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Consumers choose from a relevant set of brands. Advertising encourages consumers to consider a brand as relevant. Price and other variables influence consumer choice among relevant brands. The authors examine how the explicit consideration of competitive response affects managerial recommendations and explore the interaction between price and advertising decisions. They consider cases in which managers do and in which they do not anticipate competitive actions.

The Competitive Implications of Relevant-Set/Response Analysis

Advertising response analysis has been a key component of marketing decision support systems for more than 30 years (Benjamin and Maitland 1958; Benjamin, Jully, and Maitland 1960; Little 1966; Lodish 1986). Senior managers at General Electric (Lillis and McIvor 1985), Kodak and General Motors (Barabba 1985), Carnation (Struse 1985), and Mattel (Hatch 1980) describe how such analyses have improved advertising decisions. Ten years ago, Little (1979a) reported that more than $400 million was being spent annually on data and experiments for market response analysis. We expect that figure has more than doubled today with the advent of automated data collection by supermarket scanners and split-cable television advertising tests. Academic interest is high as a variety of researchers investigate how to estimate response curves accurately (see, among others, Assmus, Farley, and Lehmann 1984; Chakravarti, Mitchell, and Staelin 1981; Little 1975; Mahajan, Muller and Sharma 1984; Rao and Sabavala 1986).

The normal application is to prescribe how a target firm should act. The impact of competitive actions is included in some models and, in practice, judgments sometimes are made about how competitors will respond. Econometric models (Lambin 1970, 1976) model competitive response explicitly, but rarely do formal advertising response models attempt to take competitive reaction into account. For example, Hauser and Shugan (1983), Kumar and Sudharshan (1988), and Rao and Sabavala (1986) use one form of response analysis that does not model competitive reaction explicitly.

We demonstrate how explicit consideration of competitive response can influence managerial recommendations. In some cases, we find recommendations to be robust to competitive considerations. In those cases simpler models suffice. In other cases the recommendations must be changed. For example, we illustrate cases in which a noncompetitive model says to decrease advertising, but competitive considerations indicate that the net effect is to increase advertising. In those cases, to make an informed choice, the decision support user must be aware of the impact of competitive considerations.

Competitive efforts are complex. Though it is mathematically feasible to analyze these effects in very general models, such analyses obscure intuitive understandings with complicated mathematical conditions. We therefore choose a two-stage exposition. We begin with a set of expositional assumptions and present the basic results. We then relax those assumptions and show how (if) they affect the results. Where they do, we believe the reader can better interpret the complicated results once the basic results are understood.

In some cases we illustrate the results with a specific functional form. In such illustrations we select parameters that can be justified empirically. These illustrations give the reader a feel for the magnitude of the results. However, unless otherwise noted, the results apply for the more general class of advertising response functions.

We begin with a section on expositional assumptions, then present the relevant-set/response model in competitive and in noncompetitive forms. Our first set of results pertains to the advertising equilibria and our second set of results pertains to the effect of advertising on price equilibria. Finally, we illustrate whether or not the expositional assumptions affect our results.

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EXPOSITIONAL ASSUMPTIONS

The following six assumptions are made in the first two analytic sections of our article. Subsequent sections consider relaxation of the first three.

Duopoly Versus Oligopoly

In the works we have cited and in most managerial practice, competitive response is not considered explicitly. With one firm, competitive effects are not relevant; with two firms they are. When we go from two firms to three firms the equations become exponentially more complex—that is, there are 2^n relevant sets for an n-firm analysis. Analytic solutions eventually cease to be feasible. Simulation of any particular market is feasible, but such simulations require many additional assumptions and may or may not yield clear intuitive results. We believe the major insights come when we go from one firm to two firms. In a subsequent section we return to the case with more than two firms.

It is beyond the scope of our article to consider product lines in which each firm offers a number of brands, including flankers, and sizes. However, we believe much of the basic insight from single-brand firms will prove to be extendable to complex product lines.

One Variable at a Time

We look first at the competitive equilibria for advertising and then at the competitive equilibria for price. The technical conditions for these equilibria are easy to grasp and the intuition is transparent. In a subsequent section we discuss the more complicated conditions that apply when advertising and price are set simultaneously and discuss what is necessary when they are set sequentially. (Equations are given in Appendix B.)

Response Independence

We model advertising response as a function of a firm's own advertising spending. This practice is common as illustrated in most of the references cited before and is typical of many applications such as the Assessor (Silk and Urban 1978) and Defender (Hauser and Gaskin 1984) models. However, competitive advertising, say that of Colgate, will affect consideration of a firm's brand, say Crest. Our expositional assumption is based on the realization that in most cases a firm's own advertising has a greater effect on whether consumers consider that firm's brand relevant than does competitive advertising.

In a subsequent section we consider the case in which a firm's response function depends on competitive advertising and argue that the main qualitative results are not changed. A supplemental appendix (available from the authors) rederives all results for the more general case. (See also Appendix B.) We do not present it here because the technical conditions obscure the basic intuition.

Probabilistic Independence

We analyze explicitly the case in which the consideration probability of brand i is independent of whether brand j is considered. This model is in common use, say by Silk and Urban (1978). However, its use must be considered conditional on a market structure definition.

For example, consider the coffee market. Probabilistic independence is unlikely to apply to the entire market. Users of Maxwell House ground caffeinated coffee may be more likely to consider other ground caffeinated coffees as relevant than they are to consider instant decaffeinated coffees as relevant. However, the assumption might apply if we condition the analysis on the submarket of ground caffeinated coffees. Indeed, one technique to test market structure uses a related form of choice independence, the aggregate constant ratio model, to define substructures within markets (Urban, Johnson, and Hauser 1984). Our perspective is that the competitors of greatest interest are those in the same submarket; hence, to a first order, we believe probabilistic independence is relevant for this article. (Most applications of Assessor or Defender are preceded by at least a qualitative concern for market structure.)

To illustrate that probabilistic independence applies to at least one submarket, Table 1 reports data on relevant sets for plastic wraps. The data were available for the four largest brands. As the number of consumers considering zero brands is unknown, we normalized the data. Table 1 can be considered a chi squared contingency table where four variables (consideration of each brand) can take on one of two states (consider or not consider). The resulting chi squared statistic has 11 degrees of freedom, that is, 16 consideration sets minus four marginal

Table 1
TESTS OF INDEPENDENCE ASSUMPTION FOR PLASTIC WRAPS SUBMARKET

Relevant set number	*Brands considered*[a] *A*	*B*	*C*	*D*	*Predicted*[b] *number of consumers*	*Observed number of consumers*
1	1	1	1	1	8.9	6
2	1	1	1	0	27.3	22
3	1	1	0	1	2.9	4
4	1	1	0	0	8.9	11
5	1	0	1	1	14.1	9
6	1	0	1	0	43.4	46
7	1	0	0	1	4.6	5
8	1	0	0	0	14.1	9
9	0	1	1	1	12.4	11
10	0	1	1	0	38.3	35
11	0	1	0	1	4.6	6
12	0	1	0	0	12.4	20
13	0	0	1	1	19.8	26
14	0	0	1	0	60.9	70
15	0	0	0	1	6.4	6
16	0	0	0	0	No data	No data

[a]1 indicates the brand was considered and 0 indicates it was not. The identity of the major national brands is disguised for confidentiality. Brand D is the store brand.

[b]Prediction under the assumption of independence. For example, the predicted percentage for relevant set 1 is the product of the marginal consideration probabilities for brands A, B, C, and D. The marginal probabilities are determined from the data.

probabilities determined from the data, minus one degree of freedom for the normalization. As shown, the chi squared test does not reject the independence assumption at the .10 level. (Observed $\chi^2 = 14.8$, χ^2 [11 d.f.] = 17.3) A similar test in the automatic dishwashing detergent category also did not reject the assumption.

Focus on Long-Term Response

Advertising is a complex phenomenon that affects consumer response in myriad ways. Many analytical models have been proposed in marketing (see Blattberg and Jeuland 1981; Bultez and Naert 1975; Kalish 1985; Little 1979b; Lodish 1986; Mahajan, Muller, and Sharma 1984; Parsons 1975; Sasieni 1971; Simon 1982; Teng and Thompson 1983) and in economics (Dorfman and Steiner 1954; Gould 1970; Milgrom and Roberts 1986; Nelson 1970; Nerlove and Arrow 1962; Schmalensee 1978; Telser 1962; Vidale and Wolfe 1957). These models are often dynamic, concerned with the speed of response, decay, or carryover from one period to the next.

In our analysis, we are concerned with the long-term, steady-state response—that is, we model what happens if firms hold their spending constant long enough for sales to stabilize. In such a steady state the effect on sales of transient temporal phenomena such as carryover, speed of response, and forgetting dampens out and a constant sales level is obtained. For example, when spending is constant, the previous period's carryover does not vary. See Little (1979b) for discussion and illustrations with a variety of analytical models. For a general treatment, see Feinberg (1988) and Sasieni (1971).[1]

In this steady state of constant advertising spending we examine the relationship, the response function, between the steady-state sales and the stabilized advertising spending levels. We believe this focus on the long-term response is appropriate for strategic decisions involving competition. Future articles can extend the strategic insights to advertising tactics when markets are constantly in flux.

Decomposition of Advertising Effects

Advertising has many effects. Two of the most common are to influence consumers to consider brands and to influence consumers to purchase brands from the set of brands being considered. Though Nedungani (1988) has demonstrated in the laboratory that these effects can be separated, most real advertising campaigns contain elements of both forms of advertising. (He was able to modify consideration probabilities without affecting preference for condiments, fast food outlets, and alcohol mixers.) See also discussion by Ehrenberg (1988), who argues that much advertising is directed at awareness rather than positioning.

In our analyses we take the role of an engineer who separates forces into their components so that the forces can be understood. Following Hauser and Shugan (1983), Kumar and Sudharshan (1988), and Wernerfelt (1985), we analyze advertising through its effects on (1) consideration sets and (2) choice within consideration sets. Any real campaign can be built up from these components. Note that this is a logical distinction for the purpose of analysis. It does not require that two separate campaigns be developed or that the effects be separated. An actual campaign can exploit synergies between the purposes just as an engineer designs a single truss to support a bridge against vertical forces (gravity) and horizontal forces (wind).

For example, such logical decomposition is similar in spirit to that recommended by advertising models such as the hierarchy of effects (Lavidge and Steiner 1961) and AIDA (attention, interest, desire, action; Strong 1925), which have been used by advertisers and agencies for more than 60 years. See discussion by Aaker and Myers (1987, p. 105). It is also similar in spirit to that used by Arnold et al. (1987) to decompose the overall spending and the "quality" of advertising.

Hence, we focus on the component of advertising that affects consideration sets. We take as exogenous the component of advertising that affects choice within consideration sets. We illustrate how changes in the latter affect recommended spending on the former.

RELEVANT-SET/RESPONSE MODEL IN A DUOPOLY

We now use the assumptions of the preceding section to describe the relevant-set/response model. This model is used widely by major market research firms[2] and forms the basis of models such as Assessor (Silk and Urban 1978) and Defender (Hauser and Gaskin 1984).

The Relevant Set

The relevant-set/response model is applied when other variables, say price, positioning, or product features, are analyzed separately. A brand is said to be in a consumer's relevant set if it is evaluated seriously. For example, Silk and Urban (1978) operationalize a relevant set as those brands a consumer has used, has on hand at home, would seriously consider using, or would definitely not use. It is related to Howard and Sheth's (1969) concept of an evoked set, but it includes brands consumers have evaluated and rejected.

The relevant set is a much stronger requirement than awareness. For example, unaided awareness refers to those brands a consumer can name without prompting by an interviewer. Silk and Urban (1978) report that if 95% of

[1]In the case of time-varying but periodic strategies such as pulsing, we are concerned with the response to average spending rather than the details of how the budget is allocated over time.

[2]Personal communication with Karl Irons of SAMI/Burke, Stephen Needel of A. C. Nielsen & Co., and Steven Gaskin and Steven Cohen of Information Resources, Inc. (IRI) indicates a widespread use of response analysis within these, the world's first, third, and fourth largest market research firms. IRI and SAMI/Burke report an active use of relevant-set analysis. A. C. Nielsen is concerned mostly with the sales response.

the consumers are aware (unaided) of a brand, then it is in the relevant set of only about 50% of the consumers. At 70% unaided awareness, the relevant-set percentage drops to 10%. It disappears almost entirely if unaided awareness is below 60%.

There are many potential explanations for the relevant-set phenomenon. For example, we have offered a theory of the consideration-set formation process (Hauser and Wernerfelt 1989). For the purposes of this article we need only the empirical facts that (1) consumers consider a number of brands that may be more than one brand but less than the total number available and (2) the probability of consideration is a function of advertising. See, for example, Brown and Wildt (1987), Campbell (1969), Grønhaug (1973/74), Hauser (1978), Hauser, Urban, and Roberts (1983), Jarvis and Wilcox (1973), Silk and Urban (1978), and Urban (1975).

The Response Model

Response analysis is based on the notion that there is some relationship, a response function, that enables a manager to predict steady-state effects as a function of the advertising spending by both firms in the market. Such response functions are estimated by judgment, experimentation, and/or econometrics. There is much debate on how best to estimate response functions (see a review in Rao and Sabavala 1986), but there is little debate that such functions exist and are relevant.

The relevant-set/response model isolates the portion of advertising that affects the probability that a consumer will consider a brand as relevant. It usually is applied in conjunction with other procedures such as pre-test-market simulation in which other aspects of advertising, say positioning, are measured exogenously. These aspects are taken as given and the relevant-set/response model is used to decide how much a firm wants to invest in order to get consumers to consider its brand as relevant.

Basically, if firm i invests k_i dollars per annum on advertising, that firm's brand will be in the relevant set of $A_i(k_i)$ consumers, where $A_i(k_i)$ usually is stated as a fraction. (Our expositional assumptions of response independence and probabilistic independence enable us to represent $A_i(k_i)$ without explicitly noting any functional dependence on k_j or $A_j(k_j)$ for $i \neq j$. We consider dependence on k_j subsequently.)

Suppose firm 1 achieves A_1 as a relevant-set fraction and firm 2 achieves A_2. Then, as illustrated in Figure 1, A_1A_2 of the consumers consider both brands, $A_1(1 - A_2)$ consider just brand 1, $A_2(1 - A_1)$ consider just brand 2, and $(1 - A_1)(1 - A_2)$ consider neither. In other words, it is as though firm 1 competes in a product/price duopoly for A_1A_2 percent of the market and a product/price monopoly for $A_1(1 - A_2)$ percent of the market. Sales are a combination of these "duopoly" and "monopoly" sales.

In particular, if s_1 were the sales firm 1 would realize if both brands were in all consumers' relevant sets and S_1 were the sales firm 1 would realize if only brand 1 were in the relevant sets, the sales by firm 1 are given by

$$\text{sales of firm } 1 = A_1A_2\, s_1 + A_1(1 - A_2)S_1. \tag{1}$$

Sales of firm 2 are given by a similar equation.

The conditional sales, s_1 and S_1, are considered to be functions of other marketing and product design decisions.[3] Let p_1 and p_2 be the prices of brands 1 and 2, respectively, let c_1 and c_2 be the production costs, and let v_1 and v_2 represent other marketing and design variables such as positioning, advertising, promotion, and product features. Then s_1 is a function of p_1 and p_2 and of v_1 and v_2, whereas S_1 is a function of only p_1 and v_1. Without loss of generality, we define the v_j such that s_j and S_j are increasing functions of v_j. As long as the two brands are substitutable, $s_j < S_j$.

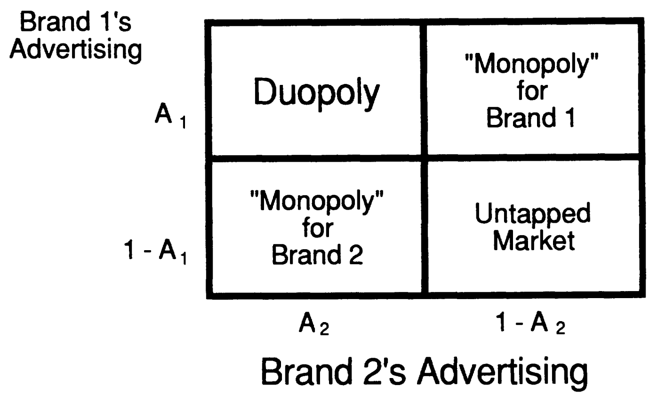
Under the stated conditions and definitions, the profit, π_1, of firm 1 is given by

$$\pi_1 = (p_1 - c_1)\{A_1(k_1)A_2(k_2)s_1(p_1,p_2,v_1,v_2) - A_1(k_1)[1 - A_2(k_2)]S_1(p_1,v_1)\} - k_1. \tag{2}$$

This, and the equivalent equation for π_2, is the relevant-set/response model. Note that the profit of brand 1 is explicitly dependent on the price and the relevant-set advertising of brand 2. As discussed in the preceding section, advertising is decomposed into its effect, k_1, on relevant sets and its effect, v_1, on choice within relevant sets. As the purpose of our article is to concentrate on the implications of the relevant-set/response model, we consider v_1 and v_2 to be set exogenously. Naturally, we consider what happens to the advertising and price equilibria when v_1 and v_2 are changed.

Because the products are considered to be differen-

Figure 1
ILLUSTRATION OF HOW CONSIDERATION ADVERTISING AFFECTS RELEVANT SETS



[3]One potential generalization would be additive errors associated with s_1 and S_1.

tiated, s_1 and S_1 are continuous functions of the prices. (In an undifferentiated market, the lower priced brand would capture all sales.)

Benchmark 1—Noncompetitive Analysis

To appreciate better the interactive nature of equation 2, consider a noncompetitive formulation in which the response function simply scales s_1, that is,

$$\text{(NC1)} \qquad \pi_1^{NC} = A_1(k_1)(p_1 - c_1)s_1 - k_1,$$

with a similar equation for brand 2. In this model the advertising response function tells a firm the fraction of consumers who will consider its brand, but the firm does not take into account the fact that competitive relevant-set advertising will affect sales among those consumers.

We do not propose NC1 as a model of firm behavior, but we do note that some recent published models (e.g., Hauser and Shugan 1983; Kumar and Sudharshan 1988; Rao and Sabavala 1986) use conditions similar to NC1. For our purposes, NC1 serves as a benchmark with which to compare the implications of the competitive analyses. (One might consider an even more restrictive benchmark: consideration-set advertising has no effect. That model is the same as NC1 with the exception that $A_1(k_1)$ is deleted. Naturally, it would imply that relevant-set advertising has no effect and that the price equilibria would be as in standard economic analysis.)

Some Technical Assumptions

To make equations 2 and NC1 empirically relevant, we must restrict the properties of the advertising and price response functions, $A(\bullet)$, $s(\bullet)$, and $S(\bullet)$. These technical assumptions are more than expositional; they endow the model with behavior that represents real markets.

Concave advertising response. Recall that we focus on the long-term effects of relevant-set advertising. We assume that such (steady-state) advertising spending increases the relevant-set proportion, but at a decreasing rate. Technically, this means that $A_j(k_j)$ is nondecreasing and strictly concave for both firms. Though this assumption may seem restrictive, as Little (1979a) and Lodish (1986) report S-shaped sales response functions, profit maximization implies that a firm should either operate on the concave portion of the response curve or not advertise at all. Our assumption implies simply that rational firms operate on the concave portion of the response curve.

Empirically, if an advertising response curve is concave, the advertising elasticity of the effect will be less than 1.0.[4] Though econometric studies typically report elasticities for sales rather than relevant sets, they do give us some insight on whether it is reasonable to expect elasticities less than 1.0.

In a summary of 128 econometric studies, Assmus, Farley, and Lehmann (1984) reported mean short-term elasticities of .22 with an ANOVA grand mean of .70. When the short-term mean was converted to a long-term elasticity, the result was .42. In a similar study of 37 European markets, Lambin (1976, p. 98) reported a long-term mean elasticity of .23 with all reported long-term elasticities less than 1.0. Thus, at least for sales elasticities, the data do not reject concavity.

Concave conditional profit functions. For the duopoly and monopoly price response we need not assume that s_1 and S_1 (s_2 and S_2) are concave. We use only the less restrictive assumption that firms consider prices in the ranges where the implied duopoly and monopoly functions, $\pi_{d1} = (p_1 - c_1)s_1$, $\pi_{m1} = (p_1 - c_1)S_1$, are concave. Such an assumption is reasonable and applies to a wide range of important markets. For example, we have shown (Hauser and Wernerfelt 1988) that the assumption applies to any market described by the Defender price and positioning model.[5]

Technical Conditions

One of the first steps in any decision support analysis or market simulation is to choose a family of parametric functional forms for modeling. For example, an analyst might choose an exponential form, a constant elasticity form, a quadratic form, or even a linear form. The analyst then estimates the parameters, runs the model, and provides managers with recommendations. There is some confidence in these predictions because the parameters are based on empirical data.

However, the seemingly innocent choice of a functional form may have implications for managerial action. One of the purposes of theoretical analysis is to identify conditions that are necessary and/or sufficient for certain actions. The analyst then can compare his or her modeling choices with the general conditions. For example, for symmetric firms, condition A1 in Appendix A holds for *all* exponential response functions, but for constant elasticity functions there is an empirical constraint related to the elasticity being less than .5.

We use two technical conditions in our results (stated in Appendix A for easy reference). Both relate to the relative impact on a firm of its own actions and those of competitors. The conditions require, in essence, that a firm's own actions influence marginal responsiveness more than do competitive actions. Technical readers will recognize the price condition as the standard economic theory price-equilibrium condition. The advertising condition is the direct analog.

[4]If A is the effect and k is the advertising spending, then for a nondecreasing strictly concave function with a non-negative intercept, it must be true that $\partial A/\partial k < A/k$. This relation implies that the elasticity is less than 1.0. Strict concavity is sufficient but not necessary for an elasticity below 1 because a linear curve, $A = a_0 + a_1 k$, has an elasticity of $a_1 k/(a_0 + a_1 k)$ which is less than 1.0 whenever a_0 and a_1 are positive. (We thank an anonymous reviewer for this insight.)

[5]The analytical proof is for uniformly distributed tastes. The Defender price response is quasiconcave. It is not concave for all prices, but the maximum profit always occurs on the concave portion of the curve.

COMPETITIVE REACTIONS

Common usage of response models is to estimate empirically the response functions and then choose the advertising (or price) level to maximize profit. This profit-maximizing action is the recommended managerial action.

We begin by deriving the optimality conditions for NC1. They are

$$\partial A_1(k_1^*)/\partial k_1 = [(p_1 - c_1)s_1)]^{-1}. \tag{NC2}$$

Because $A_1(\bullet)$ is a concave function, a smaller slope, $\partial A_1/\partial k_1$, implies a larger optimal advertising level, k_1^*. Hence, in this formulation, the larger the net dollar volume $(p_j - c_j)s_j$, the more a firm advertises. If the margins, $p_j - c_j$, are equal, the firm with the largest unadjusted unit sales, s_j, advertises more. Note that these recommendations are independent of the particular functional form or its parameters, as long as it is concave. In fact, this is just an example of the Dorfman-Steiner (1954) rule.

An equilibrium. In a competitive market this traditional analysis may not be appropriate. If firm 1 changes its advertising, firm 2 may respond. The advertising level that was optimal before firm 2's response may not be optimal after firm 2's response.

One way to extend the analysis to the competitive formulation, equation 2, is make firm 2's response endogenous and continue the analysis until an equilibrium is reached—that is, until the optimal advertising levels, k_1^* and k_2^*, are such that neither firm has any further incentive to change them. This equilibrium is called a Nash equilibrium.

This concept does not require that firms have knowledge of their competitors' advertising-response functions, budget levels, or profits. Firms act to maximize their own profits. Competitive actions affect these decisions because they affect a firm's own sales response to price and to advertising. Such responses are readily observable.

Conjectural variations. As we have described, each firm reacts to the current spending levels of competitors. This type of reaction is known as zero conjectural variations (ZCV). Econometric analysis (Gollop and Roberts 1979; Iwata 1974) suggests that it is a reasonable descriptive model. At minimum, it provides a first step beyond noncompetitive analyses.

An alternative to ZCV is for a firm to anticipate competitive reaction and act accordingly. We begin by analyzing the case of ZCV and then, in a following section, consider the implications of one or both firms anticipating the other. We show in that section that the advertising game is like a "prisoner's dilemma," that is, firms have unilateral incentives to increase advertising to levels higher than the levels that would maximize profits were they to collude.

THE ADVERTISING EQUILIBRIUM

Existence and Uniqueness

Before we analyze the equilibria implied by the relevant-set/response model, we want to know whether the market will stabilize or whether it will decay into an advertising war. If the equilibria exist, we want to know whether, for given values of p_j and v_j, the final set of advertising levels are unique. If the equilibria exist and are unique, independent of initial conditions, we can talk about the properties of the equilibria and, if necessary, find them numerically.

The following formal result addresses these issues. Formal proofs of this and all subsequent results are available from the authors. Sketches of the proofs are given in Appendix A.

> *Result 1.* For fixed prices and for fixed exogenous marketing variables, the advertising equilibrium exists and is unique.

Does Competitive Analysis Make a Difference?

The competitive formulation, equation 2, is clearly different from the noncompetitive formulation, equation NC1, but it is interesting only if it influences managerial recommendations. The following formal result shows that the formulation does make a difference and, furthermore, if one does not consider competitive reaction, managerial recommendations are consistently low. That is, a brand manager who uses the noncompetitive formulation will find that the market will consistently drive spending levels up in relation to those anticipated in a marketing plan.

> *Result 2.* The competitive formulation implies advertising spending greater than or equal to that implied by the noncompetitive formulation.

Empirical evidence for result 2 is difficult to interpret because published studies rarely separate advertising into its components. However, a meta-analysis by Aaker and Carman (1982) provides some evidence based on overall advertising. They examine 11 field studies. In 10 of those studies (91%), firms are observed to advertise more than a noncompetitive analysis would suggest. For 37 econometric studies, the results are less clear. Fifty-nine percent provided no information, 22% supported result 2, and 19% challenged result 2. Though the 19% challenging result 2 might imply that those firms are spending at the noncompetitive level, another hypothesis worth testing is that it is easier to infer steady-state response from field studies than from econometric studies.

How Do Exogenous Variables Influence the Advertising Equilibria?

The noncompetitive analysis represented by NC2 implies that optimal advertising levels, k^*, should increase whenever $(p_1 - c_1)s_1$ increases. As s_1 is an increasing function of v_1, this means that optimal advertising levels will increase whenever any exogenous variable for the target firm is set to a more favorable level. However, this advertising level is independent of competitive price or competitive marketing variables.

If the competitive formulation is to have face validity, it should generalize these simple insights. Optimal equilibrium advertising should be larger if a firm's exogenous variables are more favorable *and* if a competitor's exogenous variables are more favorable to the target firm, that is, less favorable to the competitor. The following result formalizes this insight.

> *Result 3*. The equilibrium advertising, k_1^*, increases if duopoly sales, s_1, monopoly sales, S_1, price, p_1, other marketing variables, v_1, or the competitor's costs, c_2, increase (while all else remains unchanged). It decreases if costs, c_1, the competitor's price, p_2, or the competitor's other marketing variables, v_2, increase.

Notice that result 3 depends only on rather weak conditions, notably assumption A1 and the assumption of concavity. It is true for all response functions satisfying these conditions. The implied qualitative changes can be implemented without specific knowledge of the parameters of the response functions.

We explicitly address price shortly. Here we caution the reader that result 3 indicates what happens if price changes, but sales do not. For example, result 3 applies if price changes but distribution incentives change simultaneously to maintain sales.

Should Small Share Brands Advertise More?

Suppose each firm faces the same response function and their margins, $p_j - c_j$, are equal. Suppose further that one of the two brands has a smaller market share among consumers who consider both brands. It might have this smaller share because the competitor has a better product, better positioning, or better distribution. Faced with this scenario, the smaller brand may decide to advertise more to counter the advantage of the competitor. Of course, such an aggressive move will beget aggression.

The advantage of such aggression turns out to be fleeting. Competitive reaction will overwhelm the temporary advantage and, in equilibrium, the smaller share firm will find it in its own best interests to advertise less than its competitor. We now formalize this intuition.

> *Result 4*. If greater duopoly sales ($s_1 > s_2$) implies greater or equal monopoly sales ($S_1 \geq S_2$), margins, $p_j - c_j$, are equal, and the same response functions apply to both firms, then the firm with the larger unadjusted market share, $m_j = s_j/(s_1 + s_2)$, advertises more in equilibrium.

We know of no systematic analysis of the relationship between market share and advertising; however, Clarke (1987) reports share and advertising levels for hair conditioners. For these data, the Spearman rank order correlation is .60, suggesting a positive relationship. Keithahn (1978) reports share data for the brewing industry. Corresponding advertising data are available from *Advertising Age* (November 3, 1975 and October 9, 1978) and *Beverage Industry* (May 23, 1980). For this industry the Spearman correlations for 1969–1979 range from a low of .60 to a high of 1.0. Naturally, such results might have other explanations besides result 4.

Result 4 addresses the relationship between share and absolute (relevant-set) advertising. It leaves open the possibility that smaller share brands may have larger advertising-to-sales ratios.

If Advertising is More Cost Effective, Does One Spend More?

Suppose the product category becomes more important to consumers (e.g., high fiber cereals after a Surgeon General's report) or media prices suddenly change (e.g., the 1987 tax by the State of Florida; Agnew 1987). Then, for a given spending level, a brand can get into the relevant sets of more (or fewer) consumers. We would say that the cost effectiveness of advertising has changed. It is reasonable to wonder whether this change in effectiveness causes brands to spend more or less on advertising.

To model this phenomenon we write $A_j(k_j)$ as $A_j(k_j/K_j)$ where K_j parameterizes the cost (or effectiveness) of advertising. As advertising becomes more effective, K_j decreases and less spending is needed to achieve a given consideration percentage.

The competitive model in equation 2 gives us the mechanism with which to address the cost effectiveness question for any specific set of parameters and functional forms. Because the equilibrium exists and is unique, we can readily recompute the advertising levels for any change in effectiveness.

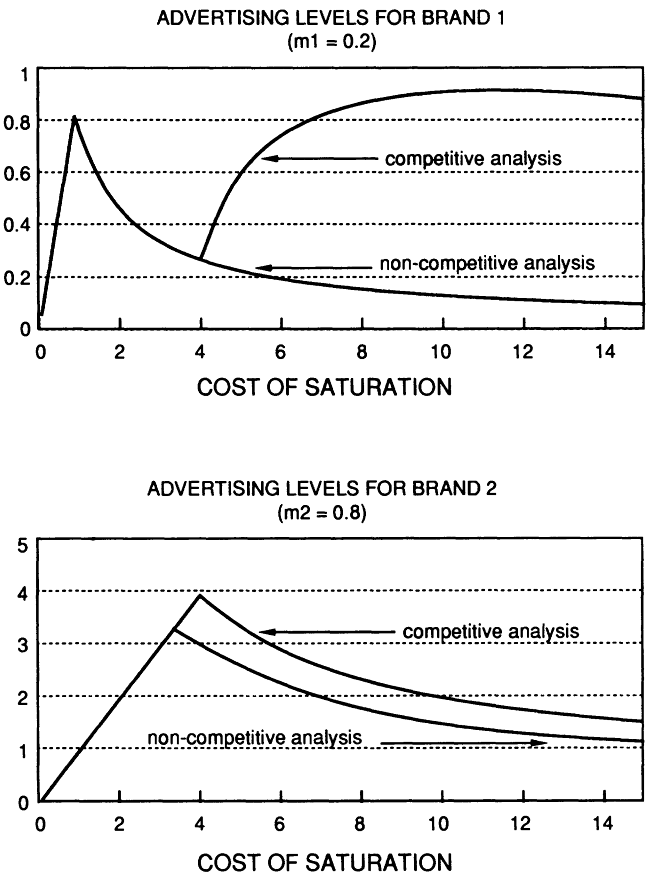
Interestingly, in general, advertising levels can either increase or decrease when effectiveness is changed, depending on current advertising levels and the exogenous parameters. In essence there are two conflicting forces: greater effectiveness gives more "bang for the buck" but it also means that less advertising is necessary for a given level of relevant-set consideration.

To give the reader a feel for how equilibrium advertising does change, we provide an "empirical" example. That is, we choose one common functional form, the constant elasticity model $A_j = (k_j/K_j)^a$, and we choose the parameter $a = .42$ on the basis of a published meta-analysis (Assmus, Farley, and Lehmann 1984).[6] This meta-analysis considers sales elasticities rather than relevant-set elasticities, but it does demonstrate that the differences between competitive and noncompetitive formulations can be large if relevant-set elasticities are of the same magnitude as sales elasticities. As a precaution we ran the analyses for other elasticities; the qualitative implications were the same. Finally, to maintain logical consistency with A_j as a percentage, we allow A_j to saturate for $k_j \geq K_j$.

The top of Figure 2 is a plot of advertising levels of brand 1, the smaller share brand ($m_1 = .2$), for both the competitive and the noncompetitive analyses. The bottom of Figure 2 is a plot of the advertising levels for brand 2, the larger share brand ($m_2 = .8$). (Recall that

[6]For illustration we set $p_1 - c_1 = p_2 - c_2 = 1.0$, $S_1 = S_2 = s_1 + s_2 = 10$, and $m_1 = .2$. These parameters scale the vertical axis, but do not change the qualitative message.

Figure 2
COMPARISON OF COMPETITIVE AND NONCOMPETITIVE ANALYSES FOR TWO BRANDS (illustration with saturating constant elasticity response function with elasticity = .42)



m_j are the unadjusted shares $s_j(s_1 + s_2)^{-1}$.)

First, compare the competitive and the noncompetitive analyses. In all cases the competitive analysis suggests higher (or equal) advertising levels. This is one illustration of result 2. Second, compare the advertising levels for brand 1 and brand 2. Brand 2, the larger share brand, advertises more. This is an illustration of result 4.

Now consider Figure 2A (brand 1). For the region with $K_j \leq 1.0$, both brands saturate at $k_j = K_j$. Around $K_j \approx 1.0$, brand 1 ceases to saturate. Brand 2 still finds it optimal to saturate; hence brand 1's increased advertising does not evoke retaliation. Both the competitive and the noncompetitive analyses suggest the same levels of advertising for this region.

Finally, around $K_j \approx 4.0$, the best strategy is for neither brand to saturate. In this region we see the difference between competitive and noncompetitive analyses. Brand 1's increased advertising evokes retaliation by brand 2, which by result 4 is at a higher level. The net result is that each firm reacts to the other, forcing advertising to even higher levels. Equilibrium advertising increases to levels almost nine times larger than a noncompetitive formulation would predict. In this region not only the quantitative recommendations, but also the quantitative implications, are different. (As the cost of saturation goes to infinity, the advertising levels of both brands ultimately approach zero.)

Figure 2 is not a pathological case. The constant elasticity functional form and the chosen value for the elasticity represent empirical practice. This example suggests that competitive considerations make a dramatic difference in managerial recommendations. At minimum, Figure 2 suggests that competitive considerations should be given close attention in practical applications.

We can summarize the implications of this example with the following formal result.

> *Result 5.* If the advertising becomes more effective (K_j decreases), equilibrium advertising may increase or decrease, depending on the exogenous conditions.

Empirically, one might be able to demonstrate the phenomenon of Figure 2. Though it is unlikely that a time series would have enough changes in cost effectiveness to observe the ups and downs of Figure 2, a cross-sectional analysis might reveal them. If a national advertiser advertises in several regions with varying cost effectiveness and if other variables can be controlled statistically, one might observe a pattern consistent with Figure 2.

Summary

Our five results indicate that competitive analysis can make a dramatic difference within the context of response analysis. We now examine how advertising affects the price equilibria.

DOES ADVERTISING AFFECT THE PRICE EQUILIBRIA?

Existence and Uniqueness

Before we discuss the price equilibria, we must establish that they exist and that they are unique (under reasonable conditions). It is well known in pricing theory that for a duopoly the price equilibrium exists and is unique if the profit functions are concave and if condition P1 (see Appendix A) holds. The following result is useful in our context because it shows that the same conditions are sufficient in the relevant-set model even though the price response interacts with the advertising response.

> *Result 6.* For fixed advertising levels, the price equilibrium exists and is unique.

On the basis of result 6, we can make statements about the price equilibria and, if condition P1 holds, we can find them easily via numerical methods.

Modified Price Elasticity

Empirically, models of unadjusted duopoly, $s_1(p_1,p_2)$, such as the Defender consumer model (Hauser and Shugan 1983), give estimates of elasticity that are very large, say 10.0 or more. However, when embedded in relevant-set/response analysis, such models predict well (Hauser and Gaskin 1984) and give reasonable elasticities. Similarly, Silk and Urban's (1978) model has been applied more than 1000 times. It appears to portray price response accurately when embedded in relevant-set/response analysis.

One explanation of empirical accuracy is that relevant-set/response analysis moderates price response. As shown in result 7, relevant-set/response analysis brings the effective elasticities of these models down to more moderate and realistic levels.

Let ϵ_{d1} be the duopoly price elasticity and let ϵ_{m1} be the monopoly price elasticity.[7]

Result 7. The effective elasticity, ϵ_{r1}, in the relevant-set/response model is a convex combination of the monopoly and the duopoly price elasticities. In particular,

$$\epsilon_{r1} = \frac{A_2 s_1 \epsilon_{d1} + (1 - A_2) S_1 \epsilon_{m1}}{A_2 s_1 + (1 - A_2) S_1}.$$

A similar equation applies to brand 2.

Result 7 states that the effective price elasticity in the relevant-set/response model will always be greater than (or equal to) the monopoly price elasticity and less than (or equal to) the duopoly price elasticity. For example, suppose that the Defender model, unadjusted for relevant sets, predicts a price elasticity of 10.0. Suppose further that s_1 is about one half of S_1, A_2 is about one half, and the monopoly price response is more or less inelastic. Then equation 3 would reduce the effective elasticity from 10.0 to approximately 2.5, a number that is much more reasonable. (For example, this value is well within the ranges reported by Lambin 1976, p. 103, in his meta-analysis of 37 econometric studies.)

We believe result 7 goes a long way toward explaining why models like Defender and Assessor work well in application even though they seem to overestimate the magnitudes of the price elasticities.

Result 7 indicates how relevant-set/response advertising affects price elasticity. It does not indicate unambiguously how an increase in total advertising affects the price elasticity. For example, on the basis of result 3, the advertising levels of the two brands are linked through competitive activity. Thus, if k_1 increases, so will k_2. This will cause A_2 to increase, which will increase the price elasticity. Thus, relevant-set advertising increases price elasticity. However, equation 3 represents only one component of the impact of advertising on price elasticity. In the relevant-set/response model, positioning advertising, a component of v_1, influences both s_1 and S_1. It could also affect ϵ_{r1} directly. Thus, the net effect of advertising is ambiguous because, empirically, advertising strategy can include both relevant-set and positioning advertising.

To measure the effect implicit in result 7, advertising field experiments must be run in such a manner that competitors can observe and react to an increase (decrease) in advertising spending. A further concern is that the researcher must measure the relevant-set and positioning components of an advertising test. We know of no published studies satisfying these criteria.

However, recognizing these caveats, we comment on three published empirical tests. Eskin and Baron (1977) report an advertising experiment in which price and advertising were varied across markets. Because it was an open, observable test, competitive brands could have responded. Eskin and Baron (1977, Table 3) found a greater price elasticity for higher advertising levels. If the advertising was relevant-set advertising (we do not know), these empirical observations are consistent with result 7.

Prasad and Ring (1976) compared the price response for matched panels (high and low advertising) within the same market. Though they found that advertising increased price elasticity, their experiment is mute with respect to result 7—competitive brands were unlikely to have responded differentially to the split panels.

Krishnamurthi and Raj (1985) also measured price sensitivity for matched panels with high and low advertising—competitive brands were unlikely to have responded. Interestingly, they measured price elasticity *within relevant sets;* hence their result—advertising decreases price elasticity—is best interpreted in terms of positioning theories. Positioning theories suggest that positioning advertising decreases price competition (e.g., Hauser 1988).

Together these experiments suggest that, with appropriate measurement, future experiments might systematically vary relevant-set and positioning advertising to resolve the otherwise ambiguous results on whether (and how) advertising increases or decreases price sensitivity.

Advertising Affects the Equilibrium Prices

If we compute the equilibrium prices for the noncompetitive formulation (equation NC1), we can easily show that they are independent of relevant-set advertising levels. In fact, Hauser and Shugan (1983, Theorem 6) and Kumar and Sudharshan (1988) formalize this result. On the basis of result 7, we expect that this independence will no longer hold in the competitive formulation. We expect that the relevant-set advertising levels could have profound effects on the equilibrium prices.

A priori, we might expect that the direction of the impact would depend on the parameters of the advertising and/or price response functions. That is, an analyst might be able to gather data, estimate the parameters of his or her model, and give recommendations to managers on whether to increase or decrease price. As result 8 shows, the impact of the analyst is more subtle.

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[7]That is, we define

$$\epsilon_{d1} = -(p_1/s_1)\,\partial s_1/\partial p_1,\ \epsilon_{m1} = -(p_1/S_1)\partial S_1/\partial p_1$$
$$\epsilon_{r1} = -(p_1/\text{Sales } 1)\,\partial(\text{Sales } 1)/\partial p_1$$

Result 8. An increase in brand 1's advertising spending, k_1, increases brand 1's equilibrium price, p_1^*, if and only if $\partial^2\pi_{d1}/\partial p_1\partial p_2 < 0$.

Result 8 suggests that qualitative recommendations may not depend on the specific parameters, but they do depend on the shape of the demand curve that the analyst chooses for data analysis. That is, the cross-derivative condition in result 8 changes sign for different, commonly used, data analysis models. For example, the analyst who chooses to use a linear model[8] will predict that an increase in advertising decreases the equilibrium price. The analyst who chooses a constant elasticity or exponential model will predict the opposite result, that an increase in advertising increases the equilibrium price. The direction does not depend on the specific estimated parameters.[9]

Result 8 provides critical information to the analyst. Its conditions indicate which functional forms give which qualitative recommendations. Armed with result 8, the analyst can make informed decisions and avoid recommendations that are driven by seemingly innocent choices in data analysis.

As result 9 shows, the impact of competitive advertising is less sensitive to the shape of the demand curve.

Result 9. An increase in brand 1's advertising spending, k_1, decreases brand 2's equilibrium price, p_2^*.

Intuitively, we interpret results 8 and 9 by recognizing that increased advertising (k_1) increases the price elasticity for brand 2 (result 7). This increased elasticity creates an incentive for brand 2 to lower its price; hence result 9. If $\partial^2\pi_{d1}/\partial p_1\partial p_2$ is negative, brand 1 has an incentive to increase its price. Result 8 states that these unidirectional shifts ultimately stabilize to an equilibrium with the price of brand 2 lower and the price of brand 1 higher than those observed before brand 1 increased its advertising.

Empirically, in markets where relevant-set advertising dominates, results 8 and 9 suggest that the correlation between price and advertising can be used to guide an analyst in the choice of the functional form with which to model price response. If observed advertising and price are correlated negatively, a linear demand curve may be appropriate. If they are correlated positively, an exponential or constant elasticity demand curve may be better.

We close with a result on how unadjusted market share affects equilibrium price. Result 10 is the price response analog to the question of whether the small share brand will be more or less aggressive, all else equal. The intuition is analogous to that of result 4.

Result 10. If $A_1 = A_2$ and $c_1 = c_2$, the brand that can achieve more sales for a given set of prices (i.e., $s_1(p,q) > s_2(q,p)$ and $S_1(p) > S_2(p)$) will choose a lower or equal price in equilibrium.

GENERALIZATIONS

The 10 formal results just derived complete the basic analysis for the advertising and pricing equilibria when neither firm anticipates the other but they react in such a way that an equilibrium is reached. With the exception of anticipation (which we cover in the next section), we believe these 10 results provide the basic intuition for the implications of the relevant-set/response model.

To state results 1 through 10 in nontechnical language, we made several expositional assumptions. We now examine what happens when some of these assumptions are relaxed.

More Than Two Brands

Most real markets contain more than two competitive brands. Though the intuition developed in our 10 results seems to be robust, it is important that we demonstrate how this intuition generalizes to more than two brands. Under appropriate conditions all 10 results generalize, but as an example we consider the relationship between relevant-set advertising and price response. Result 7 says that the effective price elasticity is a convex combination of the monopoly and duopoly price elasticities. Result 11 shows that a similar result can be derived for an oligopoly. The effective elasticity is a convex combination of the n-firm elasticities.[10]

Result 11. Let there be n firms in the market and let V_{ij} be the number of consumers who consider exactly j brands, one of which is brand i. Let ϕ_{ij} be the sales of brand i per consumer for that group and let ϵ_{ij} be brand i's elasticity for those consumers. Then, the effective elasticity, ϵ_{ri}, for brand i is given by

$$\epsilon_{ri} = \frac{\sum_{j=1}^{n} V_{ij}\,\phi_{ij}\,\epsilon_{ij}}{\sum_{j=1}^{n} V_{ij}\,\phi_{ij}}. \tag{4}$$

Equation 4 is clearly an analog of equation 3.

What if Advertising and Price Are Set Simultaneously?

We have already seen that one brand's actions, say advertising, influence the other brand's reactions. We have also seen that changes in one variable, advertising

[8] By linear we mean $s_1 = \alpha - \beta_1 p_1 + \beta_2 p_2$, where $\beta_1, \beta_2 > 0$. By constant elasticity, we mean $S_1 = p_1^{-\epsilon} f(p_2)$; by exponential we mean $s_1 = \epsilon^{-\gamma p_1} f(p_2)$, where $f(\bullet) > 0$.

[9] To prove this conjecture, recognize that the monopoly price exceeds the duopoly price, hence the optimal price exceeds the duopoly price for strictly concave $A(\bullet)$. Thus $\partial\pi_{d1}/\partial p_1 < 0$ at optimality for the constant elasticity and exponential demands. The rest is by straightforward computation. Naturally, $\epsilon_{d1} > 1.0$.

[10] When the elasticity depends on the identity of the brands rather that just the number of alternative brands, the analytical results require more complex notation.

or price, influence the equilibrium of the other variable. In general, if both variables of both brands are set simultaneously, reactions (and hence equilibria) can become very complex.

Results 1 through 11 (and the corresponding intuition) can be generalized to the simultaneous case, but the technical conditions are complicated and difficult to interpret. This complexity is not surprising because the conditions must interrelate second-order interactions among four variables simultaneously. To illustrate the complexity of the simultaneous case, Appendix B gives sufficient conditions for uniqueness of the equilibria.

> *Result 12.* If advertising and price are set simultaneously, the Nash equilibrium exists and, when AP1 and AP2 hold, it is unique.

What if Advertising and Price Are Set Sequentially?

Brands can set advertising and pricing in ways other than simultaneously. For example, one might hypothesize that consumers adjust purchasing patterns within relevant sets faster than they adjust which brands to consider.[11] In this case results 1 through 11 are still useful. For example, for given advertising levels, the price equilibria are unique. Hence, we can solve for $\hat{p}_1(k_1,k_2)$ and $\hat{p}_2(k_1,k_2)$ the equilibrium prices as functions of advertising levels. We then would rewrite the relevant-set/response model, equation 2, substituting $\hat{p}_1(k_1,k_2)$ and $\hat{p}_2(k_1, k_2)$ for p_1 and p_2. The solution procedure would be as before, though conditions guaranteeing existence and uniqueness would be very complicated. Analytic intuition is difficult but simulation is straightforward. For example, Hauser (1988) reports simulation results suggesting that sequential analysis of relevant-set advertising and positioning does not change the intuition from that of one-at-a-time analysis.

Response Dependence: What if Competitive Advertising Affects the Response to a Firm's Own Advertising?

Intuitively, the fraction of consumers who consider brand 1 will depend on the advertising of brand 2 as well as the advertising of brand 1. Of course, the impact of brand 2's advertising on brand 1's should be, in some sense, less than that of brand 1's advertising. Hence, we should not be surprised if formal results require conditions on the cross impacts of advertising.

The results for the case of response independence are easy to interpret. A supplemental appendix, available from the authors, restates and proves these results for the case of response dependence. In that appendix we show that the qualitative implications of results 1 through 10 do not change when response independence is relaxed—which is reassuring. The common practice of ignoring this dependence seems to be justified in the sense that the directional results do not change. To give the reader a feel for the conditions on cross effects we state, in Appendix B, conditions that are sufficient for uniqueness of the advertising equilibrium.

Summary of Generalizations

To derive our basic results, we made several expositional assumptions. Relaxation of any of these assumptions clearly complicates the computations. Fortunately, the assumptions of duopoly, one variable at a time, and response independence do not seem to change the qualitative intuition. For probabilistic independence we restrict the analysis to the appropriate submarket.

DOES IT PAY TO ANTICIPATE?

Our final topic is whether firms should anticipate one another. Because anticipation of price response is covered elsewhere (e.g., Friedman 1977), we concentrate on the anticipation of advertising response. To illustrate the effect, we consider the case in which brands are otherwise symmetric, $s_1 = s_2$, $S_1 = S_2$, and $A_1(k_1) = A_2(k_2)$ whenever $k_1 = k_2$. We assume, of course, that $s_1 < S_1$.

Suppose brand 1 anticipates that a change of one unit of advertising spending will evoke a response of δ_1 units in competitive advertising. Brand 1 still would choose an advertising level to maximize profit, but in doing so brand 1 would take into account brand 2's response.[12] As before, firms react, re-react, re-re-react, etc., until the market stabilizes at some set of advertising levels. (Note that the zero conjectural variations assumption simply corresponds to $\delta_1 = \delta_2 = 0$.)

For comparison we let k_1^* and k_2^* continue to denote the equilibrium levels under zero conjectural variations. We let k_1^c and k_2^c denote the collusive (or cooperative) spending that would result if the two firms agreed to set advertising at a level that would maximize joint profits. Let k_1^δ and k_2^δ be the spending levels when firm 1 anticipates a response of δ units. We will make clear from the context the value of δ_2. Finally, let π_j^*, π_j^c, and Π_j^δ be the corresponding profits.

> *Result 13.* When $s_1 = s_2$, and $A_1(\bullet) = A_2(\bullet)$, each brand anticipates the other, anticipations are equal, and $0 < \delta_1 = \delta_2 < 1$, then greater anticipation means less advertising spending and greater profits. Full anticipation ($\delta_1 = 1$) implies spending and profits at the collusive level.

In symbols, result 13 implies that $k_1^c \leq k_1^\delta \leq k_1^*$ and $\pi_1^c \geq \pi_1^\delta \geq \pi_1^*$.

Thus, if the two brands have equal anticipations, it

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[11] Of course, one might make the opposite hypothesis. In this case all the arguments of the subsection would be reversed. Our purpose here is to demonstrate the technique rather than settle which hypothesis is appropriate. That remains an empirical question.

[12] Technically, brand 1 solves the first-order conditions,

$$d\pi_1/dk_1 = (\partial\pi_1/\partial k_1) + (\partial\pi_1/\partial k_2)\,(dk_2/dk_1) = 0,$$

where dk_2/dk_1 is assumed to equal δ_1.

pays to be "smart." The result on collusive profits should not surprise us because $\delta_j = 1$ implies that each brand anticipates that the other will match its advertising level.

Result 13 seems to imply that anticipation pays—but it counts on symmetry of anticipation. Result 14 addresses the question of what happens in a case of extreme asymmetry. Assume that only one brand anticipates the other.

> *Result 14*. When $s_1 = s_2$, $S_1 = S_2$, and $A_1(\cdot) = A_2(\cdot)$, brand 1 anticipates brand 2's response but brand 2 does not anticipate brand 1's response, and $0 \leq \delta_1 \leq 1$, then greater anticipation means (1) less advertising and less profit for brand 1 and (2) more advertising and more profit for brand 2.

Interestingly, if only one brand is "smart," it pays to be "dumb" (or perhaps just clever—it depends on one's viewpoint). When brand 1 unilaterally anticipates brand 2, brand 1 is worse off and brand 2 is better off.

At first glance, result 14 seems counterintuitive—why would competitive intelligence hurt a brand? Result 14 becomes more clear when we realize that the game is being played in the context of advertising and that the advertising game is like a prisoner's dilemma (PD). That is, joint cooperation means lower spending and higher profits, whereas unilateral cuts in spending decrease profits. Thus, joint anticipation is equivalent to greater cooperation—it helps—whereas unilateral anticipation is like unilateral attempts at cooperation—it may hurt. For more discussion of cooperation in PD-like games, see Axelrod (1984) or the so-called "folk theorems" of industrial organization economics (e.g., Jacquemin and Slade 1989).

Summary of Anticipation

Like competitive advertising itself, anticipation is complex. If both brands are "smart," both are better off, but anticipation can be dangerous. Unilateral anticipation can actually hurt profits.

SUMMARY AND FUTURE DIRECTIONS

Advertising response models are applied widely and have major effects on managerial decisions. Within the context of one advertising response model, we examine how managerial action might change when competitive response is taken into account.

The equilibria exist and, under reasonable conditions, are unique. Furthermore, most exogenous variables affect these equilibria in intuitive ways. That is, for many applications, analysts can use noncompetitive approximations and be confident of their qualitative (directional) recommendations. However, for some applications, analysts must make informed decisions on whether or not to model competitive reaction explicitly. For example, noncompetitive analyses will always underestimate advertising response in comparison with competitive analysis. When effectiveness changes, even the directional implications depend on whether competitive reactions are modeled.

We also show that relevant-set/response analysis has an important moderating effect on price elasticities—a moderating effect that may explain why models such as Assessor and Defender work well in application even though their theoretical unadjusted price elasticities are high. Furthermore, the predicted impact of advertising on optimal price depends critically on the seemingly innocent decision of which functional form is chosen in data analysis. Analysts should not proceed blindly. Indeed, in some instances result 8 might give some insight into the appropriate functional form.

Finally, when we consider anticipation (as well as modeling) of competitive response, we find a prisoner's-dilemma-like game. If both firms anticipate, both are better off, but unilateral anticipation makes a film worse off.

The main limitations of our study are discussed in the Expositional Assumptions section. Overall, we believe the next critical research step is to observe markets to see whether the advertising equilibria behave as predicted by results 1 through 14. Our discussions of the previously reported empirical studies relating to results 2, 4, 5, and 7 give guidance on how this research might proceed.

APPENDIX A
SKETCH OF PROOFS

The detailed proofs to results 1 through 14 are available from the authors. We state here the equilibria conditions that are used in our proofs. We also provide a brief, verbal description of the method of proof for each result.

Equilibria Conditions

(P1) $$|\partial^2\pi_{dj}/\partial p_j^2| > |\partial^2\pi_{dj}/\partial p_1 \partial p_2|$$

(A1) $$|\partial^2 A_j/\partial k_j^2| > |\partial A_j/\partial k_j|\, |\partial A_i/\partial k_i|$$

Methods of Proof

Result 1. To prove existence of the advertising equilibrium, we show that the profit function is concave and rely on Rosen's (1965) theorem 1. We prove uniqueness by showing that the Jacobian of the pseudogradient of the profit function is negative quasidefinite whenever P1 and A1 hold. Rosen's (1965) theorem 6 then is applied.

Result 2. We prove that the competitive formulation leads to greater or equal advertising by showing that $\partial A_1(k_1)/\partial k_1$ is smaller (or equal) for the competition formulation than for NC2. Because A_1 is concave, this means that the optimal advertising is larger or equal.

Result 3. We examine the impact of sales, price, cost, and other marketing variables with the implicit function theorem (Thomas 1966, p. 68). We establish the first-order conditions for the optimal advertising, k_1^* and k_2^*. These first-order conditions are implicit functions of the variable of interest, say s_1. We next differentiate both sides of both conditions with respect to the variable of interest—in this case, s_1—and solve the resulting simultaneous equations for dk_1^*/ds_1. We then use conditions such as $s_j < S_j$ and the fact that $A_j(\bullet)$ is nondecreasing and strictly concave to establish the sign of dk_1^*/ds_1. Etc.

Result 4. We prove that the firm with the larger unadjusted share advertises more by first using symmetry to establish that equal shares imply equal advertising. Result 3, $dk_1^*/ds_1 > 0$, then gives the result.

Result 5. We show that equilibrium advertising can either increase or decrease when advertising becomes more effective by using the implicit function theorem to solve for dk_1^*/dK. Alternatively, the example in Figure 2 establishes the result.

Result 6. Analogously to result 1, we prove existence and uniqueness of the price equilibria with Rosen's (1965) theorems 1 and 6.

Result 7. We derive the price elasticity by direct computation and substitution of the formulas for ϵ_{d1} and ϵ_{m1}.

Result 8. The effect of brand 1's advertising on the equilibrium prices is obtained by implicit differentiation of the first-order conditions for the price equilibria. Result 7 and $\partial^2\pi_{d1}/\partial p_1\partial p_2 < 0$ establish the sign of dp_1^*/dA_1, which in turn establishes the results.

Result 9. The method of proof is similar to that of result 8.

Result 10. We show that the "stronger" firm selects a lower price by first using the implicit function theorem to prove a more general result that if $s_1 = \beta s_2$, then $dp_1^*/d\beta \leq 0$. Result 10 is a corollary of the more general result.

Result 11. The *n*-firm effective price elasticity is obtained by direct computation as in result 7.

Result 12 (price and advertising set simultaneously). Existence follows from concavity of the profit function as proven in results 1 and 6. Uniqueness requires Banach's fixed-point theorem (Franklin 1980, p. 228), which says that a contraction mapping leads to a unique fixed point. We define a sup norm and use the implicit function theorem to derive sufficient conditions.

Results 13 and 14. For anticipation, the relevant first-order conditions are given in footnote 12. The values of dk_2/dk_1 and dk_1/dk_2 depend on the conditions of the results. Implicit differentiation gives $dk^\delta/d\delta$. The assumption of $\delta \leq 1$, $A < 1$, $s < S$, concavity, and A1 establish the sign of $dk^\delta/d\delta$ (result 13) and $dk_j^\delta/d\delta$ (result 14). We show that π is concave in k for the conditions of the theorem. The definitions of optimality then give the result. [Note: result 14 does not require symmetry].

APPENDIX B
MORE GENERAL CONDITIONS

Advertising and Price Set Simultaneously

$$\text{(AP1)} \quad A_2[|\partial^2\pi_{d1}/\partial p_1\partial p_2| + \partial^2\pi_{d1}/\partial p_1^2] + (1 - A_2)\partial^2\pi_{m1}/\partial p_1^2 + (\partial A_2/\partial k_2)|\partial\pi_{d1}/\partial p_1 - \partial\pi_{m1}/\partial p_1| < 0$$

$$\text{(AP2)} \quad (\partial A_1/\partial k_1)^2[A_2(\partial\pi_{d1}/\partial p_1) + (\partial A_2/\partial p_2)(\pi_{m1} - \pi_{d1})] + \partial^2 A_1/\partial k_1^2 < 0$$

Response Dependence

$$\text{(A}'\text{1)} \quad (\partial^2 A_i/\partial p_i^2)S_i < (S_i - s_i)[2(\partial A_i/\partial k_i)(\partial A_j/\partial k_i) + (\partial^2 A_i/\partial k_i^2)A_j + (\partial^2 A_j/\partial k_i^2)A_i], \; i = 1,2; j \neq i,$$

and

$$\text{(A}'\text{2)} \quad \partial^2\pi_i/\partial k_i^2 < \partial^2\pi_i/\partial k_i\partial k_j < 0, \qquad i = 1,2; j \neq i.$$

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