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REPUTATION, MONITORING, AND EFFORT

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This paper studies the impact of strategic monitoring on reputation building, focusing in particular on models of the principal–agent type. The main result is that the principal always will monitor risk-neutral agents more closely early in the game, while such agents may or may not work harder as the game progresses, depending on their initial reputation. If agents are sufficiently risk-averse, they will always work harder early in the game, whereas the principal may or may not monitor such agents more closely as the game progresses.

Keywords: Employment relationship, monitoring, reputation, principal–agent model, trading relationship, adverse selection, moral hazard.

1. Introduction

In principal–agent relationships, the principal may choose to monitor an agent's effort when output is a noisy indicator of effort. Similarly, the principal may elect to measure output when effort monitoring is costly. So in general, effort monitoring and output measurement are substitutes.

The theoretical literature has focused primarily on output measurement, although there has been some work on the case where both output and monitoring are used [Mirrlees (1976), Townsend (1979), Baiman and Demski (1980), and Dye (1986)]. In contrast, the literature on pure monitoring has been quite limited and focused on applications to regulation and auditing [Sappington (1986) and Demski and Sappington (1987)]. This focus is unnecessarily narrow. In practice, firms often reward employees solely on the basis of monitored effort, particularly for jobs where output measurement is difficult. Examples of such jobs include team production (such as fast food, construction), quality control (machine maintenance, auditing), or customer service (telephone answering, small ticket retail sales). Employees in these jobs are paid by the hour and are promoted or fired on the basis of monitored effort.

In the present paper, I want to evaluate the commonly used argument that the development of 'trust' is a feature of long-term trading relationships [Williamson (1979, pp. 240 ff.)]. To keep the analysis simple, I look at a pure monitoring model with adverse selection, such that I can explore the strategic relationship between monitoring and reputation. As we will see, the argument about trust is often, but not always, true: the strategic interaction of monitoring and effort may lead to time-increasing monitoring levels (because equilibrium effort is high in early periods).

I will start by looking at the interaction between effort and monitoring in a

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static model. I find that agents with better reputations work less and are subjected to less monitoring. There is no separating equilibrium, but a unique pooling equilibrium exists, such that only one pay scheme will be offered. With risk-neutral agents, the equilibrium contract makes all payments contingent on not being caught shirking. Given this pay scheme, the expected payoffs to both principal and agent increase as the reputation of the agent goes up.

I then look at reputation building and monitoring over time.[1] If the agent is risk-neutral, the optimal contract gives him no payments if he is caught shirking at any time before the end of the last period. In such a game, the principal faces greater incentives to monitor agents earlier in the sequence, ceteris paribus, because there are more bad agents to weed out. The agent, on the other hand, faces more favorable gambles as the end payoff approaches. In equilibrium, the principal always follows her own incentives and monitors earlier in the game. If the agent has a bad reputation, he will find high initial efforts unrewarding and will prefer to expend little effort in the first period. If an agent is still around, he will find the odds more appealing later. However, agents with good reputations will work harder in the first period.

If the agents are risk-averse, optimal contracts pay them a strictly positive amount in each period. This gives the agent sharper incentives earlier in the game and, for a sufficiently high degree of risk aversion, always result in greater effort in the first period. The principal still has greater incentives to monitor agents early in the game, but if she faces an agent with great incentives to work hard early on, she might find it advantageous to play off the agent's efforts and monitor less in the first period and more in the second. Conversely, a principal who faces an agent with less incentive for early effort will follow her own incentives and monitor even earlier in the game.

2. Effort and monitoring in a static model

To prepare the way for the dynamic analysis in sections 3 and 4, I first consider a one-period model in which a monopolist principal hires from an infinitely elastic supply of risk-neutral agents without knowning their 'type'. In particular, an agent can be 'good', in which case he works all the time, or he can be 'bad', in which case he is work-averse.[2] Each agent knows his own type, whereas the principal a priori ascribes a probability δ^0 to him being good. This δ^0 is the agents' reputation. To keep the model as simple as possible, I make two simplifying assumptions. First, I assume that output, while it accrues to the principal, is difficult and costly to measure precisely until after the game. This justifies restricting the model such that compensation and type conjectures are not based on output.[3] Second, I assume that the agent cannot observe whether

[1]Abreu, Migrom, and Pearce (1987) investigated repeated games with exogenous monitoring intensity, but other work on monitoring has been in static games.

[2]The assumption that 'good' agents always work is made for expositional ease. It seems fair to conjecture that my qualitative results would hold also for the case where 'good' agents are less work-averse than 'bad' agents in a more general way than assumed here. The present formulation is simpler in the sense that we avoid type 1 errors. If an agent is caught shirking, we know that he is 'bad' because a 'good' agent never gets a bad reputation.

[3]In a more complete model, the tradeoff between stochastic monitoring and costly/noisy output observation should be analyzed. The present work is merely intended as a first step.

he has been monitored. Given this, the principal offers a mix of a flat-rate payment and bonus to those who are not caught shirking.

Prior to the game, the principal offers one or more pay schemes on a take-it-or-leave-it basis. A pay scheme is given by two non-negative real numbers (b_0, b_1) and commits the principal to pay all agents a flat fee of b_0, while those who are not caught shirking get a bonus b_1. Note that the requirement that $b_0 \geqslant 0$ implies a limited liability condition for the agent. (Without this restriction, the principal would be able to infer an agent's type from his willingness to pay a sufficiently large entry fee, thus making reputations superfluous.)

If an agent agrees to work for the principal, he receives his flat fee payment, b_0, and the parties simultaneously decide on effort and monitoring levels. As noted above, a good agent always works, while a bad agent attributes a disutility $h(q)$ where $h(0)=0$, $h'>0$, $h''(1-q)>1$, and $h'''<0$ (primes denote derivatives) to the fraction q of the period he works.[4] The principal can, without the agent knowing, observe him at a random instant in the period with probability λ if she pays a price $k(\lambda)$ where $k(0)=0$, $k'>0$, $k''(1-\lambda)\geqslant 1$, $k'''\leqslant 0$. So $(q, \lambda)\in[0,1]^2$. Depending on whether or not the agent was caught shirking, he qualifies for the agreed-upon bonus, b_1. For simplicity we normalize the value of the agent's efforts at q and assume that both players have separable and risk-neutral utility functions without discounting.[5] Furthermore, the structure of the game is common knowledge.

Before proceeding with the analysis, I would like to highlight two features of the model. First, the game proceeds in two stages, with the principal commiting to a pay scheme initially, and then the players simultaneously choosing effort and monitoring later. This structure implies that the principal cannot commit to a monitoring strategy. It is certainly possible to make the opposite assumption, but it is not very reasonable within this model: because a monitoring 'level' here is a probability of checking, commitment requires the existence of a publicly observable randomization device. While there may be real-life examples of such arrangements, they hardly abound.

Second, the pay schemes are constrained such that the agent receives the same pay if he is not checked and if he is checked and not caught shirking. This payoff arrangement is consistent with the assumption that the agent is not able to observe whether he has been checked. If the agent can observe if he has been checked, it becomes possible to make the payment contingent on observation. I will return to this restriction after Proposition 2 and argue that the essence of the results will remain unchanged if this assumption is relaxed. (On the other hand, the results could change substantially if the model permits both commitment to a monitoring strategy and observable monitoring.)

Because of the asymmetric information structure, the principal can offer two pay schemes such that all 'good' agents will select one, whereas all 'bad' agents will select the other [Harris and Townsend (1981); Rothschild and Stiglitz (1976)]. However, it will turn out that such a separating equilibrium is less advantageous to the principal than a pooling equilibrium in which only one pay scheme is offered. To show this, I will first look at some properties of subgame perfect pooling equilibria.

[4]These assumptions could be weakened but are convenient.

[5]Given that $b_0 \geqslant 0$, this implies that $b_1 \leqslant 1$.

As I have defined the game, the principal has to offer a pay scheme (b_0, b_1) in the first stage in order to maximize her expected profits:

$$\mathrm{E}(\pi) \equiv -k(\lambda^*) + \delta^0 + (1-\delta^0)q^* - b_0 - b_1[1-(1-\delta^0)(1-q^*)x^*]\,. \quad (1)$$

The pay scheme has to satisfy incentive compatibility constraints

$$\mathrm{E}(U) = -h(q^*) + b_0 + b_1[1-(1-q^*)\lambda^*] \geq \bar{U}, \text{ for bad agents and} \quad (2)$$

$$\mathrm{E}(V) = b_0 + b_1 \geq \bar{V}\,, \text{ for good agents,} \quad (3)$$

where the reservation values are

$\bar{U} = 0$, or best alternative contract (within our model) for bad agents,
$\bar{V} = 0$, or best alternative contract (within our model) for good agents,

and second stage play gives

$$\lambda^* = \operatorname{argmax}(-k(\lambda) + \delta^0 + (1-\delta^0)q^* - b_0 - b_1[1-(1-\delta^0)(1-q^*)x])\,, \quad (4)$$

$$q^* = \operatorname{argmax}(-h(q) + b_0 + b_1[1-(1-q)\lambda^*])\,. \quad (5)$$

I can now derive the following results about subgame perfect pooling equilibria in this model.

Lemma 1. If the principal has offered a single pay scheme (b_0, b_1), there is a unique equilibrium in the monitoring-effort game. In this equilibrium the agents may not take the offer.

Proof. If the contract is taken, we have a concave game and the Nikaido–Isoda theorem [Rosen (1965)] guarantees the existence of a unique equilibrium $(q^*, \lambda^*) \in [0,1]^2$. This in turn allows us to calculate expected profits and utilities and compare to $\bar{U}$ and $\bar{V}$. Q.E.D.

Proposition 1. No pay schemes where $b_0 \neq 0$ can be offered in subgame perfect pooling equilibria with risk-neutral agents.

Proof. We will try to maximize $\mathrm{E}(\pi)$ while holding $\mathrm{E}(U)$ constant. Since b_0 has no incentive effects, we can increase $\mathrm{E}(\pi)$ while keeping $\mathrm{E}(U)$ [and $\mathrm{E}(V)$] constant by increasing b_1 at the expense of b_0. Q.E.D.

The intuition behind this result is, of course, that the incentive to work increases if compensation is tied to performance. Given these results, we can evaluate the possibility of separating equilibria.

Proposition 2. There is a unique subgame perfect pooling equilibrium, but no separating equilibria.[6]

[6] See Freixas, Guesnerie, and Tirole (1985) and Aron (1987) for related results.

Proof. I prove the second part first. Assume the existence of a set of separating pay schemes (B_0, B_1), $(0, B_I)$. In this case, there will be no monitoring of the good agents. So the incentive schemes have to satisfy $B_I \geqslant B_0 + B_1$ and $B_I \leqslant B_0 + B_1(1-(1-q)\lambda) - h(q)$, which is impossible. The first part of the proposition here follows from Lemma 1 and the fact that the principal faces a simple optimization problem in the first stage of the game. Q.E.D.

The argument against separating equilibria hinges on the fact that there will be no monitoring of good agents in such equilibria. This makes it impossible for the self-selection constraints to be consistent. Coming back to the restriction on pay schemes, if the agent can observe when he has been checked, it may be possible to offer richer payment schemes. However, a pair of such schemes will also fail to satisfy the self-selection constraints for reasons analogous to those above. So the lack of separating equilibria does not depend on this aspect of the information structure.[7] However, if the principal can credibly commit to different monitoring levels, and include these into a pair of pay schemes, the above argument fails, and so separation is possible.

Given the uniqueness we can look at comparitive statics:

Corollary 1. Higher reputation leads to lower pay, expected utility, monitoring and effort.

Proof. See the appendix.

The intuition behind this result is that agents with better reputations will be subjected to less monitoring and thus will be able to work less hard, making lower pay acceptable.

I will later need:

Lemma 2. For fixed b_1, expected profits are a convex function of reputation.

Proof. See the appendix.

Based on the intuition and the results obtained for the simple one period model, I can now go to the more general case and consider reputation building over several periods.

[7]Suppose that the agent can observe whether he has been monitored, such that payment can depend on that. Let b_N be the payment if there is no monitoring and b_C be the payment to an agent who is monitored and not found shirking. The principal's problem is now

$$\max_{b_N, b_C} - k(\lambda^*) + \delta^0 + (1-\delta^0)q^* - b_N(1-\lambda^*) - b_C\lambda^*[1-(1-\delta^0)(1-q^*)],$$

subject to

$$-h(q^*) + b_N(1-\lambda^*) - b_C\lambda^* q^* = 0,$$

$$-k'(\lambda) + b_N - b_C[(1-\delta^0)(1-q^*)] = 0,$$

$$-h'(q) + b_C\lambda^* = 0.$$

Since the solution in general will not satisfy $b_N = b_C$, the principal can do better in this case.

3. Reputation building over time

To study the effects of reputation building over time, I will generalize the above model by adding an extra period. I assume that an agent who has been caught shirking in period 1 can be fired without costs before period 2. Apart from their positions in the sequence, periods 1 and 2 are identical. The principal still has the ability to commit fully on all payments. My focus will be on the difference in effort and monitoring between the two periods.

To this end we first need:

Lemma 3. There can be no separating equilibria and in pooling equilibria the principal never offers a new second contract to agents who are revealed as bad in the first period.

Proof. The impossibility of separation follows from the argument in the proof of Proposition 2. To see the second part, assume the opposite. In this case, bad agents would have lower incentives in period 1 and no additional incentive in period 2. And yet the principal would still need to satisfy their incentive constraints. Q.E.D.

This lemma allows us to focus on the properties of subgame perfect pooling equilibria. If we let subscripts identify periods, we can write a pay scheme in the form (b_0, b_1, b_2) where b_0 is paid before period 1, b_1 is paid after period 1, and b_2 is paid after period 2. Payments at time $t = 1, 2$ are only made to agents who have not been caught shirking at that time. By repeated applications of Proposition 1, it is obvious that the equilibrium pay scheme is of the form $(0, 0, b_2)$. Furthermore, it is important to note that such pay schemes force maximum risk on the agents. If the agents are risk-averse, a more even pay distribution will be optimal.

I can now state the main result.

Proposition 3. If agents are risk-neutral, the principal always monitors more early in the game, while an agent may or may not work harder early in the game.

Proof. See the appendix.

Intuitively, the principal monitors more intensively in the earlier period because the returns to monitoring are higher when more bad agents remain in the game. The agents, conversely, face greater returns to effort as the game progresses because fewer periods with negative utility stand between them and the payoff. However, the agents' actions are also influenced by the principal. If the latter has incentives to monitor quite early in the game, the agents will find high efforts in the first period very unrewarding. If an agent survives the initial period, he will find the odds more appealing later in the game.

Corollary 2. The agents are more likely to work hard early if they have a higher reputation, ceteris paribus.

Proof. See the appendix.

If the agents have higher reputations, they will be subjected to less monitoring and have higher utility to protect. Corollary 2 states that such agents will work hard early on, while agents with lower reputations have to play off the principal's incentives, work less hard in the initial period and hope to be around for better times.

I mentioned earlier that the nature of the optimal contract, in which any payment is contingent upon performance in all periods, is dependent upon the assumed risk neutrality. Following the intuition outlined above, the fact that the agents have negative utility in the first period significantly influences our results. In order to move towards more realism and investigate the importance of this, I will now modify the model by assuming that agents are risk-averse. In such a setting, the principal absorbs more of the risk and payments may be made at the start of both periods 1 and 2.

4. Reputation building over time-risk averse agents

I now assume that the agents are risk-averse in that they value each of the payments, $\tilde{b}_0$, $\tilde{b}_1$, and $\tilde{b}_2$ according to a concave utility function $w()$. I can show:

Lemma 4. There can be no separating equilibria.

Proof. By the same argument as in the proof of Proposition 2, the self selection constraints cannot be consistent. Q.E.D.

Lemma 5. If the agents are sufficiently risk averse, the optimal pay scheme is such that their expected utility in each of the periods 1 and 2 is non-negative.[8]

Proof. (a) Because the contract commits only the principal to period 2, the individual rationality constraint guarantees that the agent's expected utility in this period is positive. (b) Similarly, there is no problem for good agents. (c) The problem is only tricky for bad agents' period 1 utility. While in principle we could proceed as in Proposition 1, the above Lemma can be proved through an extreme example: consider a pay scheme $(0, 0, \bar{b}_2)$ with associated equilibrium $\bar{q}_1$, $\bar{\lambda}_1$, $\bar{q}_2,(\delta_1)$, $\bar{\lambda}_2(\delta_1)$. If $w'(\bar{b}_2) \ll w'(0)$, we can increase q_1 and $E(\pi_1 + \pi_2)$ by lowering $\bar{b}_2$ and offering a small $\tilde{b}_1$, leading to the pay scheme $(0, \tilde{b}_1, \tilde{b}_2)$. This will result in a $E_1(\tilde{U}_1) > E_1(\bar{U}_1)$. If still $w'(\tilde{b}_2) \ll w'(\tilde{b}_1)$, we could take this further and further until, given a sufficiently concave $w()$, we get $E_1(U_1) > 0$. Q.E.D.

Once risk aversion is introduced, the principal can give the agent a lower expected total payment, but higher or identical utility by increasing b_0 and b_1 above zero. This counteracts the effect described in Proposition 1: for sufficiently large risk aversion, the former effect dominates the latter.

I will now assume that the agents are 'sufficiently risk averse', in the sense of Lemma 5. For this model I get:

[8]While this very weak result is quite obvious from the analysis in Lambert (1983), I sketch a proof for completeness.

Proposition 4. If the agents are sufficiently risk-averse, they always work harder early in the game, while the principal may or may not monitor more early in the game.

Proof. See the appendix.

In this model, therefore, it is the agent who always works hardest early on, while the principal may or may not free ride on this. Intuitively, the agents' incentives are stronger if more periods and thus more rewards remain in the game. Also, the principal has greater incentive to monitor early in the game, ceteris paribus, because she can save paychecks by detecting a 'bad' agent quickly. On the other hand, if the agents have very great incentives to work hard early on, the principal may be able to relax her early monitoring. Unfortunately, I was unable to relate the relevant condition unambiguously to reputations, as in Corollary 2. The problem is that although agents with better reputations will work harder early on, the principal also has higher marginal gains from exposing such agents.

5. Conclusion

I have looked at the interaction between the time paths of effort and monitoring in models of the principal–agent type. My results show that players with less risk will 'invest' in early periods. Players with more risk, on the other hand, will only do so if they have relatively more to gain compared to their opponents.

Players with less to gain, who are subject to high risks, will find it better to play off the incentives of their opponents and make greater efforts later. In particular, I find that principals always will monitor risk neutral agents more closely in the first period, while such agents may or may not work harder early in the game, depending on their initial reputation. Conseversely, sufficiently risk-averse agents will always work harder in the first period, because of the pay scheme they are offered, whereas the principal may or may not monitor them more closely early in the game. So it is not always true that the parties in a trading relationship grow to trust each other over time. Strategic interactions may outweigh the development of trust.

In a broader sense, my results indicate that failure to consider monitoring as the other side of the reputation building process can give very misleading results. Taking into account the logical monitoring response to reputation building may modify or even reverse the conclusions obtained for fixed monitoring intensity.

Although the model is highly stylized, many of the limitations appear unlikely to influence the spirit of the results. Several details will change in response to different assumptions. In particular, non-monotone effort paths could well arise in versions with more than two periods. Overall, however, the intuition behind the propositions seems quite robust. I see no reason why the availability of noisy output measures would change the essence of the results.

Appendix

Proof of Corollary 1. In equilibrium, $\mathrm{E}(U)=0$, so b_1^* satisfies

$$0=-h(q^*(b_1^*,\delta^0))+b_1^*[1-(1-q^*(b_1^*,\delta^0))\lambda^*(b_1^*,\delta^0)]\,, \tag{A1}$$

where $q^*(b_1,\delta^0)$, $\lambda^*(b_1,\delta^0)$ are defined by (4) and (5). The Implicit Function Theorem gives

$$\partial\lambda^*/\partial\delta^0=-h''b_1(1-q^*)[k''h''+b_1^2(1-\delta^0)]^{-1}<0\,, \tag{A2}$$

$$\partial\lambda^*/\partial b_1=(1-\delta^0)[(1-q^*)h''-b_1\lambda^*][k''h''+b_1^2(1-\delta^0)]^{-1}>0\,, \tag{A3}$$

$$\partial q^*/\partial\delta^0=-b_1^2(1-q^*)[k''h''+b_1^2(1-\delta^0)]^{-1}<0\,, \tag{A4}$$

$$\partial q^*/\partial b_1=\lambda[k''+(1-\delta^0)(1-q^*)][k''h''+b_1^2(1-\delta^0)]^{-1}>0\,. \tag{A5}$$

Using this, and the Implicit Function Theorem on (A1) now gives

$$\frac{\mathrm{d}b_1^*}{\mathrm{d}\delta^0}=b_1(1-q^*)\,\frac{\partial\lambda^*}{\partial\delta^0}\left[1-(1-q^*)\lambda^*-b_1^*(1-q^*)\,\frac{\partial\lambda^*}{\partial b_1}\right]^{-1}<0\,. \tag{A6}$$

This establishes the first claim and, since $\mathrm{E}(U)=0$ and $\mathrm{E}(V)=b_1^*$ in equilibrium, also the second claim.

The third and fourth claims follow from the above as

$$\frac{\mathrm{d}\lambda^*}{\mathrm{d}\delta^0}=\frac{\partial\lambda^*}{\partial\delta^0}+\frac{\partial\lambda^*}{\partial b_1}\,\frac{\mathrm{d}b_1^*}{\mathrm{d}\delta^0}<0 \quad\text{and} \tag{A7}$$

$$\frac{\mathrm{d}q^*}{\mathrm{d}\delta^0}=\frac{\partial q^*}{\partial\delta^0}+\frac{\partial q^*}{\partial b_1}\,\frac{\mathrm{d}b_1^*}{\mathrm{d}\lambda^0}<0\,. \qquad \text{Q.E.D.} \tag{A8}$$

Proof of Lemma 2. The first two derivatives are

$$\left.\frac{\mathrm{dE}(\pi)}{\mathrm{d}\delta^0}\right|_{b_1}=(1-q^*)(1-b_1\lambda^*)+(1-\delta^0)\,\frac{\partial q^*}{\partial\delta^0}\,(1-b_1\lambda_1^*)\,, \tag{A9}$$

$$\begin{aligned}\left.\frac{\mathrm{d}^2\mathrm{E}(\pi)}{\mathrm{d}\zeta^{02}}\right|_{b_1}&=-b_1\,\frac{\partial\lambda^*}{\partial\delta^0}\left[1-q^*+(1-\delta^0)\,\frac{\partial q^*}{\partial\delta^0}\right]\\&\quad+(1-b_1\lambda^*)\left[-2\,\frac{\partial q^*}{\partial\delta^0}+(1-\delta)\,\frac{\partial^2q^*}{\partial\delta^{02}}\right].\end{aligned} \tag{A10}$$

The latter is positively proportional to

$$\begin{aligned}&(1-b_1\lambda^*)\left[2k''h''+(1-\delta)\left(k''h''\,\frac{\partial\lambda^*}{\partial\delta^0}+k''h'''\,\frac{q^*}{\partial\delta^0}\right)\right]\\&\quad+h''(1-q^*)[k''h''+b_1^2(1-\delta^0)]>0\,. \qquad \text{Q.E.D.}\end{aligned} \tag{A11}$$

Proof of Proposition 3. The expected two-period payoff to a bad agent is

$$\mathrm{E}_0(U_1 + U_2|\delta^0) = -h(q_1) + (1-\lambda_1)\mathrm{E}_1(U_2|\delta^0) + \lambda_1 q_1 \mathrm{E}_1(U_2|\delta^1)\,, \tag{A12}$$

$$\begin{aligned}\mathrm{E}_0(\pi_1 + \pi_2|\delta^0) = {} & \delta^0[1-k(\lambda_1)] + (1-\delta^0)[q_1 - k(\lambda_1)] \\ & + (1-\lambda_1)\mathrm{E}_1(\pi_2|\delta^0) \\ & + \lambda_1[\delta^0 + (1-\delta^0)q_1]\mathrm{E}_1(\pi_2|\delta^1)\,,\end{aligned} \tag{A13}$$

where

$$\delta^1 = \delta^0/[\delta^0 + (1-\delta^0)q_1]\,, \tag{A14}$$

$$\mathrm{E}_1(U_2|\delta^i) = -h(q^i) + (1-\lambda_2^i)b_2 + \lambda_2^i q_2^i b_2\,, \tag{A15}$$

$$\begin{aligned}\mathrm{E}_1(\pi_2|\delta^i) = {} & \delta^i[1-k(\lambda_2^i)] + (1-\delta^i)[q_2^i - k(\lambda_2^i)] \\ & -(1-\lambda_2^i)b_2 - \lambda_2^i[\delta^i + (1-\delta^i)q_2^i]b_2\,,\end{aligned} \tag{A16}$$

where x^i denotes the value of x in the second period equilibrium following δ^i, $i = 0, 1$.[9]

For a given value of δ, the incentives in periods 1 and 2 differ by the first-period payoffs. Unfortunately, it is not easy to compare the first and second period values of q and λ directly. I therefore use the following technical trick: define an auxiliary variable β and two functions which give one period payoffs for $\beta = 0$ and two period payoffs for $\beta = 1$. If q_1 and λ_1 are monotonic in β, their second-period values can be related to their first-period values, since we have the two-period incentives when $\beta = 1$ and the one-period incentives when $\beta = 0$. So, I discount all period 2 payoffs, including q_2, by a factor β and define the auxiliary functions

$$\begin{aligned}U(q, \beta|\delta^0) = {} & -h(q) + (1-\lambda) \\ & \times(\beta[\mathrm{E}_1(U_2|\delta^0) - b_2] + b_2) \\ & + \lambda q(\beta[\mathrm{E}_1(U_2|\delta^1) - b_2] + b_2)\,,\end{aligned} \tag{A17}$$

$$\begin{aligned}\pi(\lambda, \beta|\delta^0) = {} & \delta^0[1-k(\lambda)] + (1-\delta^0)[q - k(\lambda)] \\ & + (1-\lambda)(\beta[\mathrm{E}_1(\pi_2|\delta^0) + b_2] - b_2) \\ & + \lambda(\beta[\mathrm{E}_1(\pi_2|\delta^1) + b_2] - b_2)[\delta^0 + (1-\delta^0)q]\,.\end{aligned} \tag{A18}$$

I can now proceed to investigate the time paths of q and λ by varying β in the above. To do so, I will write the first-order conditions on q and λ, respectively, as

$$\begin{aligned}F(q^*,\ \ ) \equiv {} & -h' + \lambda^*(\beta[\mathrm{E}_1(U_2|\delta^1) - b_2] + b_2) \\ & + \lambda^* q^*(\partial\delta^1/\partial q)(\beta[\partial \mathrm{E}_1(U_2|\delta^1)/\partial\delta^1) = 0\,,\end{aligned} \tag{A19}$$

[9]Note that a carefully defended reputation increases less than a reputation supported by a small effort.

$$G(\lambda^*, \ ) \equiv -k' - (\beta[\mathrm{E}_1(\pi_2|\delta^0) + b_2 - b_2) + (\beta[\mathrm{E}_1(\pi_2|\delta^1) + b_2] - b_2) \times [\delta^0 + (1-\delta^0)q] = 0 \,. \tag{A20}$$

Furthermore, use the following shorthand for the partial derivatives of F and G:

$$F_q \equiv \partial F/\partial q \,, \tag{A21}$$

$$F_\lambda \equiv \partial F/\partial\lambda = h'/\lambda \,, \tag{A22}$$

$$G_q \equiv \partial G/\partial q = (1-\delta^0)(\beta[\mathrm{E}_1(\pi_2|\delta^1) + b_2] - b_2) + \beta[\partial \mathrm{E}_1(\pi_2|\delta^1)/\partial\delta^1] \times [\delta^0 + (1-\delta^0)q]\partial\delta^1/\partial q \,, \tag{A23}$$

$$G_\lambda \equiv \partial G/\partial\lambda \,, \tag{A24}$$

$$F_\beta \equiv \lambda[\mathrm{E}_1(U_2|\delta^1) - b_2] + \lambda q(\partial\delta^1/\partial q)[\partial \mathrm{E}_1(U_2|\delta^1)/\partial\delta^1] \,, \tag{A25}$$

$$G_\beta = -[\mathrm{E}_1(\pi_2|\delta^0) + b_2] + [\mathrm{E}_1(\pi_2|\delta^1) + b_2]\,\frac{\delta^0}{\delta^1} \,. \tag{A26}$$

F_q and G_λ are negative by the second-order conditions, while F_λ obviously is positive. Because the agent receives no pay in period 1, the first part of F_β is negative. Similarly, the second part is negative since $\partial \mathrm{E}_1/q < 0$ and $\partial \mathrm{E}_1(v_1|\delta^1)/\partial\delta^1 > 0$ (for given b_2). Further, G_β is positive by Lemma 2. This lemma also gives us that G_q is negative, since it is smaller than $\mathrm{E}_1(\pi_2|\delta^1) - \delta^1\, \mathrm{dE}_1(\pi_2|\delta^1)/\mathrm{d}\delta^1$ which is negative because expected profits have to pass below the origin when viewed as a function of reputation. [Note that $\mathrm{E}_1(\pi_2|0) = 0$.] We can thus use the Implicit Function Theorem to get

$$\mathrm{d}\lambda/\mathrm{d}\beta = (F_qG_\lambda - F_\lambda G_q)^{-1}(G_qF_\beta - F_qG_\beta) > 0 \,. \tag{A27}$$

$$\mathrm{d}q/\mathrm{d}\beta = (F_qG_\lambda - F_\lambda G_q)^{-1}(F_\lambda G_\beta - G_\lambda F_\beta) \gtreqless 0 \quad \text{as} \quad F_\lambda G_\beta - G_\lambda F_\beta \gtreqless 0 \,. \qquad \text{Q.E.D.} \tag{A28}$$

Proof of Corollary 2. Differentiation of $F_\lambda G_\beta - G_\lambda F_\beta$ using Corollary 1. Q.E.D.

Proof of Proposition 4. In this setting $\mathrm{E}_1(U_2|\delta)$ and $\mathrm{E}_1(\pi_2|\delta)$ are

$$\tilde{\mathrm{E}}_1(U_2|\delta^i) = -h(q_2^i) + w(\tilde{b}_1) + (1 - \lambda_2^i + q_2^i\lambda_2^i)w(\tilde{b}_2) \,, \tag{A29}$$

$$\tilde{\mathrm{E}}_1(\pi_2|\delta^i) = \delta^i(1 - k(\lambda_2^i) - \tilde{b}_1) + (1-\delta^i)(q_2^i - k(\lambda_2^i) - \tilde{b}_1) - (1-\lambda_2^i)\tilde{b}_2 - \lambda^i[\delta^i + (1-\delta^i)q_2^i]\tilde{b}_2^\sigma \,. \tag{A30}$$

Everything remains the same as in the risk-neutral case, except that $\tilde{F}_\beta$ now is positive because the agent expects positive utility in period 1. In particular, $\tilde{F}_\beta$ is proportional to the expected utility in period 1 given that δ is revised upwards, minus the disutility of a smaller revision due to a higher effort in a period one. This clearly dominates $\tilde{E}_1(U_1|\delta^0)$, which is assumed non-negative. Q.E.D.

List of symbols

$\delta^0 =$ probability that an agent is 'good', a priori,
$\delta^1 =$ same as δ^0, now after an observation has failed to reveal shirking,
$b_0 =$ flat-fee payment at the start of period 1,
$b_t =$ bonus at the end of period t, if no prior shirking has been detected,
$q_t =$ fraction of period t in which agent works,
$\lambda_t =$ probability that principal observes agent a random instant in period t.
$h(q) =$ cost of effort for 'bad' agents,
$k(\lambda) =$ cost of monitoring,
$\pi_t =$ profits in period t,
$U_t =$ utility for bad agents,
$\bar{U} =$ reservation value of U_t,
$V_t =$ same as U_t, but for good agents,
$\bar{V} =$ reservation value of V_t,
$w =$ utility function of risk averse agent.

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