (1981) makes the observation that the relevant concept of marginal costs has to be derived from the end-of-period costs, including experience curve effects. In a model without discounting, this gives him a constant price over the planning period, while the introduction of discounting leads to time-declining prices due to impatience in profit taking. In the marketing literature Dolan and Jeuland (1981), in a monopolistic model, obtain the same effect of discounting, but further find that a sufficient amount of repeat purchasing can make time-increasing price patterns optimal. Also looking at a monopolist, Clarke, Darrough and Heineke (1982) find that experience curve effects on fixed costs lead to time-increasing prices, while effects on variable costs, in a model with discounting, lead to time-decreasing prices. Finally, the economists Fudenberg and Tirole (1983) show that the incentives for time-increasing prices may be stronger in closed loop equilibria than in open loop equilibria. The implications of several other effects, such as brand loyalty, thought of as the ability to "invest in market share" (Spence 1981) or entry deterrence (Smiley and Ravid 1983; Eliashberg and Jeuland 1982) are yet unclear.

The purpose of this paper is to synthesize some of the open loop literature on the implications of experience curves and brand loyalty for optimal dynamic pricing policy. Specifically, §2 contains a simple model in which we look at price trends in symmetric open loop equilibria and show that they are shifted downward by discounting and exogenous declines in variable costs and up by experience curve effects on fixed costs and brand loyalty modelled as sluggishness on the part of consumers in reacting to price differences. The last result means that greater ability to invest in market share tends to make time-declining prices optimal.

Our model differs from the standard economic literature since we allow consumers to take some time to adjust their purchasing patterns to price differences in the market. This means that market share is considered a state variable, which flows in response to interfirm price differences. While the standard economic assumption of complete instantaneous adjustment may be defensible in static settings, it has long been known to marketing scholars that this is not realistic in continuous time models. Apart from the added realism (and complexity) the two major reasons for treating market share as a state variable are that we can model brand loyalty and also use price rather than

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quantity as a control variable. Other models using the same approach are, for example, Phelps and Winter (1970), Wernerfelt (1984, 1985a).

The marketing literature on this topic falls into two classes. One group of papers (Dolan and Jeuland 1981; Clarke et al. 1982; Bass and Bultez 1982; Kalish 1983) considers rather rich models but only in the context of monopoly. In contrast, another group (Thompson and Teng 1984; Rao and Bass 1985) looks at oligopoly, but makes specific assumptions about functional forms. The present paper combines these two classes in the sense that we analyze an oligopoly with less restrictive assumptions.

2. Model

In order to analyze a richer, more general model, we confine attention to symmetric open loop equilibria. While the equilibrium concept admittedly is primitive, it has by far the longest history in the literature and represents a natural first cut at a very complex problem. Asymmetric equilibria may occur even with ex ante symmetric firms (Wernerfelt 1984), but are typically difficult to characterize. Nor is it easy to find conditions which guarantee their absence (see, however, Papavassilopoulos and Cruz 1979). Similarly, equilibria with ex ante different firms pose such computational difficulties, that only the most simple models are tractable. Extension to the class of closed loop equilibria opens up an entirely new set of problems (e.g., Stokey 1985). We do, however, trust that the reader will find the intuition behind our results quite robust and we look forward to future research extending our results to asymmetric or closed loop equilibria.

2a. *Notation and Assumptions*

We begin with some notation and assumptions. Firms are indexed by i, $i = 1$ to n, and use the same technology to produce a good which may or may not be homogeneous. Each firm sets its own price and we let $p_i(t) \in R$ be firm i's price at time t. Let $s_i(t) \in [0, 1]$ be its market share, $v_i(t) \in R$ be its instantaneous variable costs, and $c_i(t) \in R$ be its instantaneous fixed costs. There are $M \in R_+$ consumers each of whom at a given time buys from one firm only, according to the positive valued, decreasing and concave demand functions $y(p_i(t))$. We assume that all demand is satisfied at all time.

We assume that variable costs decline due to experience curve effects $[g(\cdot)]$ as well as exogenous technical progress $[\alpha(\cdot)]$. Analytically:

$$\dot{v}_i(t) = g[s_i(t)y(p_i(t)), v_i(t)] + \alpha(t, v_i(t)); \qquad v_i(0) = v_{i0}; \qquad i = 1, \ldots, n. \tag{1}$$

We assume both $g(\cdot)$ and $\alpha(\cdot)$ to be twice continuously differentiable, nonpositive, increasing and concave in $v_i(t)$. Further, $g(\cdot)$ is nonincreasing in $s_i(t)$ and increasing and convex in $p_i(t)$.

Similarly, fixed costs decline following an experience curve $[h(\cdot)]$ and exogenous technical progress $[\beta(\cdot)]$:

$$\dot{c}_i(t) = h[s_i(t)y(p_i(t)), c_i(t)] + \beta(t, c_i(t)); \qquad c_i(0) = c_{i0}; \qquad i = 1, \ldots, n. \tag{2}$$

We assume $h(\cdot)$ and $\beta(\cdot)$ to be twice continuously differentiable, nonpositive, increasing and concave in $c_i(t)$. Further, we assume that $h(\cdot)$ is nonincreasing in $s_i(t)$ and increasing and convex in $p_i(t)$.

The cost function described by (1) and (2) is special in the sense that it is linear in output. On the other hand, (1) and (2) are general formulations and in particular include the ordinary experience curve effects according to which costs, say $w(\epsilon)$, depend on cumulative volume $\epsilon_i(t) = \int_0^t s_i(x)y(p_i(x))dx$, such that $\dot{w} = w'(\epsilon)s_i y(p_i) \equiv \hat{g}(s_i y(p_i), w)$.

We finally let market shares flow according to

$$\dot{s}_i(t) = f_i(s(t), p(t)); \qquad s_i(0) = s_{i0}; \qquad i = 1, \ldots, n \qquad \text{where} \tag{3}$$

$$s(t) \equiv (s_1(t), \ldots, s_n(t)), \quad p(t) \equiv (p_1(t), \ldots, p_n(t)), \quad \sum_{i=n}^{n} f_i(\cdot) = 0 \quad \text{and} \quad \sum_{i=1}^{n} s_{i0} = 1.$$

We here assume that $f(\cdot)$ is symmetric in $i, j = 1, \ldots, n$ (that is, $f(\cdot)$ is invariant to changes in the labelling of firms) and that $f_i(\cdot)$ is twice continuously differentiable, real valued, increasing in $p_j(t)$ ($j \neq i$), decreasing in $p_i(t)$ and concave in the prices. Furthermore, we endow $f_i(\cdot)$ with two properties, which need to hold only at symmetric points, that is, points where prices and market shares are identical. The first such property is:

$$(n-1)\frac{\partial f_i}{\partial p_j} = \frac{\partial f_i}{\partial p_i}; \qquad j \neq i; \qquad i, j = 1, \ldots, n.$$

This means that market shares react to a marginal unilateral price cut by firm i as they do to a marginal price raise by all other firms, excluding firm i. It is difficult to envision a reasonable $f(\cdot)$ without this property. The second property of $f(\cdot)$ (at symmetric equilibrium points) is:

$$\frac{\partial f_i}{\partial s_j} - \frac{\partial f_i}{\partial s_i} + \frac{\partial f_i}{\partial p_i} y_i n \left(\frac{\partial y}{\partial p}\right)^{-1} \geqslant 0; \qquad j \neq i; \qquad i, j = 1, \ldots, n.$$

While this condition is somewhat technical, it is not very restrictive. To see this, note that the last term on the left side is positive and may be quite large, especially if the market is large as measured by ny. Further, the difference between the first two terms will generally be positive since it should be easier to dislodge the marginal customers of a competitor, the wider a group he has. Because the loyalty of individual consumers only depends upon their own experience with a brand—not on how many others have such experiences—this says nothing about the extent of brand loyalty.

On the other hand, we argue that it is possible to interpret smaller absolute values of $\partial f_i/\partial p_i$ as representing more brand loyalty. Think of brand loyalty in the sense of user skills (Stigler and Becker 1977; Fornell, Robinson, and Wernerfelt 1985; and Wernerfelt 1985b). Products may be ex ante homogeneous, but as a consumer uses a brand, he develops user skills which make that brand more attractive for the next purchase. The stronger this learning effect is and the less the pool of consumers is renewed, the smaller will be the price sensitivity of market shares.

All firms have the same positive discount rate ρ and each seek a differentiable price path over the time interval from zero to one, trying to maximize:

$$\int_0^1 e^{-\rho t}[Ms_i(t)y(p_i(t))[p_i(t) - v_i(t)] - c_i(t)]dt; \qquad i = 1, \ldots, n. \tag{4}$$

We look for symmetric open loop Nash equilibria of the game (1)–(4).

2b. *Necessary Conditions*

For firm i, let the dual variables governing (1), (2) and (3) be $\lambda_i(t)$, $\gamma_i(t)$ and $\mu_i^j(t)$ ($j = 1, \ldots, n$), respectively. Dropping most time and firm indices and letting subscript denote derivatives, the necessary conditions for open loop Nash equilibria are:

$$e^{-\rho t}Ms[y_p(p - v) + y] + \lambda_i g_p + \gamma_i h_p + \sum_{j=1}^{n} \mu_i^j f_{jp_i} = 0; \qquad i = 1, \ldots, n, \tag{5}$$

$$\dot{\lambda}_i = e^{-\rho t}Msy - \lambda_i(g_v + \alpha_v); \qquad \lambda_i(1) = 0, \tag{6}$$

$$\dot{\gamma}_i = e^{-\rho t} - \gamma_i(h_c + \beta_c); \qquad \gamma_i(1) = 0, \tag{7}$$

$$\dot{\mu}_i^i = -e^{-\rho t}My(p-v) - \lambda_i g_s - \gamma_i h_s - \sum_{j=1}^{n} \mu_i^j f_{js_i}; \qquad \mu_i^i(1) = 0, \tag{8}$$

$$\dot{\mu}_i^j = -\sum_{d=1}^{n} \mu_i^d f_{ds_j}; \qquad j \neq i; \qquad \mu_i^j(1) = 0. \tag{9}$$

From (6) and (7) we see that λ and γ are negative and increasing. In symmetric equilibria, we can further write (5) as

$$e^{-\rho t}Ms[y_p(p-v) + y] + \lambda_i g_p + \gamma_i h_p + (\mu_i^i - \mu_i^j) f_{ip_i} = 0; \qquad j \neq i. \tag{10}$$

Subtracting (9) from (8) and using symmetry and (10) yields

$$\begin{aligned}(\dot{\mu}_i^i - \dot{\mu}_i^j) &= -e^{\rho t}My(p-v) - \lambda_i g_s - \gamma_i h_s - (\mu_i^i - \mu_i^j)(f_{is_i} - f_{is_j}) \\ &= (\mu_i^i - \mu_i^j)(f_{is_j} - f_{is_i} + nf_{ip_i}y[y_p]^{-1}) + e^{-\rho t}My^2(y_p)^{-1}.\end{aligned} \tag{11}$$

From which we see that $\mu_i^i - \mu_i^j \geq 0$ and that $\dot{\mu}_i^i - \dot{\mu}_i^j \leq 0$. By (10), this implies that $y_p(p-v) + y \geq 0$.

2c. *Analysis*

Armed with these preliminary results we can now proceed to use the implicit function theorem on (10) in order to find the time trend in the equilibrium price. Denote the Hamiltonian for (1)–(4) by $H_i(\cdot)$. We can then write (10) as $\partial H_i/\partial p_i = 0$. By the implicit function theorem we can therefore find $dp_i/dt = -\partial^2 H_i/\partial p_i \partial t(\partial^2 H_i/\partial p_i^2)^{-1}$. From the second order conditions we know that $\partial^2 H_i/\partial p_i^2$ is negative, so the sign of dp_i/dt is equal to the sign of $\partial^2 H_i/\partial p_i \partial t$. Accordingly, we can find the forces on dp_i/dt from the time derivative of the left side of (10) (signs in parentheses):

$$\underset{(-)}{-\rho e^{-\rho t}M[y_p(p-v) + y]/n} - \underset{(-)}{e^{-\rho t}My_p\alpha/n} - \underset{(?)}{e^{-\rho t}M(y_pg - yg_p)/n} + \underset{(+)}{e^{-\rho t}h_p} + \underset{(+)}{f_{ip_i}(\dot{\mu}_i^i - \dot{\mu}_i^j)}.$$

The negativity of the first two terms in this expression reflects downward pressure on the time trend of prices. The sign of the third term depends on the shapes of $y(\cdot)$ and $g(\cdot)$, whereas the positivity of the last two terms reflects pressures for time-increasing prices. Each of these five terms will disappear if and only if the model is deprived of discounting, exogenous declines in variable costs, experience curve effects on variable costs, experience curve effects on fixed costs and brand loyalty (consumer sluggishness), respectively. Accordingly, we can say that higher discount rates and more exogenous technical progress lead to time-declining prices through impatience and lowered marginal costs. Experience curve effects on variable costs on the other hand have two opposite effects, a tendency to invest in lower costs fast and a time-decline in marginal costs. Conversely, only the first of these effects is relevant for learning curve effects on fixed costs such that these lead to early "investment" in lower costs and thus time-increasing prices. Finally, the effect of brand loyalty, opening the possibility of investing in market shares, has the same effect leading to time-increasing prices. How these effects net out at a particular time in a particular market is, of course, an empirical question.

3. Conclusion

We have examined the implications of experience curves and brand loyalty for optimal dynamic pricing policies. In the class of symmetric open loop equilibria the effects of experience curves on variable costs are ambiguous: whereas discounting and exogenous declines in variable costs lead to time-declining prices, experience curves on fixed costs and brand loyalty lead to time-increasing prices.

Which of the above effects will dominate in a real market is an empirical question. However, it is interesting that we tend to see time-declining prices in markets with rapid technical progress (e.g., calculators), but less of a decline for branded products where consumers are often brand loyal. We also see less of a decline for products with high fixed costs (e.g., telecommunications services). An important area for future research is a systematic study across industries to establish whether or not our theoretical results are representative of observed phenomena.

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