Product Line Rivalry: Note

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In an important paper, James Brander and Jonathon Eaton (1984) derive several results about equilibrium product configurations when multiproduct firms compete. However, their results are derived from the assumption that each firm in fact offers multiple (two) products. The purpose of this note is to shed some light on the conditions under which such product line competition will emerge. In addition, I consider whether an industry will supply standardized or differentiated products. For expositional clarity, I will conduct the analysis in the context of an example.

I. The Example

Brander and Eaton only allow their firms to produce four product types, which are specified as two pairs of substitutes. In order to investigate whether firms will offer product lines, I need to make it possible to offer a "standardized" product, while still preserving the incentives to sell more differentiated products. To do this, I adopt a Hotelling-style framework and allow firms to locate any number of products, at a cost d per product, anywhere in the interval $[0,1]$.

I then must specify a demand structure that incorporates the tension between standardization and differentiation. For this purpose, I assume the existence of two groups of consumers, indexed by 0 and 1, according to their preferred values of the product attribute α. Consumers are only willing to buy products whose α values differ at most by one-half from their preferred values, and they will demand a price discount on products with α values other than 0 or 1. In particular, I assume that the utility functions are such that if consumers in group 0 have a choice between several products ($j = 2,3,\ldots$) located at $\alpha_j(\leq 1/2)$ and priced at p_j, they will select that which minimizes $p_j + \beta\alpha_j^2$ ($\beta > 0$). Similarly, group 1 will minimize $p_j + \beta(1-\alpha_j)^2$, $(1-\alpha_j \leq 1/2)$. If inverse demand is linear in these "attribute-adjusted" prices, it follows that product j will sell in group 0 at

$$p_{0j} = 10 - bX_0 - \beta\alpha_j^2, \quad \alpha_j \geq 1/2, \quad b > 0,$$

where X_0 is the total volume purchased by group 0 and we have normalized the intercept at 10. If group 1 is identical (except for its preferences about α), and that group is sold X_1, the price will be $p_{1j} = 10 - bX_1 - \beta(1-\alpha_j)^2$, $1-\alpha_j \leq 1/2$.

I model competition in this market as a two-stage game. In the first stage, each of n firms locates one or more products on $[0,1]$; in the second stage, these firms engage in Nash quantity competition with zero marginal costs. Given this dynamic structure, firms will make first-stage decisions with an eye to their implications in the second stage. To find the perfect equilibria of the game, therefore, I first solve the quantity game for all different location decisions and then evaluate them, given equilibrium play in the second stage. Since firms can locate any number of products over the entire interval $[0,1]$, the backwards solution procedure at first seems very complex. However, we can simplify matters a great deal. To see how, consider a product which is not located at 0, 1/2, or 1. Such a product will only sell in one group and will command a price strictly below that of the appropriately targeted product (0 or 1); therefore, the only possible equilibrium locations are 0, 1/2, and 1. Furthermore, it never pays for a firm to locate a product at 1/2 and another one at 0 or 1 at the same time, since strictly higher revenues can be obtained from locating at 0 and 1 only. Roughly following Brander and Eaton's terminology, I refer to the case where each firm specializes in one end of the market as

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TABLE 1—b TIMES PROFIT

	1/2	0	0+1
1/2	– $2(10-\beta/4)^2/9-bd$	$(10+\beta/4)^2/9-bd$ $(10-\beta/4)^2/4$ $+(10-\beta/2)^2/9-bd$	$2(10+\beta/4)^2/9-2bd$ $2(10-\beta/2)^2/9-bd$
1	– –	– $25-bd$	$325/9-2bd$ $100/9-bd$
0+1	– –	– –	– $200/9-2bd$

segmentation, while interlacing describes situations where all firms offer two products. Standardization occurs when all products are located at 1/2.

For the special case of a duopoly, it is tedious but trivial to compute the payoffs for the first stage of the game, given equilibrium play in the second stage. For example, if one firm locates at 0 while the other firm locates at 1, they will be local monopolists and can each reap profits of $25/b-d$. Similarly, if both firms locate at 1/2, we have a homogeneous duopoly and in Nash equilibrium each firm gets profits of $2(10-\beta/4)^2(9b)^{-1}-d$. The entire payoff matrix, which obviously is symmetric, is shown in Table 1. From the table, it is trivial but tedious to establish that

I. (1/2,1/2)—standardization—is a perfect equilibrium if

$$\beta<\min\{9bd/20, 40(3-2\sqrt{2}, 40-12(bd/2)^{1/2}\}$$

II. (0,1)—segmentation— is a perfect equilibrium if

$$\beta>8(17-\sqrt{189})/5;\ 100/9<bd<25.$$

III. (0+1,0+1)—interlacing—is a perfect equilibrium if

$$\beta>20-(400-18bd)^{1/2};\ bd<100/9.$$

IV. (1/2,0+1) is a perfect equilibrium if

$$\beta<\min\{20-(18bd)^{1/2}, 20-(400-18bd)^{1/2}\};\ bd<20\beta/9.$$

V. There are no other perfect equilibria.

To interpret these conditions, it also may be helpful to refer to Figure 1. As one would expect, the basic forces at work are the incentives to differentiate, measured by the heterogeneity of tastes (β), and the costs of offering additional products, measured by the scaled fixed costs (bd). Assume first that

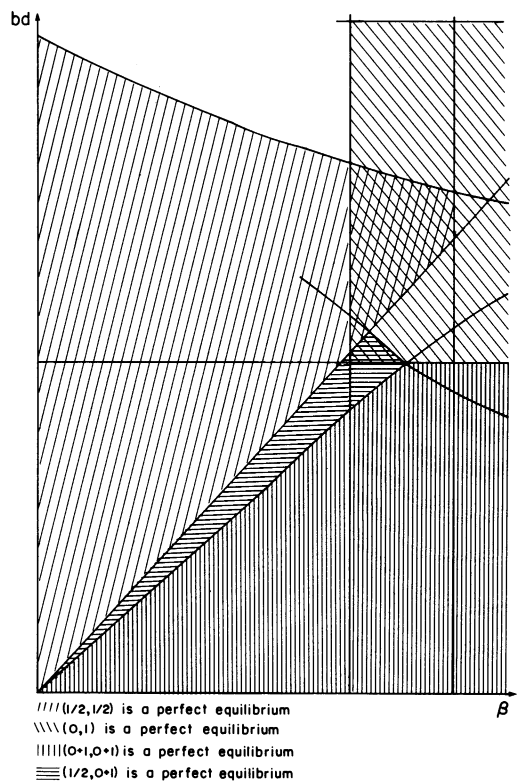

FIGURE 1

there are high fixed costs (so bd is high). If tastes are homogeneous (β is small), both firms will offer a single standardized product. If tastes are heterogeneous, the firms will segment the market and each cater to one group. In certain intermediate cases, the incentives to differentiate and the costs of differentiating balance out so that only one firm can do it profitably.

In the regions where there are multiple perfect equilibria, we can get uniqueness by letting the firms enter sequentially so that the first mover can impose the most profitable equilibrium. There are two such regions, one in which $(1/2,1/2)$ and $(0,1)$ are perfect equilibria, and one in which $(1/2,0+1)$ and $(0,1)$ are perfect equilibria. In both cases, it turns out that sequential play will lead to the configuration $(0,1)$ because this eliminates competition. (In this example, it is not advantageous to preempt the center of the market, since all demand is at the extremes.)

The assumption that only two firms compete is clearly unsatisfactory. One would expect continued entry until profits are driven to zero. If we concentrate on symmetric equilibria in industries with n firms, we can find the profits from standardization, segmentation, and interlacing as

$$b\Pi(1/2|1/2) = 2(10-\beta/4)^2/(n+1)^2 - bd$$

$$b\Pi(0|1) = 100/(1+n/2)^2 - bd$$

$$b\Pi(0+1|0+1) = 200/(n+1)^2 - 2bd.$$

From this it can be seen that the segmented structure allows the greatest number of firms, while standardization comes in second. As one would expect the interlaced product configuration is the best entry deterrent, a result also found by Brander and Eaton.

II. Concluding Remarks

In an effort to supplement the product line rivalry results obtained by Brander and Eaton, I have examined a model in which firms may or may not choose to offer multiple products. The objective is to identify those conditions under which multiple product (or "product line") offerings will emerge. The principal result is that heterogeneity in tastes and low product-specific fixed costs favor "product line rivalry" in the sense that both firms will produce two products and compete directly with each other. If tastes are more homogeneous and product-specific fixed costs are high, both firms produce a single standardized product. If both heterogeneity and fixed costs are high, each firm will produce a different single product.

Since this analysis has been conducted in the context of a specific example, the results have to be interpreted with caution. Generalization to a more realistic setting would complicate the exposition considerably. For example, if consumers are continuously distributed on $[0,1]$, it is no longer possible to establish that locations other than 0, 1/2, and 1 are dominated; therefore, the first stage of the game has to be analyzed as an infinite game. Nevertheless, I trust that the reader will find the intuition behind these results quite robust.

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References

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