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ON EXISTENCE OF A NASH EQUILIBRIUM POINT IN N-PERSON NON-ZERO SUM STOCHASTIC JUMP DIFFERENTIAL GAMES

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SUMMARY

Using the technique of Wan and Davis, we give an existence theorem for a Nash equilibrium point in N-person non-zero sum stochastic jump differential games. It is shown that if the Nash condition (generalized Isaacs condition) holds there is a Nash equilibrium point in feedback strategies. We extend the results to other solution concepts and discuss applications and extensions.

KEY WORDS Nash equilibrium Differential games Jump processes

1. INTRODUCTION

In Uchida[1] the existence of Nash equilibrium points in stochastic differential games is looked at by using the technique of Davis and Variaya.[2] It is an essential point of this technique that analogues of the time derivation of the gradient of the value function are constructed using a martingale method. Consequently, we can obtain the optimal value directly by optimizing the Hamiltonian at each point. In this paper we will give parallel results for stochastic jump differential games, using the technique of Wan and Davis.[3]

Keeping the notation close to that in Reference 3, we consider a jump process x_t specified under a basic probability measure P to which corresponds a pair of entities (Λ, λ) called the local description of the process; Λ determines the rate of occurrence of jumps while λ determines their positions. By using an indexed pair of Radon–Nikodym derivatives (α^u, β^u), we achieve control of x_t through mutually absolutely continuous transformation of the local descriptions from (Λ, λ) to (Λ^u, λ^u). Neither the jump process nor the controls need to be Markovian.

The player $i, i = 1, \ldots, N$, chooses a feedback control $u_i(t, x)$ over the finite interval $[0, T_f]$. Together, these controls determine $\mathbf{u}(t, x) = (u_1(t, x), \ldots, u_N(t, x))$. Corresponding to this choice of control, player i incurs a cost of the form

$$J_i(\mathbf{u}) = E_u\left[\int_0^{T_f} c_i(s, u_s, \omega)\, \mathrm{d}\Lambda_s + G_{if}\right] \quad \text{(see Definition 2.2)} \tag{1}$$

In Section 2 we give a mathematical formulation of the game and some related results. In Section 3 we prove the main theorem, while solution concepts other than Nash equilibrium are considered in Section 4. We devote Section 5 to applications and extensions.

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2. PRELIMINARIES

2.1. The jump process

The jump process $\{x_t\}$ is piecewise constant, takes values in a Blackwell space $(X, \mathscr{S})$ and has isolated discontinuities. If z_0 is a fixed element of X, we can define $(Y^j, \mathscr{Y}^j)$ as the copy of

$$(Y, \mathscr{Y}) = ((R^+ \times X) \cup \{(\infty, z_\infty)\}, \sigma\{\underline{B}(R^+) \otimes \mathscr{S}, \{\infty, z_\infty\}\})$$

for $j = 1, 2, \ldots$. In this case the basic measurable space is $(\Omega, \mathscr{F}^0)$:

$$\Omega = \prod_{j=1}^{\infty} Y^j \qquad \mathscr{F}^0 = \sigma\left\{ \prod_{j=1}^{\infty} \mathscr{Y}^j \right\}$$

We can now define (S_j, Z_j): $\Omega \to Y^j$ as the co-ordinate mapping such that $\{S_j\}$ are the 'inter-arrival times' and $\{Z_j\}$ the 'states' defining $\{x_j\}$. Futher, let $\omega_k = \Pi_{j=1}^{k} Y^j$ be the projection onto Ω_k. If, finally, $z_0(\omega) = z_0$ is another fixed element of X, we can let

$$T_k(\omega) = \sum_{j=1}^{k} S_j(\omega) \qquad T_\infty(\omega) = \lim_{k \to \infty} T_k(\omega)$$

such that the sample path of $\{x_t\}$ is

$$x_t(\omega) = \begin{cases} Z_j(\omega) & t \in [T_j(\omega), T_{j+1}(\omega)) \\ z_\infty & t \geqslant T_\infty(\omega) \end{cases}$$

2.2. A measure

To get a measure P on $(\Omega, \mathscr{F}^0)$, we assume that the $\{S_k\}$ are independent with survivor functions $F_t^k = P(S_k > t)$ as given by the functions

$$\Lambda^k\colon [0, d^k) \to R^+ (0 < d^k \leqslant \infty)$$

satisfying

(i) $\Lambda^k(0) = 0$, $\Lambda^k(\cdot)$ increasing and right continuous
(ii) $\Lambda^k(t) \uparrow \infty$ as $t \uparrow \infty$ if $d^k = \infty$
(iii) $\Delta\Lambda^k(s) = \Lambda^k(s) - \Lambda^k(s-) < 1$
(iv) there exist positive constants θ_1 and θ_2 such that $\Lambda^k(t) \leqslant \theta_2$ for $t \in [0, \theta_1]$ and $k \in \mathscr{K}$ where $\mathscr{K}$ is an infinite subset of the integers $1, 2, \ldots$.

Based on this,

$$F_t^k = \exp(-\Lambda^k(t) + \sum_{s \leqslant t} \Delta\Lambda^k(s)) \prod_{s \leqslant t} (1 - \Delta\Lambda^k(s))$$

for the countable set $\{s \leqslant t\colon \Delta\Lambda^k(s) \neq 0\}$.

Remark. The T_k sequence is a Poisson process if $\Lambda^k(t) = t$, but the framework applies equally to discrete time models.

We further specify the functions $\lambda^k\colon \Omega_{k-1} \times R^+ \times \mathscr{S} \to [0, 1]$ such that:

(i) $\lambda^k(\cdot, \cdot, A)$ is measurable for each $A \in \mathscr{S}$.
(ii) $\lambda^k(\omega_{k-1}, t, \cdot)$ is a probability measure on $\mathscr{S}$ for each $(\omega_{k-1}, t) \in \Omega_{k-1} \times (0, d^k]$ such that $\lambda(\omega_{k-1}, t, \{Z_{k-1}(\omega)\}) = 0$.

P can thus be defined as

$$P(S_k > t, Z_k \in A \mid F^0_{T_{k-1}}) = -\int_{(t,\infty]} \lambda_k(\omega_{k-1}, s, A)\, \mathrm{d}F^k_s$$

From this we define $\mathscr{F}_t$ as $\mathscr{F}_t^0$ completed with all P-null sets of $\mathscr{F}^0$. So $\mathscr{F}_t$ is the completed σ-field on Ω generated by x_t up to time t.

2.3. *Martingale*

To characterize the fundamental family of martingales associated with $\{x_t\}$, we define

$$p(t, A) = \sum_j I_{(t \geq T_j)}\, I_{(Z_j \in A)}$$

$$\Lambda_t(\omega) = \Lambda^1(S_1) + \Lambda^2(S_2) + \ldots + \Lambda^{k-1}(S_{k-1}) + \Lambda^k(t - T_{k-1}), \quad t \in (T_{k-1}, T_k]$$

$$\lambda(t, A)(\omega) = \sum_{k=1}^{\infty} I_{(t \in (T_{k-1}, T_k])}\, \lambda^k(\omega_{k-1};\, t - T_{k-1}, A)$$

$$\tilde{p}(t, A) = \int_{(0,t]} \lambda(t, A)\, \mathrm{d}\Lambda_t$$

$$q(t, A) = p(t, A) - \tilde{p}(t, A)$$

such that $q(t, A)$ is a local martingale of $\mathscr{F}_t$.

2.4. *Controls*

Each player $i, i = 1, 2, \ldots, N$, can influence the jump process through a control $\mathbf{u}^i(t, x)$ with values in a compact metric space U^i.

Definition 2.1. $u_i(\cdot)$ is in the class of admissible controls $\mathscr{U}_i$ if u_i is $\mathscr{F}_t$-predictable.

Remark. We could allow U_i to depend on $\mathbf{u}^i \equiv (u_i, \ldots, u_{i-1}, u_{i+1}, \ldots, u_n)$ but prefer the less general case for ease of notation.

Let us now define $U = \prod_{i=1}^N U_i$, $\mathscr{U} = \prod_{i=1}^N \mathscr{U}_i$, $\mathscr{U}^i = \prod_{j \neq i}^N \mathscr{U}_j$ and the two functions α: $R^+ \times U \times \Omega \to R^+$ and β: $R^+ \times X \times U \times \Omega \to R^+$. The measurable functions α and β satisfy:

(i) For each $(x, y) \in X \times U, \alpha(t, \mathbf{u}, \omega)$ and $\beta(t, x, \mathbf{u}, \omega)$ are $\mathscr{F}_t$-predictable.
(ii) There exist positive constants C_1, C_2, C_3 and δ' such that

$$C_1 \leqslant \alpha(\cdot) \leqslant \min C_2, \left\{\frac{1-\delta'}{\Delta\Lambda_t}\right\} \qquad \text{and} \qquad C_1 \leqslant \beta(\cdot) \leqslant C_3$$

for all $(t, x, \mathbf{u}, \omega) \in R^+ \times X \times U \times \Omega$.
(iii) $\int_X \beta(t, x, \mathbf{u}, \omega)\ \lambda(\mathrm{d}x, t, \omega) = 1$ for all $(t, \mathbf{u}, \omega) \in R_+ \times U \times \Omega$.

Remark. The conditions (ii) are unpleasantly strong. Obtaining our results without them is an important goal of future research. Given the results of Kabanov,[4] it seems that his very different techniques may give existence without (ii). For now, we need these conditions to assure mutual absolute continuity of all solution measures.

We further define $\omega^u(t,\omega)=\alpha(t,\mathbf{u}(t,\omega),\omega)$ and $\beta^u(t,\omega)=\beta(t,x,\mathbf{u}(t,\omega),\omega)$.

There now exist, for each $k\in Z^+$ and $\mathbf{u}\in\mathscr{U}$, functions α_k^u: $\Omega_{k-1}\times R^+\to R$ and β_k^u: $\Omega_{k-1}\times X\times R^+\to R$ such that

$$\alpha^u(t,\omega)=\sum_k \alpha_k^u(\omega_{k-1},t-T_{k-1}(\omega))I_{(t\in(T_{k-1},T_k])}$$

$$\beta^u(t,x,\omega)=\sum_k \beta_k^u(\omega_{k-1},x,t-T_{k-1}(\omega))I_{(t\in(T_{k-1},T_k])}$$

For a given $\mathbf{u}\in\mathscr{U}$, a measure P_u is defined on $(\Omega,\mathscr{F}_\infty)$ by the Radon–Nikodym derivative of its restriction to $\mathscr{F}_{T_M}$, $M=1,2,\ldots$, as

$$\frac{dP_u}{dP}\bigg|\mathscr{F}_{T_M}=\prod_{k=1}^{M} L_k(\omega)$$

where

$$L_k(\omega_k)=\alpha_k^u(\omega_{k-1},S_k)\beta_k^u(\omega_{k-1},Z_k,S_k)\exp\left[-\int_{(0,S_k]}(\alpha_k^u(\omega_{k-1},s)-1)\,d\Lambda^{kc}(s)\right]\pi_{S_k}^{k-}I(S_k\leqslant d^k)$$

$$\Lambda^{kc}(s)=\Lambda^k(s)-\sum_{y\leqslant s}\Delta\Lambda^k(y)$$

and

$$\pi_t^k=\prod_{s\leqslant t}(1-\alpha_k^u(\omega_{k-1},s)\,\Delta\Lambda^k(s))(1-\Delta\Lambda^k(s))^{-1}$$

2.5. *Costs*

We now define the cost structure of the game, which takes place over the finite interval $[0,T_f]$.

Definition 2.2.

For each player i, $i=1,2,\ldots,N$, the cost rate is the function c_i: $[0,T_f]\times U\times\Omega\to R^+$ such that:

(i) $c_i(t,\mathbf{u},\omega)$ is an $\mathscr{F}_t$-predictable function of (t,ω) for each $\mathbf{u}\in U$.
(ii) There is a positive constant C_4 such that $c_i(t,\mathbf{u},\omega)\leqslant C_4$ for all $(t,\mathbf{u},\omega)\in[0,T_f]\times U\times\Omega$.

The terminal costs G_{if} are non-negative $\mathscr{F}_{T_f}$-measurable random variables also bounded by C_4. Given this, the cost to player i corresponding to $\mathbf{u}\in\mathscr{U}$ is

$$J_i(\mathbf{u})=E_u\left[\int_{(0,T_f]}c_i(s,u_s,\omega)\,d\Lambda(s,\omega)+G_{if}(\omega)\right]$$

Remark. This formulation includes cases where the integral part of the cost function is of the form[3]

$$E_u\int_{(0,T_f]\times X}\varkappa(x,s,u_s)\tilde{p}^u(ds,dx)$$

where $\tilde{p}(t,A)$ is the compensation for $p(t,A)$ under P_u.

2.6. *Value functions*

From the above, the value function for player i given $\mathbf{u}^i(\cdot)$ is the process $\{W_{it}\}$ given by

$$W_{it}(\mathbf{u}^i)=\bigwedge_{u_i\in\mathscr{U}_i}E_u\left[\int_{(t,T_f]}c_i(s,u_s)\,d\Lambda_s+G_{if}\mid\mathscr{F}_t\right]$$

It further satisfies the 'principle of optimality' (theorem 4.1 in Reference 3):

Theorem 2.1. For any $u_i \in \mathscr{U}_i$, the process

$$\left[M^u_{it}(u_i) = \int_{(0,t]} c_i(s, u_s)\, \mathrm{d}\Lambda_s + W_{it}(\mathbf{u}^i)\right]$$

is an $(\mathscr{F}_1, P_u)$-submartingale. It is a martingale iff u_i is optimal given $\mathbf{u}^i$.

2.7. *Hamiltonian*

We can decompose into

$$M^u_{it}(u_i) = W_{i0}(\mathbf{u}^i) + N^u_{it}(\mathbf{u}^i) + a^u_{it}(\mathbf{u}^i) \tag{2}$$

where $\{N^u_{\mathrm{it}}(\mathbf{u}^i)\}$ is an $\{\mathscr{F}_t, P_u\}$-martingale and $\{a^u_{it}(\mathbf{u}^i)\}$ is a predictable increasing process with $a^u_{i0}(\mathbf{u}^i) = 0$. $N^u_{it}\,(\mathbf{u}^i)$ can then be written as

$$N^u_{it}(u_i) = \int_{(0,t]\times X} g_i(s, x, \mathbf{u}^i) q^u(\mathrm{d}s, \mathrm{d}x) \tag{3}$$

for some $g_i \in L^{\mathrm{loc}}_1(p)$ where $q^u(t, A) = p(t, A) - \tilde{p}^u(t, A)$. Based on this, we define the 'Hamiltonian'

$$H_i(t, \mathbf{u}, \omega) = \mathrm{c}_i(t, \mathbf{u}, \omega) + \alpha(t, \mathbf{u}, \omega) \int_X g_i(t, x, \mathbf{u}^i(t, \omega), \omega)\ \lambda(\mathrm{d}x, t, \omega)$$

The idea is now to minimize this in a pointwise fashion.

3. EXISTENCE OF NASH EQUILIBRIUM

We start by giving the conventional definition:

Definition 3.1. $\mathbf{u}^* = (u_1^*, \ldots, u_N^*)$ is a Nash equilibrium point if for each i

$$J_i^*(\mathbf{u}^{i*}) = J_i(u_i^*, \mathbf{u}^{i*}) \leqslant J(v_i, \mathbf{u}^{i*}) \quad \text{for all } v_i \in \mathscr{U}_i$$

Remark. Elliott and Davis[5] have looked at equilibria where $u_i(t, x)$ only can depend on $\mathbf{u}^i(s, x)$, $s \leqslant t$. While this seems to be a natural requirement, it is not the conventional Nash concept.

Definition 3.2. The Nash condition holds if for each i there exists a function u_i^*: $[0, T_\mathrm{f}] \times X \times \Omega \times U^i \to U_i$ such that for each $(t, x, \omega, v_i) \in [0, T_\mathrm{f}] \times X \times \Omega \times U_i$

$$H_i(t, u_i^*, \mathbf{u}^{i*}, \omega) \leqslant H_i(t, v_\mathrm{i}, \mathbf{u}^{i*}, \omega)$$

Remark. Since u_i^* depends on both past and future values of $\mathbf{u}^{i*}$, the Nash condition is quite complicated and less innocuous than it first appears. Perhaps the dynamic programming methods of Reference 5 can improve on this.

Our main result is now:

Theorem 3.1. If the Nash condition holds, there is a Nash equilibrium point.

Proof. We prove that the controls u_i^* above are optimal in $\mathscr{U}_i$ given $\mathbf{u}^{i*}$ for all i by following the proof in Reference 3 (theorem 4.2). By (2) and (3)

$$M_{it}^u(\mathbf{u}^i) = J_i^* + \int_{(0,t]\times X} g_i(s,x,\mathbf{v}^i)q^u(\mathrm{d}s,\mathrm{d}x) + a_{it}^u(\mathbf{u}^i) \tag{4}$$

Letting * stand for $\mathbf{u}^*$,

$$\begin{aligned} M_{it}^*(\mathbf{u}^{i*}) &= \int_0^t c_i(s,x,s,\mathbf{u}^*)\ \mathrm{d}\Lambda_s + W_i(t) \\ &= M_{it}^u + \int_0^t (c_{is}^* - c_{is}^u)\ \mathrm{d}\Lambda_s \\ &= J_i^*(\mathbf{u}^{i*}) + \int_{(0,t]\times X} \mathrm{d}q^u + a_{it}^u + (c_{is}^* - c_{is}^u)\ \mathrm{d}\Lambda_s \\ &= J_i^*(\mathbf{u}^{i*}) + \int_{(0,t]\times X} g_i\ \mathrm{d}q^* + a_i^*(\mathbf{u}^{i*}) \end{aligned}$$

where

$$a_i^*(\mathbf{u}^{i*}) = a_{it}^u(\mathbf{u}^{i*}) - \bar{a}_{it}^u(\mathbf{u}^{i*}) = a_{it}^u(\mathbf{u}^{i*}) - \int_0^t [c_{is}^u - c_{is}^* + \int_X g_i(\alpha^u\beta^u - \alpha^*\beta^*)\ \mathrm{d}\Lambda_s$$

By theorem 4.1 in Reference 3, if $a_{iT_s}^* = 0$ a.s. then u_i^* is optimal and $\mathbf{u}^*$ is a Nash equilibrium. Hold $\mathbf{u}^{i*}$ constant; this follows from the proof on p. 219 of Reference 3. Since this holds for all $u_i^*(\mathbf{u}^{i*})$, we see that $\mathbf{u}^*$ thus constructed is a Nash equilibrium. □

Remark. It may well be possible to obtain this result under less restrictive assumptions if attention is confined to the Markovian case. In the context of control theory this is, of course, of considerable practical interest. However, in many modern applications of game theory, such restrictions are not natural.

4. OTHER SOLUTION CONCEPTS

It is easy to extend our main result to other solution concepts. Let us first give the conventional definitions:

Definition 4.1. $\mathbf{u}^* \in \mathscr{U}$ is efficient if there is no $\mathbf{u} \in \mathscr{U}$ such that $J_i(\mathbf{u}) < J_i(\mathbf{u}^*)$ for all $i = 1, 2, \ldots, N$.

Definition 4.2. $\mathbf{u}^* \in \mathscr{U}$ is the core if there is no $S \subset \{1, 2, \ldots, N\}$ and no $\mathbf{u} \in \mathscr{U}$ such that $J_i(u_{\bar{S}}^*, u_S) < J_i(\mathbf{u}^*)$ for all $i \in S$, where $u_{\bar{S}}^* = \{u_{\bar{S}}^*, i \in \bar{S}\}$, $u_S = \{u_i, i \in S\}$ and $\bar{S}$ is the complement of S.

Theorem 4.1. There is an efficient point if there exist a non-negative $\boldsymbol{\lambda} = (\lambda_1, \ldots, \lambda_N) \in R^N, \boldsymbol{\lambda} \neq \mathbf{0}$, and a function u_i^*: $[0, T_f] \times \Omega \to U_i$ for all i such that for each $(t, \omega, \mathbf{u}) \in [0, T_f] \times \Omega \times \mathscr{U}$

$$\sum_{i=1}^N \lambda_i H_i(t, \mathbf{u}^*, \omega) \leqslant \sum_{i=1}^N \lambda_i H_i(t, \mathbf{u}, \omega)$$

Proof. Proceeding as in the proof of theorem 3.1, we can prove that the $\mathbf{u}^*$ and the λ above satisfy for each $(t, \omega, \mathbf{u}) \in [0, T_f] \times \Omega \times \mathcal{U}$

$$\sum_{i=1}^{N} \lambda_i J_i(\mathbf{u}^*) \leqslant \sum_{i=1}^{N} \lambda_i J_i(\mathbf{u})$$

so there is no $\mathbf{u}^*$ such that $J_i(\mathbf{u}^*) > J_i(\mathbf{u})$ for all i. □

Theorem 4.2. If the Nash condition holds and for each $S \subset \{1, 2, \ldots, N\}$ there exist constant $\lambda_i^S \geqslant 0$, $i \in S$, not all zero, such that for each $(t, \omega, \mathbf{u}) \in [0, T_f] \times \Omega \times \mathcal{U}$

$$\sum_{i \in S} \lambda_i^S H_i(t, \mathbf{u}^*, \omega) \leqslant \sum_{i \in S} \lambda_i^S H_i(t, u_{\bar{S}}^*, u_S, \omega)$$

then the core is non-empty.

Proof. Again proceeding as in the proof of theorem 3.1, we can prove that the $\mathbf{u}^*$ and the $\boldsymbol{\lambda}^S$ above satisfy for each $(t, \omega, \mathbf{u}) \in [0, T_f] \times \Omega \times \mathcal{U}$

$$\sum_{i \in S} \lambda_i^S J_i(\mathbf{u}^*) \leqslant \lambda_i^S J_i(\mathbf{u})$$

so there is no S and no $\mathbf{u}$ such that $J_i(u_{\bar{S}}^*, y_S) < J_i(\mathbf{u}^*)$ for all $i \in S$. □

5. APPLICATIONS AND EXTENSIONS

We will confine our discussion to social science applications. In this context, a prime example is matching games where players team up for longer or shorter periods and try to control the switching behaviour of each other. The present paper is motivated by a model in which firms use prices to influence the brand-switching behaviour of consumers.[6] Another class of examples, with very significant practical implications, can be found in research and development races between firms in cases where technology follows a jump process. An application on another level is the theory of incentive contracts in cases where a central player (a manager) looks for reward schemes which will induce other players (workers) to maximize the net output of a team production process of the jump category.

As is always the case, these and other examples pose the need for more powerful results. One promising avenue which might allow one to drop the conditions (ii) could be suggested by the technique of Kabanov.[4] Alternatively, the deterministic piecewise Markov processes of Vermes[7] may be sufficiently general to help us in many applications. It should be quite easy to prove existence of equilibrium for games played with such processes. At the other end of the spectrum, the very general processes considered by Jacod and Protter[8,9] seem to have many potential applications (in fact, they were directly motivated by a variation of the brand-switching problem alluded to above). It should finally be pointed out that many applications suggest the desirability of a theory of competitive impulse control. Unfortunately, the author has been completely unsuccessful in his attempts to make progress in this direction. Perhaps the 'Dynkin' games of Reference 10 will be a good starting point.

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