General Equilibrium with Real Time Search in Labor and Product Markets

Birger Wernerfelt

Northwestern University

The paper is concerned with economies in which agents find sellers and employers in a time-consuming search process while they simultaneously trade with their current partners. A symmetric steady-state equilibrium does not exist, but asymmetric steady-state equilibria exist and are such that larger firms offer higher wages and charge lower prices than smaller firms, but still make more profits. These profits can be seen as rents from a superior market position.

I. Introduction

The Walrasian auctioneer and tatonnement process, which eliminates out-of-equilibrium trades, is central to traditional microeconomics. Except for very few organized markets, it is, however, not a realistic conception of actual market processes. Nor is it representative of the way economic agents view modern society: businesspeople often talk about market share as an asset in itself and sometimes look at advantageous factor market positions as the key to their competitive advantage. To the extent that it is appropriate to study social processes in the categories of the involved individuals, this could be seen as an undesirable feature of the Walrasian tradition.

While it is widely recognized that real market processes exhibit a lot of trade on the way to Walrasian equilibria, there is very little work on the reasons for and implications of this. In fact, most models of search do not operate in "real time" in the sense that agents are assumed to finish collecting information before the trade. The literature on truly

The helpful suggestions of an anonymous referee are gratefully acknowledged.

[*Journal of Political Economy*, 1988, vol. 96, no. 4]

dynamic effects has been based on the original insight of Arrow (1959), according to which the time needed to search out alternatives produces lags in buyer shopping responses such that sellers have "dynamic monopoly power" (see also Diamond 1971). This has been investigated in partial equilibrium models of employment turnover with on-the-job search (Burdett 1978; Jovanovic 1984) and (simultaneously with the present effort) in a static partial equilibrium model by Mortensen (1985).

This paper extends the arguments above to a general equilibrium model in which each agent has both a labor and a consumer side. I find that symmetric steady-state equilibria do not exist whereas asymmetric steady-state equilibria exist and are such that larger firms offer higher wages and charge lower prices than smaller firms but still make more profits. These profits can be seen as rents from a superior market position.

II. Model

I consider the properties of steady-state equilibria in a production economy in which firms post wage and price offers and agents engage in time-consuming but otherwise costless search. I will be able to characterize the viable wage-price offer strategies, the profits associated with each, and the size distribution of firms. The model is based on ex ante identical consumers and endogenous firm formation, such that only the randomness of the search processes generates strategic heterogeneity.

I advise the reader that the product side of the economy would be identical to that of Mortensen (1985) if the discount rate were zero.

A. *An Atomless Production Economy*

Let us look at an atomless economy with overlapping generations in infinite-horizon discrete time. In all periods $t = 1, 2, \ldots$, there is a unit measure of identical agents, each of whom is endowed with a single indivisible unit of effort per period. This effort may be spent either on labor or on leisure. The utility of leisure is normalized to the rate one, and an individual's life expectancy is independent of how he spends his time. The only other good in the economy is also valued at one utile per unit and is produced by firms from units of labor according to a differentiable function $q(\cdot)$, which expresses the measure of labor units required to produce one unit of the good as a function of the contemporaneous measure of labor units employed in the firm. Assume that $q(\cdot)$ is decreasing and convex. As its argument goes to infinity, q approaches $1/\gamma$, which is strictly less than one, such that the economy is productive. Assume that individual agents can be regarded as wage and price takers. Firms may be formed costlessly by any agent, and each firm produces at full capacity and satisfies as much demand as possible while the owning laborer consumes any surplus units.[1] Assume that firms are passed on to an inheritor of the owner such that they can exist in perpetuity.

Newborn agents are ignorant of labor and buying opportunities but can costlessly receive random, independent offers of each at rates μ and λ. The sequence of events in a given period is as follows. First, agents who have received both a wage and a price offer in the past select a prospective employer and prospective seller to go to. Second, each firm announces a single wage to the agents who turned up to consider working and a single price to those who turned up to consider buying. Third, agents who work and buy become "active" laborers and buyers if and only if the wage announced by their employer is at least as large as the price announced by their seller. Fourth, a randomly chosen fraction τ of all agents die. Fifth, a measure τ of agents are born. Sixth, each agent observes one additional randomly chosen wage with probability μ and one additional randomly chosen price with probability λ. (Since the more attractive prices and wages will be offered by more productive firms, price search needs to be made faster than wage search, so assume that $\lambda > \mu$.) Note that observing a firm's wage does not entail observing its price, and vice versa. Agents can recall all wage and price offers they have observed and decide which firms to approach in a given period on the basis of their expectations about the firms' actions in the period. All agents maximize expected lifetime utility using the discount factor $\delta < 1$, and it is not possible to store either units of labor or units of the consumption good.[2]

Firms will not be required to offer constant wage-price pairs, but we will focus on equilibria in which they do this. In such equilibria, an agent's situation in any period is summarized by the highest wage and the lowest price, if any, he has observed so far. Because search is costless, all agents will search all the time, working themselves up the wage distribution and down the price distribution. Since these processes are furthermore independent, the period t density of agents at wage w and price p can be expressed as the product of the marginal densities $h(w)$ and $k(p)$.

The situation of a firm in any period is summarized by the measure

[1] The objective of the firm will then be maximization of discounted surplus, which differs from discounted profits if firms are heterogeneous. While this formulation is convenient, it is, as far as I can tell, not crucial for the essence of the results.

[2] In the steady-state equilibrium, storage will never be optimal, but this is deceptive since the equilibrium is supported by a no-storage assumption.

of its pool of laborers, l_t, and its pool of buyers, b_t. Because search is costless, each firm has the same distribution of agents in these pools, and the translation to active laborers $\tilde{l}_t$ and active buyers $\tilde{b}_t$ is given by

$$\tilde{l}_t = l_t \int_0^{w_t} k_t(x)dx, \; \tilde{b}_t = b_t \int_{p_t}^{\infty} h_t(x)dx$$

if the firm offers w_t and p_t, respectively, and agents believe that these offers are constant over time. Since an active buyer who makes $w_t \geqslant p_t$ will buy w_t/p_t units, the revenues of the firm are $b_t\int_{p_t}^{\infty} xh_t(x)dx$, while production costs are $w_t l_t \int_0^{w_t} k_t(x)dx$. The fact that the owner consumes surplus output can be expressed by the budget balance condition

$$w_t l_t \int_0^{w_t} k_t(x)dx = b_t \int_{p_t}^{\infty} xh_t(x)dx. \tag{1}$$

Before we look at the properties of steady-state equilibria of this economy, it is helpful to note that the perfect information equilibrium is one in which all firms operate at minimum efficient scale while the wage-price ratio is γ. Prices will be normalized such that the full-information price is one.

B. *Existence Results for Steady-State Equilibria*

A steady-state equilibrium of the model described above is a situation in which (i) agents follow the optimal labor-buyer-search strategies, (ii) the distribution of laborers and buyers is time invariant, (iii) firms make time-invariant wage and price offers that maximize discounted surplus production, assuming constant wage-price offers of other firms, and (iv) no new firms are formed.

As the search process is specified, higher wages and lower prices will command larger steady-state pools of laborers and buyers, so that if two firms offer the same wage (price), they will also offer the same price (wage) in equilibrium. Accordingly, a steady-state equilibrium is completely characterized by four time-invariant functions from R_+ to R_+: (i) the density of the laborers over wages, $h(w)$; (ii) the density of buyers over prices, $k(p)$; (iii) the measure of firms over wages, call it $f(w)$; and (iv) the measure of firms over prices, call it $g(p)$. The linkage between wage and price offers can be found from the market level of aggregation of (1):

$$wh_t(w) \int_0^{w} k_t(x)dx = k_t(p) \int_{p}^{\infty} xh_t(x)dx,$$

which implicitly defines $\hat{w}(p|h, k)$ or $\hat{p}(w|h, k)$. We therefore have the equilibrium condition

$$g_t(\hat{p}(w|h_t, k_t)) = f_t(w) \text{ or } g_t(p) = f_t(\hat{w}(p|h_t, k_t)).$$

In a steady state, in which laborers and buyers expect constant offers, the optimal search strategy entails constant search and myopic adoption of any improvement. In this case, the market-level implications are

$$h_{t+1}(w) - h_t(w) = \left[\frac{(1-\mu)\tau}{(1-\mu)\tau+\mu} + \int_0^w h_t(x)dx\right]\mu f(w) - h_t(w)\left[\tau + \mu \int_w^\infty f_t(x)dx\right] \quad (2)$$

and

$$k_{t+1}(p) - k_t(p) = \left[\frac{(1-\lambda)\tau}{(1-\lambda)\tau+\lambda} + \int_p^\infty k_t(x)dx\right]\lambda g(p) - k_t(p)\left[\tau + \lambda \int_0^p g_t(x)dx\right] \quad (3)$$

such that the steady-state conditions are

$$\left[\frac{(1-\mu)\tau}{(1-\mu)\tau+\mu} + \int_0^w h_t(x)dx\right]\mu f(w) = h_t(w)\left[\tau + \mu \int_w^\infty f(x)dx\right]$$

and

$$\left[\frac{(1-\lambda)\tau}{(1-\lambda)\tau+\lambda} + \int_p^\infty k_t(x)dx\right]\lambda g(p) = k_t(p)\left[\tau + \lambda \int_0^p g(x)dx\right].$$

From (1) the surplus of a firm characterized by l and b can be expressed as

$$b_t \int_{p_t}^\infty x h_t(x)dx\left(\left\{w_t q\left[l_t \int_0^w k_t(x)dx\right]\right\}^{-1} - p_t^{-1}\right) \equiv \pi(l_t, b_t, w_t, p_t).$$

Accordingly, the maximization problem facing firms is to find sequences $w(t|l_0, b_0)$ and $p(t|l_0, b_0)$ that satisfy (1) and

$$\max \sum_{t=0}^{\infty} \delta^t \pi(l_t, b_t, w_t, p_t)dt, \quad (4)$$

$$l_{t+1} - l_t = \left[\frac{(1-\mu)\tau}{(1-\mu)\tau+\mu} + \int_0^{w_t} h_t(x)dx\right]\mu \frac{1}{n} - l_t\left[\tau + \mu \int_{w_t}^\infty f_t(x)dx\right] \equiv L_t(l_t, w_t), \quad (5)$$

$$b_{t+1} - b_t = \left[\frac{(1-\lambda)\tau}{(1-\lambda)\tau+\lambda} + \int_{p_t}^\infty k_t(x)dx\right]\lambda \frac{1}{n} - b_t\left[\tau + \lambda \int_0^{p_t} g_t(x)dx\right] \equiv B_t(b_t, p_t), \quad (6)$$

where n is the measure of firms in the economy. For this model the following theorem applies.

THEOREM 1. There are no steady-state equilibria in which $f(\cdot)$ and $g(\cdot)$ have (*a*) mass points or (*b*) unconnected support.[3]

Proof. See the Appendix.

An important implication of this is the following corollary.

COROLLARY. Symmetric steady-state equilibria do not exist.

Inspection of the proof of theorem 1 gives an idea about the critical assumptions: if there are direct or opportunity costs of search or switching costs, the argument does not go through. If it is assumed that δ, λ, μ, τ, and $q(\cdot)$ are such that the control problem (1) and (4)–(6) satisfies its second-order conditions, the following theorem applies.

THEOREM 2. There exists an asymmetric steady-state equilibrium.

Proof. See the Appendix.

We can find n from the requirement that no more firms are formed. This can be done by considering the maximization problem for a firm for which l and b initially are zero. Let us denote the smallest and largest wages (prices) by α and β, respectively, and use subscripts to denote partial derivatives. We then get, after considerable manipulation,

$$\pi_b h(\beta)\lambda + \pi_l h(\alpha)\mu = 0. \tag{7}$$

C. *Properties of Asymmetric Steady-State Equilibria*

The first important property of equilibrium is that the firms offering low wages offer high prices, and vice versa. Heuristically, this happens because firms with large (small) buyer pools need large (small) labor pools. Note further that the highest (lowest) price in the market is equal to the highest (lowest) wage such that the optimal strategies lie on a curve in the box $[\alpha, \beta]^2$. This is illustrated in figures 1 and 2. This wage and price pattern implies that the largest firms lie on the highest cost curve while smaller firms lie on lower cost curves but are so inefficient that their net costs are higher. This is illustrated in figure 3. We can use equation (1) to compute the profit margins for various (w, p) pairs and to convince ourselves that markets will clear at each price, taking into account the surplus consumed by the laborer who owns the firm.

It is possible to interpret the equilibrium in almost Newtonian terms because of analogies to the constancy of energy. Consider the

[3] If $q(\cdot)$ is U-shaped, we need to be concerned about a degenerate equilibrium on the upward-sloping part of q.

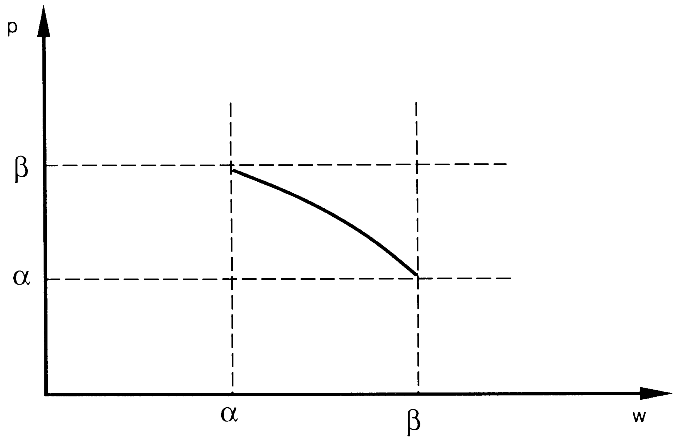

Fig. 1.—Equilibrium strategies

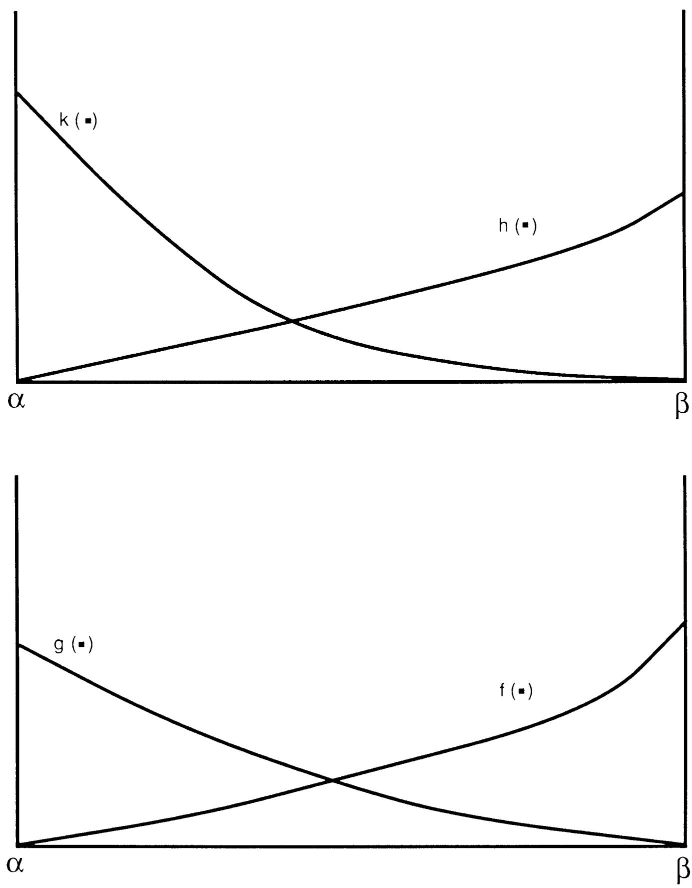

Fig. 2.—A sample solution

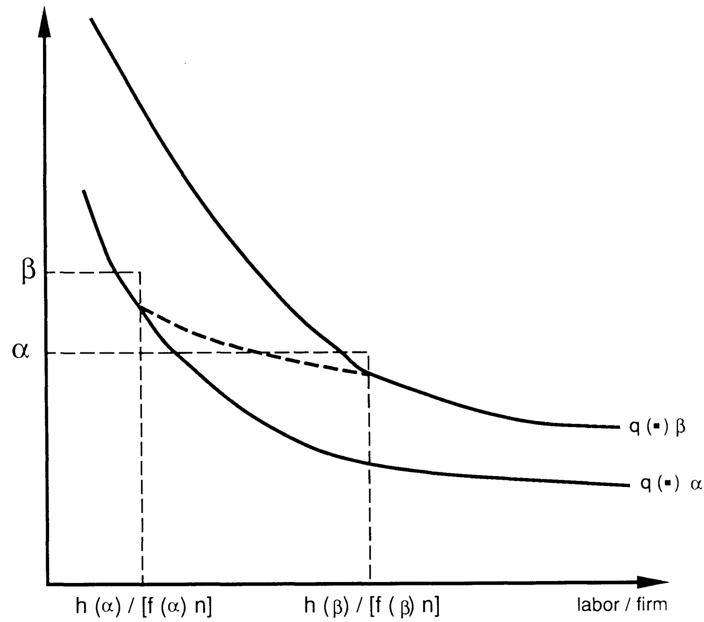

FIG. 3.—The equilibrium cost functions $q(\cdot)w$

equilibrium curve in figure 4. The largest firms have the highest "potential energy" by virtue of large l's and b's. They could milk this position by lowering wages or increasing prices. The differential surplus from having this position is, however, sufficiently high to make such a milking strategy unappealing. On the other hand, it is so expensive to move up to this position that the involved costs just prevent smaller firms from moving up. So the costs or benefits from moving along the equilibrium curve are such that no one wants to move. The value of being a firm of type x plus the costs or benefits of moving to position y are equal to the value of being a firm of type y. In case the interest rate goes to zero, these mobility barriers disappear and the profits go to zero throughout.[4] Note finally that (7) gives the entry condition such that ex ante surplus is zero for an outsider. So while incumbent firms earn surplus, entrants would have to pay a fair price for it in the entry process.

III. Conclusion

Since the results of the paper are summarized in the Introduction, I will not repeat them here. Instead, I will emphasize that models of

[4] In this case $\alpha \rightarrow 1$ and $\beta \rightarrow \gamma$, while the size of the largest firms approaches minimum economic scale.

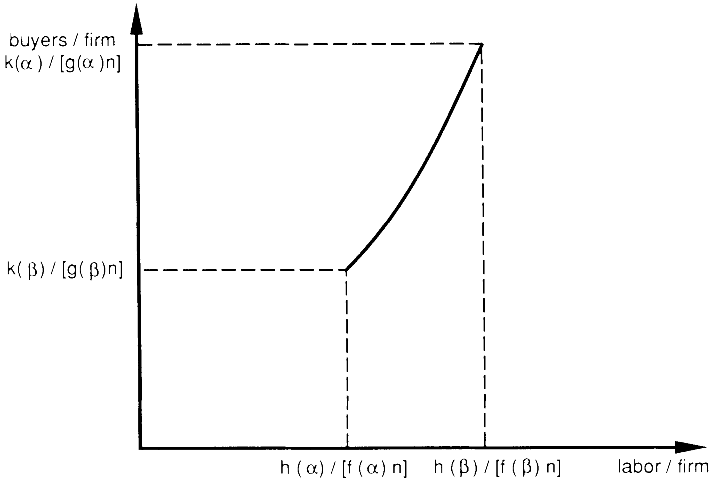

FIG. 4.—The equilibrium curve

real time search depict a good market position (be it in product or factor markets) as an asset that may yield very significant quasi rents. Markets are networks of more or less durable long-term relationships rather than auctions. Since one does observe such trading patterns in the economy and since the equilibria in these models give rise to quite reasonable macro predictions, there would seem to be a case for continued work in this area.

The wage and price dispersion in this paper is generated by the random nature of the search process even though all agents are identical ex ante. The same idea is exploited in a recent model by Albrecht, Axell, and Lang (1986), which exhibits quite different results. In particular, they show the existence of several equilibria with a finite number of wage-price pairs. This result follows from their assumption that there is no on-the-job search and no utility of leisure. Accordingly, if a firm deviates a bit from, say, a two-price, two-wage equilibrium, it need not get more buyers or laborers. It draws only from the pool of uncommitted agents, and if the opportunity costs of search are high and "few" firms are expected to offer better deals, the members of this pool may not search more because of the deviation. It should also be mentioned that Albrecht et al. constrain firms to constant wage-price offers, so the possibility of exploiting dynamic monopoly power is ignored.

While the mathematical difficulties may be nontrivial, it would be desirable to consider the possibility of non-steady-state equilibria in the model. On a different level, the present model could be generalized to incorporate switching costs, costly search, advertising, and so forth.

Appendix

Proof of Theorem 1

(*a*) If f has a mass point, an infinitesimal wage increase by any of the involved firms will give that firm a discrete upward jump in its steady-state pool of active laborers. Since q is decreasing, such an action will increase surplus. (*b*) If there is a gap in the support of f, firms immediately above the gap could benefit from moving immediately below it. Q.E.D.

Proof of Theorem 2

In a steady-state equilibrium agents maximize expected lifetime utility by behaving as described in (2)–(3) or (4)–(5). We can show existence by assuming this behavior and demonstrating that the game (1) and (4)–(6), played between a fixed measure of firms, has a steady-state equilibrium.[5]

The proof is structured as follows. We will first use (1) and various accounting identities to simplify the game such that the players select functions $w_t(l_t|h_t^+)$, where h_t^+ is an infinite sequence $h_t, h_{t+1}, \ldots$ of densities of laborers over wages. Consider a sequence of identical such densities, called h^0. Given h^0, there is a function $\bar{w}(l|h^0)$ that will keep a firm's l constant. Similarly, there is a function $w^*(l|h^0)$ that will maximize (4) given (1), (5), and (6). The idea in proof is that one can select h^0 such that $\bar{w}(l|h^0) = w^*(l|h^0)$. Since (2) and (3) are aggregates of (5) and (6), these strategies will generate the sequence h^0.

For starters, theorem 1 states that all relevant functions are jointly continuous. In particular, this is true of h, k, π, L, and B. Now let us use (1) to find $p_t(w_t|l_t, b_t, h_t, k_t)$. Further, knowing that $\bar{w}(l)$ is monotonous, we can construct its inverse $\bar{l}(w)$ and use the accounting identity $fn = h/\bar{l}$ to rewrite the steady-state conditions

$$\left[\frac{(1-\mu)\tau}{(1-\mu)\tau+\mu} + \int_0^{w} h(x)dx\right]\frac{\mu}{n} = \bar{l}(w)\left[\tau + \int_{w}^{\infty}\frac{h(x)}{\bar{l}(x)}\,dx\,\frac{\mu}{n}\right]. \tag{5'}$$

Similarly, we can construct $\bar{b}(p)$ to get

$$\left[\frac{(1-\lambda)\tau}{(1-\lambda)\tau+\lambda} + \int_p^{\infty} h(x)dx\right]\frac{\lambda}{n} = \bar{b}(p)\left[\tau + \int_0^{p}\frac{k(x)}{\bar{b}(x)}\,dx\,\frac{\lambda}{n}\right]. \tag{6'}$$

Given h and k, for any l' there exists a unique b' for which the solution to (5'), $\bar{w}(l')$, is identical to the solution to (6'), $\bar{w}(p(b'))$. In the following we will restrict our attention to pairs l, b that satisfy this relationship. Hence we can suppress b. Similarly, the relationship $b'(l')$ defines a unique k for any h, so we can work with sequences of h alone and suppress the associated k's. The problem is now somewhat simpler since we can perform the analysis on w, l, and h.

Define M as the space of integrable functions from R_+ to R_+, each of which is positive on a single interval $[\bar{\alpha}, \bar{\beta}] \subset R_+$ ($\bar{\alpha} < 1 < \bar{\beta}$) and zero elsewhere; M_m

[5] For games of this type, Jovanovic and Rosenthal (1986) show the existence of an equilibrium in which the state-action distribution is constant. Unfortunately, this does not rule out "cycling," so although their equilibrium is stationary in one sense, it is not steady state in this sense.

is the subset of M that is continuous and monotonous on $[\bar{\alpha}, \bar{\beta}]$. Given any member of M, say h', (5′) identifies a unique $l(w|h')$. For any $\bar{l}(w|h') \in M_m$, (5′) is an ordinary differential equation of the first order in h. So (5′) maps M onto M_m. We can denote the inverse of $\bar{l}$ by $\overline{w}(l|h')$. This is clearly continuous in h.

If we substitute $f(x) = h(x)/[\bar{l}(x)n]$ into (5) and (6), the game (1) and (4)–(6) becomes a set of control problems for any infinite sequence of identical densities $h^0 = h^*, h^*, \ldots, h^* \in M$. Given h^0, standard results tell us that (1) and (4)–(6) has a solution $w_t^*(l_t|h^0)$, $t = 1, 2, \ldots$. Similarly, because w_1^* is a continuous function of h^* and integrals of h^*, we can find densities $h^{**} \in M$ such that $w_1^*(l|h^{**}, h^{**}, \ldots) = \overline{w}(l)$ for all $\bar{l}(w) \in M_m$. To do this, take the interval $[\bar{\alpha}, \bar{\beta}]$ from $\bar{l}(w)$. After this, $w^*(l|h) = \overline{w}(l)$ becomes another ordinary differential equation in h. We can define C as the correspondence from $\bar{l}(w)$ to h^{**}. From the second-order conditions and the monotonicity of $\overline{w}(l)$, we know that the control problems are well behaved such that the optimal policy is continuous in the exogenous functions. Therefore, C is upper semicontinuous. Any fixed point of $C \circ \bar{l}(w|h)$ identifies an h for which the optimal actions of each player (a) keep the state of that player constant and (b) preserve the aggregate state distribution.

The existence of such a fixed point can be guaranteed by the Fan-Glicksberg fixed-point theorem (Fan 1952; Glicksberg 1952). We have already established upper semicontinuity, and it is straightforward to verify nonemptiness and that C is closed- and convex-valued. To check that $C \circ \bar{l}(w|h)$ maps M into M, start in M and apply $\bar{l}$ to get into M_m, from which C maps into M again. Since M is convex, we are done. Q.E.D.

References

Albrecht, James W.; Axell, Bo; and Lang, Harald. "General Equilibrium Wage and Price Distributions." *Q.J.E.* 101 (November 1986): 687–706.

Arrow, Kenneth J. "Toward a Theory of Price Adjustment." In *The Allocation of Economic Resources,* by Moses Abramowitz et al. Stanford, Calif.: Stanford Univ. Press, 1959.

Burdett, Kenneth. "A Theory of Employee Job Search and Quit Rates." *A.E.R.* 68 (March 1978): 212–20.

Diamond, Peter A. "A Model of Price Adjustment." *J. Econ. Theory* 3 (June 1971): 156–68.

Fan, K. "Fixed Point and Minimax Theorems in Locally Convex Topological Linear Spaces." *Proc. Nat. Acad. Sci.* 38 (1952): 121–26.

Glicksberg, I. L. "A Further Generalization of the Kakutani Fixed Point Theorem with Application to Nash Equilibrium Points." *Proc. American Math. Soc.* 3 (1952): 170–74.

Jovanovic, Boyan. "Matching, Turnover, and Unemployment." *J.P.E.* 92 (February 1984): 108–22.

Jovanovic, Boyan, and Rosenthal, R. W. "Anonymous Sequential Games." Mimeographed. New York: New York Univ., Dept. Econ., 1986.

Mortensen, Dale T. "Closed Form Equilibrium Price Distribution Functions." Mimeographed. Evanston, Ill.: Northwestern Univ., Dept. Econ., 1985.