On the Function of Sales Assistance

Birger Wernerfelt


Stable URL:
http://links.jstor.org/sici?sici=0732-2399%28199424%2913%3A1%3C68%3AOTFOSA%3E2.0.CO%3B2-L

*Marketing Science* is currently published by INFORMS.
ON THE FUNCTION OF SALES ASSISTANCE

BIRGER WERNERFELT
Massachusetts Institute of Technology

The paper provides a formal rationale for the common practice of using sales assistance. The argument is made in a model where a consumer's situation determines his needs. The products are differentiated and the map identifying the best match between a consumer's situation and a product is known only to the sales assistant. I look at procedure which involves the consumer revealing his needs and the sales assistant identifying the best matching product. This is compared to several other procedures and found to be best when the number of possible consumer situations is large. It is shown that reputational considerations may keep the sales assistant honest. I look at the impact of guarantees and other modifications to the model and discuss normative implications.

(Retailing; Personal Selling; Comparative Institutions; Game Theory; Retailing and Wholesaling)

1. Introduction

The average American firm spends more on personal selling than advertising or promotion (Pederson et al. 1988). Reflecting this, there is a very rich literature on how sales assistants best can behave. However, I know of no very precise literature on the more fundamental question of when one should use sales assistance at all.

One reason for this is undoubtedly that there is a lot of heterogeneity among jobs usually described as "selling positions." For example, "milkmen" are primarily delivering and taking orders, medical detailers mostly educate MDs about new products, and insurance sales assistants are tailoring products to the buyer's need. In this paper I am focusing on the latter type of selling, often called "creative selling." To motivate this, note that many products are sold without any communication about nonmonetary matters between seller and buyer. Examples include products sold by newsstands and supermarkets such as glue or detergents. For other products (e.g., stereo components, prescription medicine, autoparts, hardware and most industrial products), "sales assistance" is normally available. Since it is expensive to make such sales assistance available, one might ask what function it serves.

I pursue the idea that sales assistance improves the quality of matches in markets where consumers have different ideal points (so-called horizontally differentiated markets). Specifically, suppose that a consumer's situation determines his needs, but that the number of possible situations is very large. For horizontally differentiated products, I will assume that the map identifying the best match between a consumer's situation

1 Chu et al. (1992) is a recent contribution.
ON THE FUNCTION OF SALES ASSISTANCE

and a product is known only to the sales assistant (the seller). So once the buyer reveals his situation, the seller can identify which product is best for him.

This theory is closely related to Philip Kotler's (1991, p. 671) statement that an effective salesperson knows "how to listen and question in order to identify customer needs and come up with good product solutions." It is also consistent with AMA's (1960) definition of personal selling as "oral presentation in a conversation with one or more prospective purchasers for the purpose of making sales." It is finally compatible with Barton Weitz's (1978) findings that salespersons do better the better they understand customer decision making.

However, the story gives rise to at least two questions. First, one may ask why the firm doesn't just print a table of the map between situations and best matches and make it available. There are several scenarios under which this would not happen: I here assume that the number of situations is very large, such that a complete table would be prohibitively expensive. For example, suppose that the buyer wants to purchase a speaker for his stereo system. In some situations the relevant dimensions of situations would be few (the amplifier, the type of music, the size of the room, etc.), but it would be effectively impossible to list all the dimensions which could play a role (there may be a particular kind of curtains in the room, there may be a possibility of later using another type of music, etc.). In a simple model with a two-product seller and a single buyer, I demonstrate how the parties can solve this problem by engaging in dialogue.² The buyer has a good match with one of the products, but she cannot herself identify which (so the product is in some sense an experience good). While the parties can identify the good match in several ways, I characterize circumstances where the most efficient solution is a dialogue in which the buyer reveals her situation, such that the seller can direct her to the appropriate product.

The second question concerns the truthfulness of the sales assistant. Put differently, what gives the sales assistant the incentives to "tell the truth"? There is a lot of anecdotal evidence (Fisher, Business Week 8/3/92, 46–48) that sellers worry about getting reputations for "dishonesty." The bulk of the paper is concerned with modelling this issue in a three-period version of the model. It is assumed that each product may be of either high or low quality and that the buyer is unable to distinguish a low-quality product from a bad match. Suppose that a seller with two low-quality products will sell in the first period but be subject to boycott later. In this case a seller with one product of each quality has a tradeoff if the buyer's situation indicates the low quality product. He can either make the sale and face boycott or he can refuse to sell (be honest) and hope for better luck later. In Proposition 1, I give a set of sufficient conditions for existence of a sequential equilibrium in which the seller will be honest and refuse to sell. Proposition 2 is concerned with the uniqueness of the equilibrium.

I want to emphasize that the second point only has interest because it is made in the context of the first. It is very well known that concerns for reputation may discipline sellers to refrain from short term opportunism. The essential contribution of the paper is to show an example where honest two-way communication is preferable and feasible.

Before presenting the model, I would like to briefly survey the economic literature on communication. The first thing to note is that there is very little of it. Economists often lament the shortage of work on communication costs (Arrow 1979, Tirole 1988, p. 49) and the few models which do exist are often concerned with the complexity (the amount) of communication (Hurwicz 1960, Saijo 1988). There is no economic literature on written versus oral communication and only one paper (Kofman and Ratliff 1991) on unilateral versus bilateral communication. The latter paper compares the quality of coordination a team can achieve with a given amount of communication. For example, if

² To avoid agency problems it is assumed that the seller provides the sales assistance himself.
a buyer is looking for paint, the seller could tell him about all latex paints and all oil paints and their uses. These would presumably be two very long lists. However, if the buyer initially announces that he wants to paint woodwork in a bathroom, (a rather short message) then the seller can skip everything about latex paints. So if it is equally expensive for both parties to communicate, one can describe situations where dialogue is more efficient than monologue. The comparison with one-way written communication which is relevant in this paper, can be seen as a more general case where one side can communicate with a very cheap technology. The less trivial difference between the present paper and that of Kofman and Ratliff, is that I will adopt a game theoretic, as opposed to team theoretic, perspective.

Section 2 of the paper contains the formal model. The rather lengthy proofs have been relegated to the Appendix and the intuition is presented in the body of the paper. Modifications to the model are discussed in §3 and some normative implications are offered in §4.

2. Model

I consider a 2 person-3 period matching game. Since the argument about the superiority of honest dialogue can be made in the (1-period) component game, I will do so in the interest of simplicity. (A list of symbols is supplied at the end of the paper.)

In the one-period model, I compare several procedures for solving a matching problem and identify cases where the “sales assistant” solution is superior. The essential intuition is that if the number of possible buyer situations is very large, it is expensive to list them all. Using a sales assistant avoids this.

In the three-period model I focus on an equilibrium which is driven by the incentives of sellers who do not have the best match in high quality. Such sellers can refuse to sell in the first period, hoping to get a (for them) more favorable match later. Since the buyer is unable to penalize the seller beyond the last period, we need three periods such that refusal to sell in period one can be rewarded in period 2. Let me now proceed with the one-period model.

The seller has two products (a and b) in his assortment and may sell one of either or nothing. Throughout the game, each of these products is of high (H) or low (L) quality and the seller knows his “type” \( T \in \{(L, L), (L, H), (H, L)\} \), while the buyers prior over the type-space is \( \{1 - 2 \eta, \eta, \eta\} \). (H, H) types are omitted from the model because they add nothing to it. One could imagine that the existence of an (H, H) type would make separation harder since it would act as a “smoke screen” for the (L, L) type. However, if (H, H) types are added to the model, they separate by low first period prices. Since the core results concern mixed types who initially have bad match, the (H, H) types clutter up the model. A small aesthetic problem is that the nonexistence of (H, H) types makes the game more sensitive to extensive form. For example, in the current formulation, if the buyer could force the seller to make a claim ex ante about which of his products is of high quality, then the equilibrium would be quite different.

Any product costs \( c > 0 \) to produce. The buyer’s situation \( s \in S \) is such that he has a good match with one product and a bad match with the other. Further, the good match is a with probability \( \theta > \frac{1}{2} \). The buyer knows that there are two products (a and b) and that one (a) is the good match with probability \( \theta \), while the other is the good match with complementary probability. However, she cannot identify the good match herself. We also make the technical assumption that she cannot tell whether an individual product is an a or a b, or if it is the same type as any used earlier.\(^3\) (So if one insists on taking

\(^3\) This assumption could be relaxed at the expense of making the model richer. In particular, if the assumption is false, one could investigate the possibility of the seller making claims about the type of the product when selling it. Under the current assumption such claims would be cheap talk.
this literally, medical products may be an example.) In contrast, the seller is very well-informed about the products. By listening to the buyer's description of her situation, he can identify the good match \( m \in \{ a, b \} \). We assume that the seller makes a take-it-or-leave-it offer \( p \) for the recommended product.\(^4\) (Because the seller has the informational advantage, it is natural to give him most of the bargaining power.) After receiving an offer, the buyer decides whether to buy or not and payoffs are realized. Payoffs for the seller are 0 if no trade occurs and else \( p - c \). The buyer gets \( \tilde{v} - p \) for a product which is simultaneously a good match and a high quality. Other combinations are indistinguishable for him and nets him utility \( v - p \). With a slight abuse of notation the buyer's experience is indicated by \( v \in \{ \tilde{v}, v, 0 \} \) where 0 refers to the case in which no purchase is made. It is assumed that the structure of the game is common knowledge.

Let us now compare different communication arrangements in terms of their costs and ability to solve this information problem. Concerning communication costs, we assume that it costs \( k_p \) to identify an element of \( S \) in print, and \( k_o \) to do so orally. Further, it costs \( l_p \) to identify and element of \( \{ a, b \} \) in print and \( l_o \) to do so orally. We assume that each of the \( |S| \) elements of \( S \) occurs with equal probability.

As benchmarks, note that when there is no communication, the buyer can make a random choice giving her \( v \) with probability \( 1 \times (1 - \theta) + 1 \times \theta \) and else \( v \). This compares with the full information outcome giving her \( \tilde{v} \) with probability \( \eta \) and else 0. So the value of information is \( 1/2 \eta(\tilde{v} + v - 2c) - (v - c) \) if \( 1/2 \eta(\tilde{v} - c) + (1 - 1/2 \eta)(v - c) < 0 \) or else \( \eta(\tilde{v} - c) \). One possibility is to print a brochure listing all elements of \( S \) and the associated match. In the above notation, this will achieve the full information outcome at cost \( (1 - \theta) |S| k_p + 2l_p \). Since all elements of \( S \) are equally likely, you will want to list all \( s \) for which \( b \) is the best match while referring others to \( a \). A number of products for which \( |S| \) is small are sold this way, e.g., glue, paints, or detergents.

Another possibility is sales monologue, in which the seller transmits the same information orally. Once again it is optimal to list all \( s \) for which \( b \) is the best match, and the costs are \( (1 - \theta) |S| k_o + 2l_o \). Given economies of scale in printing, one would suspect that the oral solution is better for small markets only.

A third possibility is for the seller to identify his high-quality product, in print at cost \( l_p \), or orally at cost \( l_o \). This would increase the probability of getting \( \tilde{v} \) to \( \eta(= \theta + \eta(1 - \theta)) \), while the probability of getting \( v \) would be the same and the probability of getting 0 would be \( (1 - 2\eta) \). The full information outcome is better than this because it allows the buyer to avoid \( v \) in all cases.

The final option is a dialogue, in which the buyer describes his situation at cost \( k_o \), after which the seller recommends a product or remains quiet, at expected cost \( 2\eta l_o \).\(^5\) If the seller is honest, this option will yield the buyer \( \tilde{v} \) with probability \( \eta \) and also allows him to avoid \( v \) in favor of no purchase (which is efficient if \( y < c \)). Comparing these, it is clear that dialogue dominates exhaustive listings when \( |S| \) is large. Further, if \( y < c \), the expected gains from dialogue are \( \eta(\tilde{v} - c) + 2\eta l_o + k_o \), while the expected gains from the seller identifying his high-quality product are \( \eta(\tilde{v} - c) + \eta(y - c) + l_o \). So dialogue, a. k. a. sales assistance, is the dominant alternative when the number of buyer situations is large, a bad match is costly, and description of one \( s \) is cheap (high values of \( |S| \) and low values of \( y \) and \( k_o \)).\(^6\) So under the stated assumptions

\(^4\) It is natural to think of the process as consisting of the physical showing of a product accompanied by a single price quote.

\(^5\) We focus on oral dialogue on the presumption that printed dialogue could take too long a time.

\(^6\) There are clearly other alternatives which we have not considered. One class of alternatives consists of expert systems, such as "Design Center" described in Little (1990), which help the buyer identify the good match.
dialogue (sales assistance) is more efficient than alternative ways of solving the information problem, as long as the dialogue is honest. We will now demonstrate how reputational concerns may discipline the seller to communicate honestly.

To do so, consider a 3-period version of the above component game with sales assistance, recalling that we abstract from agency problems by assuming that the seller provides the sales assistance. The seller’s type is fixed throughout the game, but the buyer’s situation \( s_t (t = 1, 2, 3) \) is determined as a random independent draw at the beginning of each period. (Instead of thinking of this as one buyer coming three times, one could think of a sequence of three buyers who communicate.) The good match \( m_t \in \{ a, b \} \) may therefore also change form period to period. After identifying the good match, the seller recommends that the buyer purchase nothing \((r_t = 0)\) or a specific product \((r_t = 1)\). We use \( q_t \in \{ 0, a, b \} \) to indicate the seller’s information about the recommendation, while the buyer’s information is \( r_t \) (recall the assumption that he cannot tell whether the product is \( a \) or \( b \)). In case purchase is recommended, the seller makes a take-it-or-leave-it price offer \( p_t \), which we normalize to zero if \( r_t = 0 \).

Before defining the strategy spaces, it is useful to summarize the players’ information sets. After period \( t \), the seller’s information about play in that period consists of \((m_t, q_t, p_t, f_t)\), while the buyer knows \((r_t, p_t, f_t, v_t)\). So the seller has perfect information, while the buyer does not know which product was recommended to her (if any was), nor whether bad quality or poor match was responsible for low utility (if \( v_t = y \)).

The sequence of events is

1. **(0)** Seller learns his type \((T)\), for \( t = 1, 2, 3 \).
2. **(t.i)** Buyer learns and communicates his situation \((s_t)\),
   - Seller learns best match \((m_t)\),
   - Seller makes recommendation \((q_t)\),
   - Seller makes price offer \((p_t)\).
3. **(t.ii)** Buyer follows or rejects \((f_t)\),
4. **(t.iii)** Payoffs.

This is illustrated in Figure 1.

Total payoffs are \( \sum_{t=1}^{3} f_t (v_t - p_t) \) for the buyer, and \( \sum_{t=1}^{3} f_t (p_t - c) \) for the seller. A strategy for the buyer is three functions

\[
\{ f_t(r_t, p_t); f_2(r_t, p_t, f_t, v_t, r_t, p_t); f_3(r_t, p_t, f_t, v_t, r_t, p_t, f_t, v_t, r_t, p_t) \}
\]

and a strategy for the seller is three function pairs

\[
\{ q_t(T, m_t); p_1(T, m_t); q_2(T, m_t, q_t, p_t, f_t, m_t), p_2(T, m_t, q_t, m_t); q_3(T, m_t, q_t, p_t, f_t, m_t, q_3, p_3(T, m_t, q_t, p_3, m_3) \}
\]

We will look for sequential equilibria defined in the usual way.

The central question concerns the actions of a seller who in period 1 finds that the buyer’s “best” match is with a product he supplies in low quality. I will establish the existence of a sequential equilibrium, called the “honest equilibrium,” in which (1) sellers with two low-quality products recommend purchase and are boycotted thereafter, (2) sellers with a high quality product, but a poor match, recommend no purchase in period 1 but may sell later, and (3) buyers follow recommendations until they experience

Another example, due to an anonymous referee, is the “Cosmetic Computer” introduced by Clarion Cosmetics. A different possibility is to reduce \(| S | \) by developing a (technical) language capable of capturing almost all relationships between \( S \) and \( \{ a, b \} \) in lower dimension. This is what “technically skilled” buyers can use.
ON THE FUNCTION OF SALES ASSISTANCE

<table>
<thead>
<tr>
<th>Event</th>
<th>Seller learns his type</th>
<th>Buyer learns and communicates his situation</th>
<th>Seller infers best match</th>
<th>Seller makes recommendation and price offer</th>
<th>Buyer follows or rejects recommendation</th>
<th>Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Symbol $T$, time $t = 0, 1, 2, 3$

$T \rightarrow (t, i)$

$T \rightarrow (t, ii)$

$T \rightarrow (t, iii)$

FIGURE 1. Sequence of Events.

\[ p_1^*(T, m_1), \quad p_2^*(T, m_1, m_2) \quad \text{and} \quad p_3^*(T, m_1, m_2, m_3), \]

which drives the buyer’s expected period surplus to zero.

Given this, the honest equilibrium can be described as follows, omitting irrelevant arguments from the functions. In period 1

1.1 $q_1^*(L, L, m_1) = a$ (sellers with two low-quality products recommend a),

1.2 $q_1^*(H, L, a) = a$,

1.3 $q_1^*(L, H, b) = b$ (sellers with a high-quality product recommend purchase of that product),

1.4 $q_1^*(H, L, b) = 0$,

1.5 $q_1^*(L, H, a) = 0$ (sellers with a bad match and a high-quality product recommend against purchase)—this is the behavior which motivates the label “honest,”

1.6 $f_1^*(r_1^* \neq 0) = 1$,

1.7 $f_1^*(r_1^* = 0) = 0$ (buyers follow recommendations),

1.8 $p_1^* = [\eta \bar{u} + (1 - 2\eta)\bar{v}](1 - \eta)^{-1}$ (price is set as high as possible given equilibrium expectations).

So $L, L$ types are “dishonest,” while all other types make “honest” recommendations which are followed by the buyer.

In period 2:

2.1 $q_2^*(L, L, m_1, m_2) = a$ (sellers with two low-quality products recommend a),

2.2 $q_2^*(H, L, m_1, m_2) = a$,

2.3 $q_2^*(H, H, m_1, m_2) = b$ (sellers with a high-quality product recommend it),

2.4 $f_2^*(v_1 = \bar{v}) = 0$,

Specifically, mixed types hope to get good matches in period 2 such that the buyer will assign a higher probability to their being the $(H, L)$ type. In the honest equilibrium, the seller will be boycotted in period 3 if he follows $b_1 = 0$ with $v_2 = \bar{v}$, because he then is more likely to be an $(L, H)$ type. This also explains why we have a two-product model. If there is only one product, it cannot be rational to punish $(0, \bar{v})$ in our separating equilibrium. On the other hand one could clearly develop efficient equilibria in repeated versions of one-product games as well. So the existence of an “assortment” is not critical to the argument.

This is the reason we need three periods.
(2.5) \( f^*_2(v_i \neq v) = 1 \) (buyers boycott iff they received low utility in period 1),

(2.6) \( p^*_2(v) = [1 - 2\theta(1 - \theta)]\bar{v} + 2\theta(1 - \theta)v \),

(2.7) \( p^*_2(0) = 2\theta(1 - \theta)\bar{v} + [1 - 2\theta(1 - \theta)]v \) (prices are set as high as possible given expectations based on first-period outcomes),

(2.8) \( p^*_2(v) = \bar{v} \) (price of product which will not be purchased).

So consumers boycott sellers who were "dishonest" in period 1.

Finally, in period 3

(3.1) \( q^*_3(L, L, m_1, m_2, m_3) = a \) (sellers with two low-quality products recommend a),

(3.2) \( q^*_3(H, L, m_1, m_2, m_3) = a \),

(3.3) \( q^*_3(L, H, m_1, m_2, m_3) = b \) (sellers with a high-quality product recommend it),

(3.4) \( f^*_3(v, \cdot) = 0 \) (buyers boycott if they received low utility in period one),

(3.5) \( f^*_3(0, v_2 \neq \bar{v}) = 0 \),

(3.6) \( f^*_3(0, v_2 = \bar{v}) = 1 \) (buyers boycott unless a recommendation not to purchase is followed by high utility),

(3.7) \( f^*_3(\bar{v}, \cdot) = 1 \) (buyers follow recommendations from sellers who delivered high utility in period one),

(3.8) \( p^*_3(\bar{v}, \bar{v}) = [(1 - 3\theta(1 - \theta))\bar{v} + \theta(1 - \theta)v][1 - 2\theta(1 - \theta)]^{-1} \),

(3.9) \( p^*_3(0, \bar{v}) = \frac{1}{2} \bar{v} + \frac{1}{2} v \),

(3.10) \( p^*_3(\bar{v}, v) = \frac{1}{2} \bar{v} + \frac{1}{2} v \) (prices are set as high as possible given expectations based on prior outcomes),

(3.11) \( f^*_3(v, \cdot) = \bar{v} \) (price of product which will not be purchased).

So buyers boycott sellers who delivered \( v \) in the first or the second period, but reward sellers who followed \( v_1 = 0 \) with \( v_2 = \bar{v} \) (because they believe that these sellers are more likely to be of the \( H, L \) than of the \( L, H \) type). It is the possibility of this last event which induces honesty in period 1.

**Proposition 1.** Given the assumptions

\[
(1 - \theta)[(2\theta + \frac{1}{2}(1 - \theta) - \eta(1 - \eta)^{-1}]^{-1} \leq (\bar{v} - v)(c - \bar{v})^{-1} \\
\leq [1 - 2\theta(1 - \theta)][\theta(1 - \theta)]^{-1}, \quad (A1)
\]

\[
(2\theta^2 + \frac{1}{2})(1 - \theta) \leq \eta(1 - \eta)^{-1} \leq (2\theta + \frac{1}{2})(1 - \theta), \quad (A2)
\]

the honest equilibrium is a sequential equilibrium.

**Proof.** See Appendix.

While (A1) and (A2) look formidable, they can be weakened considerably by adding more periods to the model. Essentially, the first part of (A1) and the second part of (A2) make it pay off for an unlucky mixed type to be honest and recommend no trade in period one. The second part of (A1) makes it rational for a buyer to boycott after \( (0, v) \). The first part of (A2) prevents a \( (L, L) \) type from mimicking an unlucky mixed type. Intuitively, the mixed type forgoes sales in period 1 because it can hope to be lucky in period 2, such that it will sell in period 3 as well. The \( (L, L) \) type can harbor no such hopes. As a check on consistency, note that these assumptions and (A3) below are satisfied for \( c = 0, \theta = \frac{1}{2}, \bar{v} = \frac{2}{3}, \eta = \frac{1}{4} \).

We have now shown that honest sales assistance may be useful and that partial honesty may be supported in equilibrium. While it is hard to assess the "reasonableness" of the assumptions (A1) and (A2), they can clearly be weakened if one adds more periods to the model. However, it remains to assess the reasonableness of the equilibrium given

---

\(^9\) Since the point is to show that these behaviors can occur, it is not interesting to examine cases where (A1) and (A2) do not hold.
(A1) and (A2). For example, if there are infinitely many equilibria or other more reasonable equilibria, our story has less merit.

As is common in signalling games, we are faced with a nontrivial issue of uniqueness. Note first that one can construct a few equilibria which are payoff-equivalent to the honest equilibrium, by specifying (3.1), (3.2), and (3.3) differently. That is, if the buyer cannot get \( \tilde{v} \) in period 3, the seller can give him \( \tilde{v} \) in different ways. Further, there are a few equilibria in which the seller is less benevolent in period 3 than in the honest equilibrium. For example, one can specify (3.2)–(3.3) such that the buyer only gets \( \tilde{v} \) in some of the cases where it is possible. Existence of such equilibria will require stricter assumptions than (A1)–(A2), but they are qualitatively similar. So the most we can hope for is uniqueness in the sense that no equilibria exist outside the class consisting of the honest equilibrium and those described above. For a suitably strong equilibrium concept one can in fact get such a result. Economists have defined divine sequential equilibria as those where out-of-equilibrium signals are not interpreted as coming from a type \( T \), if for any interpretation of the signal for which \( T \) wishes to defect or is indifferent, another type \( T' \) strictly wishes to defect (Banks and Sobel 1987). We can show that the honest equilibrium is divine, and that there are no divine pure strategy equilibria with separation on prices or pooling on recommendations for purchase.\(^{10}\)

**Proposition 2.** Given (A1), (A2) and

\[
[2\theta(1 - \theta)]^{-1}(1 - \eta)^{-1} > (\tilde{v} - \tilde{v})(c - \tilde{v})^{-1},
\]  

(A3)

the honest equilibrium is a divine pure strategy equilibrium and there are no such equilibria with separation on prices or pooling on recommendations for purchase.

**Proof.** See Appendix.

So if reasonable equilibria are conceived of as pure strategy divine, then all reasonable equilibria have the same qualitative structure as the honest equilibrium.\(^{11}\) The interpretation is that “honest” recommendations from the sales assistant are “reasonable,” that all “reasonable” behaviors feature honest recommendations, and that no other means of revealing the information are reasonable.

One might object that our model is “rigged” by the assumption that the sales assistant has free, yet valuable, information. However, we may get identical results from the assumption that it is cheaper to endow a few sales assistants with matching skills than it is to educate all buyers. The central points in the paper are slightly different, though. We established (1) that honest two-way communication is better than any other mechanism (one-way written communication, price signalling, etc.) for utilizing this information and (2) that honesty may be sustained in equilibrium. It is these two points which give us a formal, comparative institutional rationale for the commonly observed use of sales assistance.

To conclude this section, I would like to make two comments on the model. First, as noted above, it is obvious that the assumptions can be weakened if more periods are added. A more interesting way to accomplish the same is to allow for several buyers who can communicate about their experiences with the seller. Secondly, one could contrive a price signalling mechanism in which the seller announces two prices: \( p^* \) for the recommended product and some prohibitively higher price for the other product. Such a game would obviously be subject to the same analysis as that above.

\(^{10}\) Among the known equilibrium refinements, divinity is the weakest to give uniqueness in this sense.

\(^{11}\) It should be noted that these claims are dependent on the restriction to pure strategy divine equilibria. One must expect that other equilibrium concepts will give different results. In particular, mixed strategy equilibria (i.e., partial pooling) normally exist in signalling games.
3. Modifications

I here consider several less trivial modifications to the model. The effects of these changes throw interesting light on the drivers of the results.

1) Suppose that we look at a two-period version of the model. In this case, honesty cannot be supported in equilibrium. To see this, note that all types will be opportunistic in the last period. So purchase depends on the buyer's probability distribution over types. Honesty consists in refusing to sell in the presence of a bad match. If honesty in the first period is interpreted as indicating the best type \((H, L)\), it will lead to sales in period 2. However, the payoffs from this (no sale, sale) sequence are the same for all types. If \((H, L)\) types will do it, so will \((L, L)\) types. So the above “optimistic” beliefs are not supported in equilibrium. By going to a three-period model the buyer gains an intermediate observation, in which “better” types are more likely to give \(\bar{v}\). It is this difference which allows credible separation. As more periods are added, this effect grows.

2) Consider next a situation where the buyer is better informed in the sense that he can tell whether \(v\) is due to a poor match or low quality. In this case, it is less damaging to sell a poor match than low quality. The match may change, while quality is constant. For example, if an \((H, L)\) type finds that \(b\) is the best match, he may be tempted to recommend \(a\), such that the buyer can learn its high quality. This obviously makes honesty less attractive, because the incentives to refuse a sale are stronger for the low quality types.

If the product line is very wide and there is imperfect correlation between the qualities of individual products, this effect weakens. With only two products, the effect is strong.

3) A very interesting modification involves assuming that no products are of low quality but that product lines are incomplete. (This corresponds roughly to the exclusive retailing format used for example in the automobile industry.) Suppose that product lines are either just \(a\) or just \(b\). In this case the two types correspond to \((H, L)\) and \((L, H)\) when \(\eta\), the probability of \((L, L)\), is zero. As can be seen from (A2), honesty cannot be supported in this case. Intuitively, this happens because \(g\) is less damaging without the possibility of \((L, L)\). With more periods, the \((H, L)\) type would, however, be able to separate. So in choosing between nonexclusive and exclusive retailing, other pros and cons have to be traded off against this incentive problem.

4) Another class of modifications involves guarantees. To distinguish guarantees from the reputational enforcement mechanism used in §2, we look only at explicit contracts. Since such contracts can take many different forms, we will look at several examples.

4a) Suppose first that the seller can offer a perfect unconditional “money-back” guarantee. So the buyer is sure to get \(\bar{v}\) or 0, since if he gets \(v\), the trade is costlessly voided. In this case honesty is not a problem, and even the buyer is indifferent to it.

4b) For many products it is difficult to write “money-back” guarantees, because part of the consumption utility is consumed in the evaluation process. Food products are a good example. If the buyer can get the entire price refunded after tasting/eating half the product, he will clearly do so (ad infinitum); as long as it cannot be legally verified whether the match in fact is bad.\(^{12}\) For other products, the main difficulty lies in verifying the goodness of the match. However, even with such contractibility problems, reputational concerns may well force the seller to give unconditional guarantees as analyzed in (4a). In many cases, however, this does not happen.

We therefore also look at guarantees which are based on the product quality only. Suppose again that the guarantee is complete in the sense that the trade is voided (costlessly) if the product is of low quality. This implies that the \((L, L)\) type never makes any sale, while the two other types may. So we have come back to the case discussed under

\(^{12}\) The fact that many candy bars offer such guarantees does not contradict this assertion. The cost of returning such items is quite high and the consumer does not end up being fully compensated. (See further (4c) below.)
(3) above, where only \((H, L)\) and \((L, H)\) types "exist." As mentioned there, this actually makes it harder to get honesty in the sense that more than three periods are needed.

(4c) For a variety of reasons, including legal costs, time costs, mailing costs, etc., guarantees rarely leave the buyer as well off as if he had never bought. So it is useful to think about guarantees which get partial reimbursement for the buyer, but leave the seller as if no trade had occurred (or worse). Assuming that reimbursement is sufficient to always get claims, these guarantees have the same action implications as "more perfect" guarantees and the analyses in (4a) and (4b) apply.

(4d) Suppose finally that the seller can commit himself to incurring a very large penalty, e.g., by offering a $1000 "reward" for each "poorly tasting" product, or for each product of verifiably low quality. If steep enough, these incentives will be forcing, but not beyond the level discussed in (4b). That is, if only quality is contractible, sellers may still be dishonest about match.

In conclusion, guarantees of both match and quality will solve the incentive problem and eliminate the need for the reputational mechanism discussed in §3. However, guarantees on quality only will not solve the problem but make it harder for reputation to work (because they eliminate the "bad" \((L, L)\) types and therefore make \(p\) less damaging).

(5) An empirically important extension is to the case where the salesman and the seller are two different players. The problem is then that the seller has the right to profits from later sales, while the salesman has to forego current sales to build sufficient reputation to generate those later sales. For a variety of reasons it is often hard for the seller to commit to an incentive contract which guarantees that the salesman will get appropriate credit for those later sales. This makes honesty more difficult and may prevent it.

A very good example (due to a referee) is Sears's recent decision to abort incentive compensation for service advisors at Sears Auto Centers (Fisher 1992). In light of some highly publicized examples where customers were sold unneeded parts, Sears decided to discontinue all incentives and goal-setting for these employees. A less dramatic alternative would be to obtain some sort of leading indicator of future sales, e.g., by measuring customer satisfaction. This problem—how you can use measures of customer satisfaction in incentive systems to combat employee short-termism—is the focus of another paper (Hauser et al. 1992) and beyond the scope of the present discussion.

4. Conclusion

The purpose of this paper was to show that one can formulate a simple model in which sales assistance, conceived of as two-way nonprice communication between buyer and seller, has relatively attractive efficiency properties.

The discussion of model modifications, in §3 and elsewhere, allows us to develop several normative implications.

(1) Sales assistance is more likely to prevail when products are complicated and many buyers are unsophisticated. This follows directly from the sales assistants' matching function. In terms of evidence, Kim (1992) found that first-time computer buyers were less likely to use mail-order (do without sales assistance) than second time buyers (who presumably are more knowledgeable). Another example is tropical fish, where the buyer's situation consists of the fish she already has, as well as the size of the tank. Since different fish match up differently, a novice buyer needs help. Other examples are building contractors, repair services, medicine, cameras, audio equipment and wine.

(2) Sales assistance is more likely to prevail when the cost of the sales assistant's time is low relative to that of the buyer. This is based on the argument that either the sales assistant or the buyer has to take time to learn the matching function. As positive evidence, I suggest that sales assistance is more prevalent for "high-end" products. The contrast between department stores and discounters may be a good example.
78

(3) **Sales assistance is more likely to prevail when product lines are wider.** A wider product line makes matching more difficult and allows for a wider spectrum of types (such that separation is “easier”). Once again using positive evidence, one normally sees wider product lines in department stores than in discounters.

(4) **Sales assistance is more likely to prevail when bad buyer experiences can affect later sales more.** As discussed above, the reputational mechanism works better when the number of periods (the purchase frequency) is higher or when communication between buyers allows for transmission of past experiences. In support of this, I suggest that most people will rely on trust in frequently used sellers, but will seek more information (word-of-mouth, the Better Business Bureau, general product education) about infrequently used sellers. Note that there is an interesting tension between this effect and the “novice buyer” effect discussed under point 1 above.

(5) **Sales assistance is more likely to prevail when the sales assistant can be given stronger long term incentives.** As brought out by the Sears example in §3, it is important that the relative weight given to current sales not be too big relative to the weight given to later sales. Although the model contains no discounting, this point could easily be made by capturing relative weights in a discount factor. The severity of these agency problems depends on the ease with which efforts to increase future sales can be monitored. One would expect some cross-sectional variation in this. For example, multoutlet retailers have a harder time assigning specific credit for goodwill than single outlet stores. Another example is products, such as repair services, where it takes the customer longer to evaluate the quality and the match. (Indeed, the Sears example illustrates this point.)

If one is willing to look at these implications as positive, that is if one assumes that most surviving sellers use the best selling format, empirical testing should be possible. This is a difficult, but important area for future research.

In conclusion, I would like to make the methodological point that the paper offers an explanation for a common marketing practice in a comparative institutional framework. In a companion paper (Wernerfelt 1992), I have tried to do this for other practices as well.\(^\textsuperscript{13}\)

Acknowledgements. Comments from two anonymous referees, seminar participants at MIT, Wujin Chu, and Jim Hess are gratefully acknowledged. The usual disclaimer applies.

\(^{13}\) This paper was received May 8, 1992 and has been with the author 2 weeks for 2 revisions. This paper was processed by Brian Ratchford.

List of Symbols

- \(a\) product,
- \(b\) product,
- \(c\) costs,
- \(f \in \{0, 1\}\) indicates whether buyer follows recommendation,
- \(H\) high-quality product,
- \(k_p\) cost of identifying an element of \(S\) in print,
- \(k_o\) cost of identifying an element of \(S\) orally,
- \(L\) low-quality product,
- \(l_p\) cost of identifying \(a\) or \(b\) in print,
- \(l_o\) cost of identifying \(a\) or \(b\) orally,
- \(m \in \{a, b\}\) best match,
- \(p\) price,
- \(q \in \{0, a, b\}\) seller’s information about his recommendation,
- \(r \in \{0, 1\}\) buyer’s information about seller’s recommendation,
- \(s \in S\) buyer’s situation,
- \(S\) set of possible situations,
- \(t \in \{0, 1, 2, 3\}\) period,
- \(T \in \{(L, L)(H, L)(L, H)\}\) seller’s type,
ON THE FUNCTION OF SALES ASSISTANCE

\( \forall \in \{ \varepsilon, \bar{\varepsilon}, 0 \} \) buyer's experience,
\( \eta \) probability of seller being \((H, L)\) (same as probability of \((L, H)\)),
\( \theta \) probability of match being \(a\).

\( \phi \) buyers' beliefs about seller's type on equilibrium path (proof of Proposition 1)
\( \tau, \mu, \nu, \gamma \) buyers' beliefs about seller's type off equilibrium path (proof of Proposition 2),
\( \lambda, \omega \) functions (proof of Proposition 2),
\( \Omega \) set of beliefs (proof of Proposition 2).

Appendix

PROOF OF PROPOSITION 1. As usual, we will start with the last period, assume that the honest equilibrium is played, and check incentive compatibility and individual rationality. To set us up for this, it is convenient to write out the buyer's beliefs \( \phi(T) \) about the seller's type \((T)\) for different histories. Starting with on-the-equilibrium-path beliefs in period 3,

\[ \phi_{LL}(\varepsilon, \bar{\varepsilon}) = 1, \]
\[ \phi_{HL}(0, \bar{y}) = (1 - \theta)^2(1 - 2\theta)(1 - \theta)^{-1} = 1 - \phi_{LM}(0, \bar{y}), \]
\[ \phi_{HL}(0, \bar{\varepsilon}) = \frac{1}{2} = \phi_{LM}(0, \bar{\varepsilon}), \]
\[ \phi_{HL}(\bar{\varepsilon}, \bar{y}) = \frac{1}{2} = \phi_{LM}(\bar{\varepsilon}, \bar{y}), \]
\[ \phi_{HL}(\bar{\varepsilon}, \bar{\varepsilon}) = \theta^2(1 - 2\theta)(1 - \theta)^{-1} = 1 - \phi_{LM}(\bar{\varepsilon}, \bar{\varepsilon}). \]

In period 2,

\[ \phi_{LL}(\varepsilon) = 1, \]
\[ \phi_{HL}(0) = 1 - \theta = 1 - \phi_{LM}(0), \]
\[ \phi_{HL}(\bar{\varepsilon}) = \theta = 1 - \phi_{LM}(\bar{\varepsilon}). \]

Finally, in period 1, if trade is recommended, \((\tau, 1)\),

\[ \phi_{LL} = (1 - 2\eta)(1 - \eta)^{-1}, \]
\[ \phi_{HL} = \eta(1 - \eta)^{-1}, \]
\[ \phi_{LM} = \eta(1 - \theta)(1 - \eta)^{-1}. \]

As usual, off-the-equilibrium-path beliefs can be specified arbitrarily. For now, use the principle that deviations are considered proof that the seller is of the least favorable type possible. So any deviation in price indicates \((L, L)\) or, if \(0\) has been observed, \((L, H)\). Further, \(\phi_{LL}(\varepsilon, 0) = \phi_{LM}(\bar{\varepsilon}, 0) = 1.\)

Given these beliefs, let us first check the buyer's third period strategy given by Equations (3.4)–(3.7). Equation (3.4) stipulates no trade when it is known that \(T = (L, L)\). The expected utility of trading at some price \(p_3\) is \(\varepsilon - p_3\) and the seller cannot make this positive and still make a profit if

\( c > \varepsilon. \) (I)

Consider next Equation (3.5): \( f_3(T, 0, \bar{\varepsilon}) = 0. \) The history suggests beliefs such that the expected utility of purchase at the lowest possible price \(c\), is

\[ (\bar{\varepsilon} - c)(1 - \theta) + (\bar{\varepsilon} - c)((1 - \theta)^3 + \theta^3)[1 - 2\theta(1 - \theta)^{-1}]. \]

This is negative if

\[ (\bar{\varepsilon} - y)(c - y)^{-1} < [1 - 2\theta(1 - \theta)](1 - \theta)^{-1}. \) (II)

If the history is \((0, \bar{\varepsilon})\) Equation (3.6) suggests trade. To check this out, the expected utility of purchase is

\[ \left[ \frac{\bar{\varepsilon} - p_3(0, \bar{\varepsilon})}{2} + \frac{\bar{\varepsilon} - p_3(0, \bar{\varepsilon})}{2} \right] , \]

which is zero for \( p_3(0, \bar{\varepsilon}) = (\bar{\varepsilon} + y)/2 \) as given in (3.9). So this is feasible if

\[ (\bar{\varepsilon} - y)(c - y)^{-1} > 1. \) (III)

Finally, Equation (3.7) refers to the histories \((\bar{\varepsilon}, \bar{\varepsilon})\) and \((\bar{\varepsilon}, \bar{\varepsilon})\). The former case gives the same expected utility of purchase as \((0, \bar{\varepsilon})\) and this leads to (III). The latter case gives the expected utility of purchase
BIRGER WERNERFELT

which is zero by Equation (3.8).

Because the third period is the last one, the seller is indifferent between recommending \( a, b, \) or \( m_3 \). So Equations (3.1)–(3.4) are weakly incentive compatible. This concludes the examination of period 3.

In period 2, it is trivial to see that the buyer will refuse to buy after \( v_1 = \bar{v} \). In case \( v_1 = 0 \), the expected utility of purchase is

\[
\bar{v}2\theta(1 - \theta) + (\bar{v} - 2\theta(1 - \theta)) - p_3(0).
\]

This is zero when \( p_2(0) = p_2^*(0) \) as given in Equation (2.7) and the seller may make a profit at that price if

\[
(\bar{v} - \bar{v})(c - \bar{v})^{-1} > [2\theta(1 - \theta)]^{-1}.
\]

Equation (2.6) for \( v_1 = \bar{v} \) checks out similarly and leads to a weaker condition on payoffs.

Concerning the seller, there is no problem showing that \((L, L)\) types weakly prefer to behave as depicted in Equations (2.1) and (2.8). Also mixed types with “unlucky” matches, \((H, L, b)\) and \((L, H, a)\) are indifferent. Finally, “lucky” mixed types clearly behave as required by Equations (2.2)–(2.3).

In period 1, if

\[
p_1^* = \left[\bar{v} + (1 - 2\eta)\bar{v}\right](1 - \eta)^{-1}
\]

as suggested by Equation (1.8), the expected utility of purchase is zero and the price is feasible for

\[
(\bar{v} - \bar{v})(c - \bar{v})^{-1} > (1 - \eta)(\eta)^{-1}.
\]

On the seller side, there is no problem checking the recommendations of “lucky” mixed types. To look at the incentives for unlucky mixed types to recommend against trade, consider the least attractive such case \((T, m_i) = (L, H, a)\). The expected profits from selling now and being boycotted later are \( p_1^* - c \). The expected profits from being “honest,” selling in period 2, and hoping to sell in period 3 are

\[
p_2^*(0) - c + (1 - \theta)[p_2^*(0, \bar{v}) - c].
\]

The latter dominates the former if

\[
(\bar{v} - \bar{v})(c - \bar{v})^{-1} > (1 - \theta)[(2\theta + \frac{1}{2})(1 - \theta) - \eta(1 - \eta)]^{-1}, \quad \text{and}
\]

\[
(2\theta + \frac{1}{2})(1 - \theta) > \eta(1 - \eta).\]

An \((L, L)\) type will not mimic this if \( p_1^* > p_2^*(0) \) or

\[
2\theta(1 - \theta) < \eta(1 - \eta).\]

It remains to be shown that (I)–(VIII) are implied by (A1)–(A2). Note first that (IV) is stronger than (I), and (III). Further, given (VIII), (IV) is stronger than (V). If we replace (VIII) with the stronger condition

\[
(2\theta^2 + \frac{1}{2})(1 - \theta) < \eta(1 - \eta)^{-1}
\]

then (IX) implies that (VI) is stronger than (IV). (A1) consists of (II) and (VI), while (A2) is (IX) and (VII).

Q.E.D.

PROOF OF PROPOSITION 2. We show that (1) the honest equilibrium is divine, (2) there are no divine pure strategy equilibria with price separation, and (3) there are no divine equilibria with pooling on recommendations to purchase.

(1) We need to check the out-of-equilibrium beliefs that (i) any deviation in price signals \((L, L)\) or \((L, H)\), and (ii) \(\phi_{LH}(0, 0) = \phi_{HH}(\bar{v}, 0) = 1\). Since the suggested actions in (ii) are dominated for all types, that part is easy. We can therefore concentrate on (i).

Suppose that \( \hat{p}_i < p_1^* \) is observed and that this is taken to imply that the deviator is of the \((H, L, a)\) type with probability \( \tau(\hat{p}_i) \) and else \((L, H, b)\).\(^{13}\) If so, deviators can charge prices:

\[
\hat{p}_2(\bar{v}) = \left[\tau\theta + (1 - \tau)(1 - \theta)\right]\bar{v} + \left[\tau(1 - \theta) + (1 - \tau)\theta\right]v,
\]

\[
\hat{p}_3(\bar{v}, \bar{v}) = \left[(\tau\theta^2 + (1 - \tau)(1 - \theta)^2)\bar{v} + (1 - \theta)\theta\bar{v}\right]\tau\theta + (1 - \tau)(1 - \theta)]^{-1},
\]

\[
\hat{p}_3(\bar{v}, \bar{v}) = \left[\theta(1 - \theta)\bar{v} + (\tau(1 - \theta)^2 + (1 - \tau)\theta^2)\bar{v}\right]\tau(1 - \theta) + (1 - \tau)\theta^{-1}.
\]

An \((H, L, a)\) type will prefer the deviation if

\[
\hat{p}_1 + \hat{p}_2(\bar{v}) + \theta\hat{p}_3(\bar{v}, \bar{v}) + (1 - \theta)\hat{p}_3(\bar{v}, \bar{v}) > p_1^* + p_2^* \bar{v} + \theta p_3^*(\bar{v}, \bar{v}) + (1 - \theta)p_3^*(\bar{v}, \bar{v}).
\]

and an \((L, H, b)\) type will prefer it iff

\[
\hat{p}_1 + \hat{p}_2(\bar{v}) + (1 - \theta)\hat{p}_3(\bar{v}, \bar{v}) + \theta\hat{p}_3(\bar{v}, \bar{v}) > p_1^* + p_2^* \bar{v} + (1 - \theta)p_3^*(\bar{v}, \bar{v}) + \theta p_3^*(\bar{v}, \bar{v}).
\]
In order to rule out the inference that the deviation comes from \((H, L)\), I need to establish that one cannot find a \(\tilde{p}_1 < p^*_1\) and a \(\tau\) such that \((X)\) but not \((XI)\) holds. Simple reorganization shows that \((X)\) is weaker than \((XI)\) iff

\[
\lambda_1(\tau) = \left[ \tilde{p}_1(\tilde{v}, \tilde{y}) - \tilde{p}_1(\tilde{v}, y) \right] - \left[ p^*_1(\tilde{v}, \tilde{y}) - p^*_1(\tilde{v}, y) \right] > 0, \quad (XII)
\]

and that \((X)\) holds for \(\tilde{p}_1 < p^*_1\) if

\[
\lambda_2(\tau) = \theta \left[ \tilde{p}_3(\tilde{v}, \tilde{y}) - \tilde{p}_3(\tilde{v}, y) \right] - \theta \left[ p^*_3(\tilde{v}, \tilde{y}) - p^*_3(\tilde{v}, y) \right] + \tilde{p}_2(\tilde{v}) - p^*_2(\tilde{v}) > 0. \quad (XIII)
\]

The function \(\lambda_1\) is concave and takes its maximum at \(\tau = \frac{1}{2}\), while \(\lambda_2\) is increasing. If we define \(\tau_{12}\) by \(\lambda_{12}\) = \(\lambda_{22}\), tedious but trivial algebra yields \(\tau_{12} = \theta\). Furthermore \(\lambda_1(\tau_{12}) = 0\). So \((XII)\) is only true for \(\tau < \tau_{12}\) while \((XIII)\) demands \(\tau_{12} < \tau\). Divinity therefore suggests that deviators be of \((L, H)\) or \((L, L)\) types. Given that the deviations do not pay off and the honest equilibrium is divine.

Let us now look at the possibility of price separation. First, \((H, L, a)\) types cannot separate since they will be imitated by \((L, H, b)\) types. Secondly, these two types have no incentive for joint separation, since \(\tilde{v}\) identifies them. That is, for a hypothetical separating equilibrium a deviation to \(p^*_1\) could (by divinity) come from these types and they will therefore prefer it. Thirdly, types for whom \(v_1 = \tilde{v}\) cannot persuade the buyer to purchase if they separate at \(\tilde{p}_1 > \tilde{v}\). They will therefore either not sell (so we are in the honest equilibrium) or prefer to imitate others (so the conjectured equilibrium unravels). Since this exhausts the possibilities we conclude that there are no divine pure strategy equilibria with price separation.

It should be noted that there is a separating mixed strategy equilibrium in which all \((H, L, a)\) types and some \((L, H, b)\) types charge a low initial price and the latter earn zero expected profits. Furthermore, if the model is enriched with \((H, H)\) types, these will generally be able to separate with a low \(p_1\).

We finally look at the possibility of equilibria in which sellers pool on recommendations for purchase. The only nontrivial argument concerns equilibria in which all types recommend purchase in all events. The prices would be

\[
p^*_1 = \left[ \eta\tilde{v} + (1 - \eta)\tilde{y} \right],
\]

\[
p^*_2(\tilde{v}) = \left[ 1 - 2\theta(1 - \theta)\tilde{v} + 2\theta(1 - \theta)\tilde{y} \right],
\]

\[
p^*_3(\tilde{v}) = \left[ (1 - \theta)\theta\eta\tilde{v} + (1 - \eta)(1 - \theta)\theta\eta\tilde{y} \right](1 - \eta)^{-1} \quad (\text{<c by (A3)}),
\]

\[
p^*_3(\tilde{v}, \tilde{y}) = \frac{1}{2} \tilde{v} + \frac{1}{2} \tilde{y}.
\]

Suppose now that a seller refuses to trade in period one and that the buyer assigns probabilities \((\mu, v, 1 - \mu - v)\) to his type being \((H, L)\), \((L, H)\), and \((L, L)\) respectively. In period 2, this seller can charge

\[
p^*_2 = \left[ \theta\mu + (1 - \theta)v \right]\tilde{v} + \left[ 1 - \theta\mu - (1 - \theta)v \right]\tilde{y},
\]

and later

\[
p^*_2(\tilde{v}) = \left[ \theta(1 - \theta)(v + \mu)\tilde{v} + (1 - \mu)(2 - \theta) - v(1 - \theta)^2)\tilde{y} \right](1 - \nu - \theta(\mu - v))^{-1} \quad (\text{<c)}).
\]

\[
p^*_2(\tilde{v}) = \left[ \mu(1 - \theta)^2\tilde{v} + (\mu + v)(1 - \theta)^2\tilde{y} \right](1 - \theta)^{-1}.
\]

Since I would like to get rid of the above equilibrium, I would like to show that divinity requires that the deviation is taken to imply that the deviator is not a \((L, L)\) type \((1 - \mu - v = 0)\).

Assume first that \(\mu, v\) are such that \(p^*_2(\tilde{v}) > c\). In this case the deviation pays off for an \((L, L)\) type in

\[
\{(\mu, v)| p^*_2 + p^*_3(\tilde{v}) - 2c > p^*_1 - c\} = \Omega_1,
\]

and for an \((H, L, b)\) type in

\[
\{(\mu, v)| p^*_2 + \theta p^*_3(\tilde{v}) + (1 - \theta)p^*_3(\tilde{v}) - 2c > p^*_1 - c\} = \Omega_2.
\]

Divinity rules out the \((L, L)\) type if \(\Omega_1 \subset \Omega_2\). We therefore need to check if the boundary of \(\Omega_1\) is contained in \(\Omega_2\). We therefore define two increasing functions of \(\mu\):

\[
\omega_1(\mu) = p^*_2(\mu) + p^*_3(\tilde{v}; \mu) - p^*_1 - c, \quad (XIII)
\]

\[
\omega_2(\mu) = \omega_1(\mu) + \theta[p^*_3(\tilde{v}; \mu) - p^*_3(\tilde{v}; \mu)]. \quad (XIV)
\]

As \(\mu\) goes from zero to one, \(\omega_1\) and \(\omega_2\) increase from negative to positive values. Define \(\mu_1, \mu_2\) by \(\omega(\mu_1) = 0\), \(\omega(\mu_2) = 0\). Since \(\omega_2 > \omega_1\), we conclude that \(\mu_2 < \mu_1\) and \(\Omega_1 \subset \Omega_2\). We are therefore done with this step.

The other case, where \(p^*_3(\tilde{v}) < c\), is easier, because a \((L, L)\) deviator will get \(p^*_2\) only. So divinity rules out \((L, L)\) types under weaker conditions.
Let us finally look at \((L, H)\) types.

Given that \((L, L)\) types are ruled out, suppose that a seller who deviates by refusing to sell is believed to be a \((H, L)\) type with probability \(y\), and else a \((L, H)\) type. In periods 2 and 3, such a seller can charge

\[
p_f^P = (1 - \theta - \gamma + 2\theta \gamma) \bar{v} + (\theta + \gamma - 2\theta \gamma) \gamma.
\]

\[
p_f^P(\bar{v}) = \left( (1 - \theta)^2 + \gamma (2\theta - 1) \right) \bar{v} + \theta (1 - \theta) \gamma (1 - \theta - \gamma + 2\theta \gamma)^{-1}.
\]

\[
p_f^P(v) = \left( 1 - \theta \right) \theta \bar{v} + (\gamma (1 - \theta)^2 + (1 - \gamma) \theta^2 \gamma \gamma) (\gamma + \theta - 2\theta \gamma)^{-1}.
\]

As above, we concentrate on the difficult case in which \(p_f^P(v) < c\). Corresponding to \((L, H)\) and \((H, L)\) deviators, we can define two monotonic functions

\[
\omega_1(\gamma) = p_f^P(\gamma) + \theta p_f^P(1; \gamma) + (1 - \theta) p_f^P(1; \gamma) - p_f^P - c, \quad (XV)
\]

\[
\omega_2(\gamma) = \omega_1(\gamma) + (2\theta - 1) [p_f^P(1; \gamma) - p_f^P(1; \gamma)]. \quad (XVI)
\]

Define \((\gamma_1, \gamma_2)\) by \(\omega_1(\gamma_1) = 0\), \(\omega_2(\gamma_2) = 0\). Since \((\omega_2 > \omega_1)\), we conclude that \(\gamma_1 < \gamma_2\). Since the case with \(p_f^P(v) < c\) requires weaker conditions, we are done with the argument. So divinity requires that no-sell deviations from the hypothetical pooling equilibrium be interpreted as coming from \((H, L)\) types for sure. They are therefore profitable and the equilibrium is not divine. Q.E.D.

References


