THE EXISTENCE AND NATURE OF MULTI-BUSINESS FIRMS: DOUBLE SPECIALIZATION AND NEIGHBORING BUSINESSES

By

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ABSTRACT

We propose a theory of the scope of the firm and offer supporting evidence. The theory suggests that multi-business firms exist because they allow better deployment of factors that, because of sub-additive bargaining costs, cannot be traded in fractions or rented for short periods. It predicts that the businesses making up multi-business firms are “neighbors” in two senses: their service needs are correlated and factors specialized to one business are also productive in the other. We then look at a sample of acquisitions and document three regularities. First, input intensities of targets change to more closely resemble those of their acquirers. Second, the performance of targets does not catch up to that of their acquirers. Third, acquirers tend to select targets in industries that are similar to their own.

JEL Codes: D02, D23
Key Words: Theory of the firm, multi-business firms
I. Introduction

Multi-business firms dominate the global economy and an understanding of them is an important component of the theory of the firm. Inspired by influential work in the management literature, we here develop a new explanation for the existence of these firms. The theory shows how multi-product firms, like markets, allow greater specialization. To see how, consider the maintenance needs of an apartment building. If the building is a stand-alone business, it probably employs a superintendent who performs minor repairs of several different kinds. A single building does not have large enough needs in any particular area to warrant the hiring of a specialized plumber, carpenter, etc. However, a landlord who owns several buildings could utilize such specialists on a full time basis by rotating them among his properties on an as-needed basis. Under suitable conditions, we show that this kind of specialization can be more efficient than that achieved if several one-building landlords engage in sequential contracting with different tradesmen through the market.

The theory implies, among other things, that targets, post-acquisition, change their input-intensities in the direction of their acquirers. At the same time, it also predicts that the target’s post-acquisition performance does not catch up to the pre-acquisition performance of the acquirer.

Let us now walk through the intuition behind the model and the hypotheses.

Intuition - labor inputs only

A worker’s productivity when performing a particular service for a particular business depends on the narrowness of his human capital and the match between it and the (service, business) pair in question. His productivity is higher the more narrowly his human capital is focused on the service, but lower if he has to perform a service outside his area of expertise. Analogously with the business; his productivity is higher the more narrowly his human capital is focused on the business, but lower if he has to work for someone else.

Everything else being equal, a worker would prefer his job description to be as narrow as possible in both the service and business domains. If he can perform the same service for the same business in every period, his human capital investments can exhibit the identical dual
specialization, he will be maximally productive, and he will make more money. However, since the services needed by an individual business change from period to period, workers often select mixed mode human capital investments to protect themselves from having to work outside their areas of expertise. For example, a worker who is an expert in a single service, but has general business skills, can follow the demand for that service from business to business, like e.g. an independent plumber. Conversely, if a worker is an expert on a single business but has general service skills, he can take a job performing odd services for the business in question, like e.g. a building superintendent.

Workers can, however, approximate dual specialization by taking advantage of business “neighborhoods”. This concept captures the observation that some pairs of businesses are more “similar” than others. There are two aspects of similarity: Business specialized human capital and services needed. First, the human capital used to work in some pairs of businesses are more similar to each other than to those required to work in other businesses. Second, some pairs of businesses tend to need some of the same services, while others need very different services. Continuing the example with apartment buildings, we can think about maintaining two 1920’s brownstones vs. one brownstone and a modern high-rise. Both the building specific information necessary and the typical problems are likely to be more similar in the two brownstones.

To see the role of neighborhoods in efficient production, consider a worker whose human capital is focused on a business neighborhood and an individual service. This worker may be able to spend most or all of his time providing the chosen service to businesses in the neighborhood. When these businesses are independently operated, they could be seen as a local market. More importantly, if all the neighborhood businesses are operated by a single entrepreneur and the workers are employees, they would constitute a multi-business firm.

To find conditions under which the scope of firms change, we need a theory of employment. We take this component of the model from Wernerfelt (1997, 2015) and model distortions brought about by bilateral price-determination by the reduced form assumption that a worker incurs bargaining costs if his per-service payment is negotiated with a single entrepreneur. The crucial assumption is that these costs are sub-additive in the breadth of the job description - the number of possible service/business pairs covered by the negotiation. For

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1 As shown in the Corollary in Section III, it turns out that we need both properties to make our argument.
example, if the contract gives the entrepreneur the right to ask for any one of $S$ services in a specific business, then the cost of negotiating it are less than $S$ times the cost of negotiating a price for a single specific service. (In other examples, the contract gives the entrepreneur the right to ask for a specific service in any of $B$ businesses, or any one of several service/business pairs.)

The sub-additivity is key because it is that which makes it meaningful to compare several alternative contracts: The parties may bargain over individual prices on an as-needed basis, they may eliminate all later bargaining by once-and-for-all agreeing on a price for which they would perform any service in any business the entrepreneur asks for, or they may negotiate over a price for any element of a set of service/business pairs with intermediate cardinality - hoping to avoid additional negotiations later on. We use the term “employee” to describe a worker who has agreed to follow orders in the sense that he will perform any of several service/business pairs “on demand” with no further negotiation, thus making adaptation costless. Employees are more efficient than market contractors if needs change frequently.

These ingredients enable the model to explain the existence of multi-business firms. Think again of a business neighborhood consisting of several apartment buildings and a number of workers who have human capital focused on the business neighborhood and individual services, such plumbing, carpentry, etc. No single building will need a plumber on any given day, but on many days, at least one of them will. For simplicity, assume exactly one. If the plumber is an independent contractor, he will negotiate a [one business, plumbing] contract every day. But if all the businesses are managed by the same entrepreneur, our plumber can negotiate an employment contract of the form [business neighborhood, plumbing]. Because of the sub-additivity, this may well be cheaper. So the model suggests that multi-business firms will be preferred over markets if needs change frequently and neighborhood effects are strong.

**Intuition – more broadly**

Taken literally, the model with labor inputs would appear to explain only very small multi-business firms. Large, high profile mergers are typically not justified by the desire to leverage the time of individual workers, but by other factors. Fortunately, we can readily modify the theory to apply more broadly. The extensions are mostly a matter of reinterpreting labels, though the theory in some cases predicts “ownership” rather than employment. This does not
require a major change in the forces driving the decision to expand: As pointed out by Grossman and Hart (1986), ownership confers the right to make residual use decisions by fiat. That is, just like employment, it allows low cost adaptations. So the theory posits that multi-business firms are built to share human and non-human factors for which the bargaining (transactions) costs involved in partial sale or short-term rental of would be large. Indeed, many merger announcements cite “synergies” from shared factors with exactly this characteristic. Examples include skills embedded in teams of employees, specialized machinery, brand names, etc.

To see how the neighborhood concept applies to these factors, consider brand names. You transfer a brand name if the new business needs branding and if the “meaning” of the name fits the new business. (A defense contractor has little need for a consumer brand. Among businesses that do, “Disney toys” is OK, while “McDonald’s shampoo” is not.) Similarly with new product development teams. You transfer the skills of an existing team to a new business if the new business needs new products and if the team’s skills apply. (A real estate broker has little need for new products. Among businesses that do, clothing designers might be OK in shoes, though probably not in software.)

Tests

Most tests of theories of multi-business firms are cross-sectional, comparing more or less diversified firms along different dimensions (e.g. Lang and Stulz, 1994; Montgomery and Wernerfelt, 1988; Wernerfelt and Montgomery, 1988). Aiming to extract more information, we will here look at what happens when one firm acquires another and thus becomes more diversified. Since the firms in our model decide on their scope from the beginning of the first period, it does not predict any changes of scope. So we have to make the usual appeal to an unanticipated shock, such that the pre-merger state is in disequilibrium, while the merger restores equilibrium. With this interpretation, the acquirer has excess capacity of a specialized factor in the disequilibrium state, and the merger is undertaken to make it possible to transfer this excess capacity to another business. We test three predictions.

First, suppose that the would-be acquirer has a team of employees that are good at advertising and that this team has more time on its hands. Prior to the merger, we would expect the acquirer to spend a lot of money on advertising (an input that is complementary to
advertising skills) and less on alternative ways to enhance revenues, such as R&D. After the merger, when the target gets access to the strong advertising skills, we would expect it to adopt the same expenditure pattern. So both input intensities (advertising-to-sales and R&D-to-sales) of the target should move towards those of the acquirer.

Second, and continuing with the same example, since the advertising skills were developed in the acquirer’s business, we would expect them to be slightly more productive there than in the target. So while the target’s performance should improve when it gets access to the advertising skills, it should not improve to the level of the acquirer’s. This means that, on a per-business basis, the performance of the merged firm falls short of the pre-merger performance of the acquirer.

Third, since the neighboring business has to be similar to that of the acquirer in terms of both the fit of the advertising skills and the returns to advertising in general, we should be able to make some broad predictions about the relationship between the two industries. In this context, the “closeness” between two industries is defined with respect to a specific factor and the neighborhood concept. So if we have even a rough measure of inter-industry distance “averaged” over individual factors, we would expect targets to be closer to their acquirers than predicted by chance.

**Literature**

It is useful to organize the literature review by first discussing our theory of the firm per se and then looking at theories of multi-business firms.

Our model is based on sub-additive bargaining costs and advantages of specialization. Sub-additive bargaining costs were first introduced by Wernerfelt (1997) and given a microfoundation in Wernerfelt (2015). Beyond that, the most closely related literature is the theory of the firm as proposed by Coase (1937) and the view of employment pioneered by Simon (1951). Coase talks about the costs of determining prices and Simon views the essence of employment as order-taking and introduces the idea that workers might be willing to comply if they are more or less indifferent between the possible orders. The primary driver in our model is not indifference, but the costs of negotiating at every turn. The model shares this focus on ex post adaptation with,
among others, Bajari and Tadelis (2001), Bolton and Rajan (2001), Matouschek (2004), and Dessein, Galeotti, and Santos (2016). However, none of these papers use the sub-additivity assumption to differentiate between employment and sequential contracting.

The idea that there are advantages of specialization and that markets make it possible originates with Adam Smith (1776) and is developed further by Stigler (1951). We here add to this literature by introducing the multi-business firm as another way of sustaining specialization.

Moving on the theories of multi-business firms, our model can be seen as a microfoundation for a lot of management literature, but in particular the “resource-based view”. The book by Penrose (1959) suggested that firms will diversify into industries that require human capital similar to that used in their existing industries, thereby allowing them to leverage excess capacity of managers. Later literature has generalized the argument to other factors of production, using terms like “resources” (Wernerfelt, 1984; Barney, 1991), “competencies” (Prahalad and Hamel, 1990), or “capabilities” (Kogut and Zander, 1992; Grant, 1996; and Teece, Pisano, and Shuen, 1997). It is an informal validation of the theory that this literature resonates deeply with managers – the actors that make real-world decisions about the scope of their firms.\(^2\)

Seen as a theory of multi-business firms, the management literature makes two key contributions: One is the suggestion that firms expand by entering businesses that are “related” (or “neighboring”) in the sense that the same factors apply and are important. The second is the claim that this expansion should be based on “hard to trade” factors. However, the latter point is made with reference to factors conveying competitive advantages only. The fact that mundane factors may be hard to trade as well is of little interest to this literature. Rather, the idea is that a resource only can be a source of competitive advantage if it is hard to trade, since competitors otherwise could imitate simply by buying the resource. In such cases, money can be made by trading, rather than by using, factors. We add to this literature in two ways. First, by making the simple point that excess capacity of all factors should be used if possible. Second, and much more importantly, by providing a micro-foundation for deciding whether a specific factor should be leveraged inside or outside the firm’s boundary.

\(^{2}\) As a rough gauge of the influence of this literature, the combined citation counts of the six above-mentioned papers exceed 190,000 (scholar.google.com, February, 2019)
In fairness, the strategy field has developed some qualitative elements of such a microfoundation in Rumelt (1974) and Teece (1980, 1982). Specifically, Rumelt (1974) defines a “core factor”, as a factor of production that is indivisible and subject to transactions costs. When individual markets are of limited size relative to the capacity of a core factor, we have “economies of scope” and the basis for multi-product firms. This theory is in many ways similar to ours except for two points. First, our factors do not have to be ontologically indivisible. What matters is that it is economically unattractive to sell or rent fractions of them. Secondly, transactions costs do not matter except in the way described above.

Plan of the paper

We describe an economy with labor as the only input in Section II. In Section III, we first analyze the labor model before generalizing it to other factors. We derive several intuitively appealing predictions in Section IV and test the theory in Section V. The paper concludes with a brief discussion in Section VI.

II. Model with Labor Input

Basic economic environment

We will think of a large economy. Businesses, operated by entrepreneurs, produce by using workers to perform services. Each business $b \in \Omega_B$ needs one service, $s'_b \in \Omega_S$, in each period $t$, and if a needed service $s$ is performed by the worker $w$, it results in $q_{bsw}$ units of output. Any worker can perform any service at zero cost, but only one per period and output cannot be expanded by using two workers or by performing an unneeded service. We assume that the number of services $S$ is smaller than the number of workers $W$, and that the latter is at least as large as the number of businesses $B$. (So some workers may be idle.)

The model covers two time periods, $t = 1, 2$, and $\delta > 0$ is the weight on second period payoffs, representing both the long run and the rate at which things change. (So it is possible, and

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3 We will define $q$ as quantity, but it can also be interpreted as quality. Alternatively, the model could be reformulated to focus on costs.
perhaps even natural, to think of $\delta > 1$. Most of the important action in the model will concern the way in which changes between periods are handled. We first analyze the cases in which each entrepreneur is constrained to operate one business, and then move on to consider the choice between one and two, and thus the attractiveness of multi-business firms.

It will be an important feature of the model that businesses come in neighborhoods such that each is in exactly one neighborhood. In a sense we will make precise below, neighboring businesses have some of the same needs and use “similar” human capital. We use $N(b)$ to denote the neighborhood of which $b$ is a member. Each business neighborhood is associated with one common service, $s^*_b$, which in any period is needed by one member of the neighborhood. We model this as follows:

**Assumption 1:** In any business neighborhood, $N(b) = \{b, b'\}$: $s^l_b = s^*_b$ and $s^l_{b'} = s^2_{b'} \neq s^*_b$ with probability $\rho/2$, $s^l_b = s^2_{b'} = s^*_b$ and $s^*_b \neq s^l_{b'} \neq s^2_{b'} \neq s^*_b$ with probability $1/2 - \rho/2$.

So neighborhoods consists of two businesses. In each period, exactly one of the two businesses needs the common service. With probability $\rho$, needs do not change between periods, such that the same business needs the common service in both periods. With probability $1 - \rho$, the two businesses need the common service in one period each and then need two different services in the other periods.

The idea behind the neighborhood construct is that businesses are different and that some are more similar than others. Assumption 1 guarantees that a worker can perform the same service in every period and still do all his work within a single neighborhood.\(^4\) This “double specialization” does, in turn, form the basis for multi-business firms.

Given Assumption 1, we can describe $s^l$ and $s^2$, the first and second period distributions of needs. In the first period, each business has a 50% chance of needing the common service associated with its neighborhood and an equal chance of needing any of the other $S - 1$ services.

\(^4\) Assumption 1 is obviously very strong. It rules out two interesting cases; that in which the members of a neighborhood sometimes do not need the common service, and that in which both of them sometimes need it in the same period. The first of these allows us to explain why employment relationships occasionally break down and the second could be used to introduce waiting time as an additional cost of in-house specialists. Unfortunately, both generalizations complicate the formulas and give rise to several new “cases”. So rather than obscuring the main message of the present paper, we leave them to future research.
Exactly one business in each neighborhood needs the common service in either period and therefore also in period 1. The second period distribution depends on the first period realizations in three ways. First, with probability \( \rho \), there will be no change in the needs within a neighborhood. Second, a business that did not need its neighborhood’s common service in period 1 will need it in period 2 with probability \( 1 - \rho \). Third, to keep the analysis uncluttered, we will assume that the number of businesses needing each service is the same in both periods. Beyond these constraints, second period needs are random draws.

\textit{Production costs}

After observing \( s' \), workers choose their \textit{human capital profiles} by costlessly acquiring more or less narrow business and service skills.\(^5\) In the service domain, the skills may be focused on an individual service or services in general. Similarly, in the business domain, the skills may be focused on an individual business or businesses in general. Because no misunderstanding should be possible, we use the subset of \( \Omega_B \times \Omega_S \) in which a worker is invested as shorthand for his human capital profile. So \( w \)’s human capital is summarized in his profile \( (h_{wB}, h_{wS}) \in \{\{b\}_{b \in \Omega_B}, \Omega_B \} \times \{\{s\}_{s \in \Omega_S}, \Omega_S \} \equiv H_B \times H_S \). To keep things simple, we assume that these investments are publicly observable.

A worker’s productivity is higher the more narrowly invested he is and lower as he works further from his area of expertise. Specifically, when \( w \) performs \( s \) for \( b \), his production per period is \( q_{bw} = q_{bw} + q_{sw} \) where \( q_{bw} \) depends on \( h_{wB} \) and the match between it and \( b \), while \( q_{sw} \) depends on \( h_{wS} \) the match between it and \( s \). Since the productivities have finite supports, we can maximize generality by defining them in terms of a few parameters: Both business and service components are \( 1 \) if you have specialized human capital and work in that exact area, they are \( q_B \) and \( q_S \), respectively, if you have general human capital, and the business component is \( q_B^* \) if you have human capital at the level of one business and works for the other business in its neighborhood. The productivities are finally \( q_{B}^- \) and \( q_{S}^- \) if you work beyond the neighborhood or the service in which you are specialized. As suggested by the notation and the idea of gains from focus and match, we posit that \( 0 < q_{B}^- < q_B < 1, q_{B}^- < q_B^* < 1, \) and \( 0 < q_{S}^- < q_S < 1 \).

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\(^5\) We do not model how this happens, it could be by education or experience (prior to period 1).
The notation is summarized in Tables 1 and 2 below.

**Table 1**

**Business Component of Productivity** ($q_{bw}$)

<table>
<thead>
<tr>
<th>$h_{wb} \setminus Business$</th>
<th>$b = b'$</th>
<th>$b \in N(b') \setminus b'$</th>
<th>$b \in \Omega_B \setminus N(b')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b'$</td>
<td>1</td>
<td>$q_b^*$</td>
<td>$q_b^-$</td>
</tr>
<tr>
<td>$\Omega_B$</td>
<td>$q_B$</td>
<td>$q_B$</td>
<td>$q_B$</td>
</tr>
</tbody>
</table>

**Table 2**

**Service Component of Productivity** ($q_{sw}$)

<table>
<thead>
<tr>
<th>$h_{ws} \setminus Service$</th>
<th>$s = s'$</th>
<th>$s \in \Omega_s \setminus s'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s'$</td>
<td>1</td>
<td>$q_s^-$</td>
</tr>
<tr>
<td>$\Omega_s$</td>
<td>$q_s$</td>
<td>$q_s$</td>
</tr>
</tbody>
</table>

Together with Assumption 1, the productivities described in Table 1 mean that business neighborhoods are defined by two properties: Needs are correlated and human capital is partially transferable from individual businesses to their neighbors. We will later show (in the Corollary in Section III) that both properties are necessary to explain the existence of multi-business firms.
Trading mechanisms

Trades are governed by mechanisms. If we think of markets as examples, the idea is that a mechanism provides a forum in which workers and businesses meet and arrive at a price for labor. More formally, each worker can enter one mechanism per period, and each business can be entered, by the entrepreneur who operates it, in one mechanism per period. A mechanism specifies two sets: \((m'_B, m'_S) \subseteq (\Omega_B \times \Omega_S)\). By entering any mechanism other than \((\emptyset, \emptyset)\), a worker agrees to perform any \(s \in m'_S\) for any \(b \in m'_B\) in exchange for a price determined by the number of workers and businesses who enter the mechanism. Similarly, entrepreneurs agree to choose one \(s \in m'_S\) for each \(b \in m'_B\) they operate and pay the price to any worker who performs it. Players enter \((\emptyset, \emptyset)\) in period 2 to indicate that they want to trade a service whose price has been agreed upon in period 1. (In this case there is no need to bargain over any more prices.) Before trading, businesses and workers are matched randomly within each mechanism.

Assumption 2: A mechanism produces agreement on exactly one unit price per period.

Since the players are risk neutral, this is a simple and natural way to eliminate complete contracts.

Assumption 3: Contracts are binding as long as both parties want to trade an included service.

Apart from being reasonable in our setting, this keeps the analysis simple by ruling out renegotiation. Another important implication is that any price agreed upon on period 1 binds the parties in period 2. That is, if the parties agreed to a price for a specific service in period 1 and find themselves wanting to trade that service in period 2, neither side can ask for renegotiation. Assumption 3 is necessary to make “employment-like” mechanisms viable. In these mechanisms, parties enter a blanket contract in period 1 and the worker thereby agrees to “take orders” without any further negotiation. That is, the worker will perform any service the entrepreneur asks for, as long as it is covered by the period 1 mechanism.
We do not have to specify how mechanisms work, but can simply make mild assumptions directly on their payoffs. We will say that a mechanism clears iff it is entered by the same number of workers and businesses.

Assumption 4: If a mechanism clears, the unit price gives all workers and entrepreneurs at least $\epsilon > 0$ net payoffs in all states and both sides get higher payoffs if workers are more productive.\(^6\)

The first part of Assumption 4, which we soon will make more precise, requires that the business valuation of each unit of output, $v$, is sufficiently high and that gains are shared. It serves to avoid situations in which workers want to quit after learning which service they are to perform for which business. The second part ensures that the choices maximizing the payoffs of individual players and those maximizing joint surplus are the same. This, quite reasonable, property makes the analysis much simpler.

Assumption 5: If a mechanism does not clear, players on the long side get zero payoff. Workers who do not enter any mechanism, and players who do not want to trade any service for which they have agreed on a price, get positive payoffs a bit below $\epsilon$ per period.

Together, Assumptions 4 and 5 imply:

**Lemma 0**: All mechanisms in both periods clear in all equilibria

It is useful to define three special mechanisms. A worker is an employee if he negotiates with a single entrepreneur in the first period and the contract gives the entrepreneur the right to choose one of several second period assignments for the worker (give an order) with no additional negotiation, such that $(m^2_B, m^2_S) = (\emptyset, \emptyset)$. If a worker enters $(m^t_B, m^t_S) = (b, s^t_b)$ in both periods, he is a contractor, and a he is a market worker if $(m^t_B, m^t_S) = (\Omega_B, s)$.

Reflecting the idea that larger markets are more efficient, participation in mechanisms is assumed to be costless, except when only one entrepreneur enters. In such cases, we make the non-standard assumption that the parties face costs of bilateral bargaining and that these are sub-additive in the number of items bargained over. Formally:

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\(^6\) The “net” refers to the bargaining costs that are introduced in Assumption 7 below.
**Assumption 6:** In mechanisms entered by more than one entrepreneur, neither side incurs any bargaining costs.

**Assumption 7:** In mechanisms entered by a single entrepreneur, both sides incur bargaining costs totaling $K(m')$ per worker, where $K(0) = 0$, and $K(\cdot)$ is weakly increasing and sub-additive.\(^7\)

While this is an unusual premise, it is not unreasonable: Most people prefer not to bargain, but if they have to, would rather bargain once over a $300 pie than 30 times over $10 pies. Consistent with this, Maciejovsky and Wernerfelt (2011) report on a laboratory experiment in which bargaining costs are found to be positive and sub-additive. In the context of the current paper, Assumption 7 makes it possible for the employment mechanism to be more efficient than sequential contracting, since their bargaining costs are $(1 + \delta)K(1)$ and $K(S)$, respectively.\(^8\) To keep the number of parameters down, we here posit that $K(1) < K(2) = K(S) < 2K(1)$.

It is not important how the bargaining costs are split between entrepreneurs and workers, but to keep things simple we assume that they split evenly. This means that Assumption 4 requires both unit prices and unit valuations minus unit prices to be at least $(e + K(S)/2)/(q_B + q_S)$.

Since bargaining costs are controversial, we will assume that they are “small”. This eliminates some unappealing equilibria in which players take on extra production costs simply to save on bargaining costs. Formally,

**Assumption 8:** $\min\{K(S), (1 + [1 - \rho]\delta)K(1)\} < (1 + \delta)v(1 - q_B)$.

We finally assume that $\delta[qs - \rho - (1 - \rho)q_S]$ and $\delta[q_B - \rho - (1 - \rho)q_B]$, the discounted values of the expected loss in second period service and business productivities for a worker who specializes in a single non-common service or a single business in order to do well in the first period with no regard to the second period consequences, are so large that workers find it

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\(^7\) A possible micro-foundation for Assumption 7 is given in Wernerfelt (2015, 2016) where prospective bargainers invest to get information about each other’s reservation prices (and thus improve their bargaining power).

\(^8\) The assumption was first introduced in Wernerfelt (1997).
unattractive to engage in such gambles. This gets us around a possible artifact of the two period setting and holds if $\delta$ is sufficiently large:

Assumption 9: $\delta > \text{Max}\{[1 - qs]/[qs - \rho - (1 - \rho)qs], [1 - qB]/[qB - \rho - (1 - \rho)qB]\}$

Sequence of events

We will evaluate the attractiveness of multi-business firms by comparing total surplus when entrepreneurs operate single businesses to that when they operate two.

0. A social planner allocates one or two businesses to every entrepreneur. The allocation is denoted $\Pi$. Workers are numbered from 1 to $W$.

1. Business needs for periods $I$, $s^I$, are realized and publicly observed.

2. Workers sequentially choose their human capital profiles with lower numbers going first $\Pi \times s^I \times (H_B \times H_S)^{w-1} \rightarrow H_B \times H_S$ and their choices are publicly observed as they make them.

3. Businesses simultaneously select mechanisms for period 1: $\Pi \times (H_B \times H_S)^W \times s^I \rightarrow \{0, 1\}^B \times \{0, 1\}^S$. These are publicly observed.

4. Workers sequentially decide whether to enter a mechanism and if so which: $\Pi \times s^I \times (H_B \times H_S)^W \times (\{0, 1\}^B \times \{0, 1\}^S)^B \times (\{0, 1\}^B \times \{0, 1\}^S)^{w-1} \rightarrow \{0, 1\}^B \times \{0, 1\}^S$. Their choices are publicly observed as they make them.

5. Businesses and workers are randomly matched within each mechanism and all workers perform the indicated services.

6. Business needs for periods $2$, $s^2$, are realized and publicly observed.

7. Businesses simultaneously select mechanisms for period 2: $\Pi \times M \times (H_B \times H_S)^W \times s^2 \rightarrow \{0, 1\}^B \times \{0, 1\}^S$, where $M$ is the set of possible first period mechanism choices by all players. The second period mechanism choices of businesses are publicly observed.\(^9\)

8. Workers sequentially decide whether to enter a mechanism and if so which: $\Pi \times M \times s^2 \times (H_B \times H_S)^W \times (\{0, 1\}^B \times \{0, 1\}^S)^B \times (\{0, 1\}^B \times \{0, 1\}^S)^{w-1} \rightarrow \{0, 1\}^B \times \{0, 1\}^S$. Their choices are publicly observed as they make them.

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\(^9\) Note that $(\{0\}^B, \{0\}^S) = (\emptyset, \emptyset)$. 

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9. Businesses and workers are randomly matched within each mechanism, all workers perform the indicated services, and all payoffs, net of any bargaining costs, are distributed.

Since there are a lot of equilibria, we will be looking for the most efficient subgame perfect equilibria.

III. Equilibria

We will first look at the model with labor inputs described in Section II and then generalize to other productive factors.

Analysis – labor inputs

We start by looking at single-business firms. This analysis mirrors that in Wernerfelt (2015) with the existence of business neighborhoods as an added feature. When businesses are very different in the sense that the value of business specific human capital is high or changes between businesses are expensive, the most efficient equilibria are those in which workers stay with the same business in both periods. The use of firms depend on the frequency with which adaptations are needed. If changes are frequent, employment is best, and otherwise sequential contracting. When services are very different, the most efficient equilibrium is that in which market workers stay with the same service in both periods. The existence of business neighborhoods creates a fourth, asymmetric, option in which workers specialize in two neighboring firms, taking advantage of the fact that one of them needs the common service in each period. We will soon see the same idea exploited in multi-business firms. Formally, we have

**LEMMA 1**: One of the following is the most efficient subgame perfect equilibrium when each entrepreneur operates exactly one business:

---

10 This is tested in Novak and Wernerfelt, 2012.
**Employment**: \((h_{wB}, h_{wS}) = (b, \Omega_S), (m^1_B, m^1_S) = (b, \Omega_S),\) and \((m^2_B, m^2_S) = (\emptyset, \emptyset),\) with (total two period per worker) net surplus \((1 + \delta)v(1 + q_S) - K(S)\)

**Sequential Contracting**: \((h_{wB}, h_{wS}) = (b, \Omega_S), (m^1_B, m^1_S) = (b, s^1_b)\) and \((m^2_B, m^2_S) = \left(b, s^2_b \right)\) if \(s^2_b \neq s^1_b\) and \((m^2_B, m^2_S) = (\emptyset, \emptyset)\) if \(s^2_b = s^1_b\). The expected net surplus is therefore \((1 + \delta)v(1 + q_S) - [1 + (1 - \rho)\delta]K(1)\).

**Global Market**: \((h_{wB}, h_{wS}) = (m^1_B, m^1_S) = (\Omega_B, S),\) with net surplus \((1 + \delta)v(q_B + 1)\)

**Local Market**: Label two neighboring businesses such that \(s^1_B = s^*_B\) and \(N(b) = \{b, b'\}\). One of the first \(B/2\) workers sets \((h_{wB}, h_{wS}) = (b, s^*_B), (m^1_B, m^1_S) = (b, s^*_B), (m^2_B, m^2_S) = (b, s^*_B)\) if \(s^2_B = s^*_B\) and \((m^2_B, m^2_S) = (\emptyset, \emptyset)\) if \(s^2_B = s^*_B\), while one of the second \(B/2\) workers sets \((h_{wB}, h_{wS}) = (b', \Omega_S), (m^1_B, m^1_S) = (b', s^1_b), (m^2_B, m^2_S) = (b, s^1_b)\) if \(s^2_b = s^*_b\) and \((m^2_B, m^2_S) = (\emptyset, \emptyset)\) if \(s^2_b = s^*_b\). Expected net surplus for the first worker will be \(v[2 + \delta(1 + \rho + \{1 - \rho\}q_B^*)] - [1 + (1 - \rho)\delta]K(1)\), while that for the second worker will be \(v[1 + q_S + \delta(1 - \rho)q_B^* + q_S)] - [1 + (1 - \rho)\delta]K(1)\).

**Hybrid Market**: Label two neighboring businesses such that \(s^1_B = s^*_B\) and \(N(b) = \{b, b'\}\). One of the first \(B/2\) workers becomes a market specialist, setting \((h_{wB}, h_{wS}) = (m^1_B, m^1_S) = (\emptyset, \emptyset)\), \(s^1_B = s^*_B\) in period 1, and working for another business needing \(s^1_B\) in period 2. Expected net surplus for the first worker is \(v[2 + \delta(1 + \rho + \{1 - \rho\}q_B^*)] - [1 + (1 - \rho)\delta]K(1)\), while that for the second worker is \((1 + \delta)v(q_B + 1)\).

**Proof**: See Appendix A

In both the Employment and Sequential Contracting equilibria, workers are fully specialized in the business dimension and narrowly specialized in the service dimension, resulting in per period productivity \(1 + q_S\). So the question is simply whether the bargaining costs in Employment or Sequential Contracting are lower. In the Global Market equilibrium, workers are fully specialized in the service dimension and generalists in the business dimension, resulting in per period productivity \(q_B + 1\). The comparison between market and non-market equilibria largely rests on the relative advantages of service versus business specialization.
In the Local Market equilibrium, the workers specialize in one business each, start with that and then serve it or its neighbor in period 2. One is almost doubly specialized and the other is a bit less efficient than under Sequential Contracting. In the Hybrid Market, one worker is again almost doubly specialized, while in each period a different market worker meets the need that is not $s^*_b$.

To develop some intuition for why the sets of strategies described in Lemma 1 are equilibria, note first that any single deviation at a mechanism choice stage would risk putting the deviator on the long side of a mechanism (and thus with zero payoff). Since entrepreneurs choose first and workers move sequentially, some deviations would be “corrected” by a later moving player. However, the latter could not do any better than simply restoring the equilibrium by making whatever choice the deviator “should” have made in the first place.

Because the first three equilibria are symmetric, it is impossible for any individual worker to find higher productivity in any other mechanism in either period. So there are no gains to be had. In the last two equilibria, the almost doubly specialized workers do better than the others serving the same neighborhood. So the first $B/2$ workers to select human capital will each specialize in one common need, while the last $B/2$ will select according to the less attractive role.

To see that these are subgame perfect equilibria, one has to ask two questions. First, given the human capital investments in each of them, are there any continuation equilibria in which workers can be even more productive? Second, are there any other human capital profiles for which that would be possible? We go through the various possibilities in the Appendix, but it should be clear that the only serious contenders involve significant amounts of specialization, leaving relatively few good alternatives to those postulated in Lemma 1.

We next look at the case in which each entrepreneur operates two businesses. Since it is obvious that the most efficient such firms consist of neighboring businesses, we will focus on that case. To facilitate comparison with the single business case, we express productivity on a two-period per worker basis.

**Lemma 2:** When each entrepreneur operates a pair of neighboring businesses, three new neighborhood subgame perfect equilibria appear. They are similar to, but dominate, the
Local and Hybrid Markets. In all three of these, one worker again chooses to “almost doubly specialize”, but now only negotiates once (with the owner of both the neighboring businesses). Depending on parameter values, the other becomes a second employee, a contractor, or a market worker. If we again label two neighboring businesses such that \( s'_b = s^*_b \) and \( N(b) = \{b, b'\} \), the new equilibria are:

**Multi-business with Dual Employment:** One of the first B/2 workers sets \((h_{WB}, h_{WS}) = (b, s^*_b), (m^1_B, m^1_S) = (N(b), s^*_b), (m^2_B, m^2_S) = (\emptyset, \emptyset)\), and one of the second B/2 workers sets \((h_{WB}, h_{WS}) = (b', \Omega_S), (m^1_B, m^1_S) = (N(b), \Omega_S), (m^2_B, m^2_S) = (\emptyset, \emptyset)\). Expected net surplus for the first worker is \[v[2 + \delta(\rho + \{1 - \rho\}q^*_B + 1)] - K(1),\] while that for the second worker is \[v[1 + q_S + \delta(\rho + \{1 - \rho\}q^*_B + q_S)] - K(S).\]

**Multi-business Employment with a Contractor:** One of the first B/2 workers sets \((h_{WB}, h_{WS}) = (b, s^*_b), (m^1_B, m^1_S) = (N(b), s^*_b), (m^2_B, m^2_S) = (\emptyset, \emptyset)\), and one of the second B/2 workers sets \((h_{WB}, h_{WS}) = (b', \Omega_S), (m^1_B, m^1_S) = (b', s^*_b), (m^2_B, m^2_S) = (b, s^*_b)\) if \( s^*_b = s^*_b \) and \((m^2_B, m^2_S) = (b', s^*_b)\) if \( s^*_b = s^*_b \). Expected net surplus for the first worker is \[v[2 + \delta(\rho + \{1 - \rho\}q^*_B + 1)] - K(1),\] while that for the second worker is \[v[1 + q_S + \delta(\rho + \{1 - \rho\}q^*_B + q_S)] - [1 + (1 - \rho)\delta]K(1).\]

**Multi-business Employment with Market Workers:** One of the first B/2 workers sets \((h_{WB}, h_{WS}) = (b, s^*_b), (m^1_B, m^1_S) = (N(b), s^*_b), (m^2_B, m^2_S) = (\emptyset, \emptyset)\). One of the second B/2 workers becomes a market specialist, setting \((h_{WB}, h_{WS}) = (m^1_B, m^1_S) = (\Omega_B, s^*_b), (m^2_B, m^2_S) = (b', \Omega_B)\), serving \( b' \) in period 1, and working for another business needing \( s'_b \) in period 2. Expected net surplus for the first worker is \[v[2 + \delta(\rho + \{1 - \rho\}q^*_B + 1)] - K(1),\] while that for the second worker is \((1 + \delta)v(q_B + 1).\)

**Proof:** See Appendix A

We can now easily evaluate the efficiency of multi-business firms by comparing the payoffs to the first B/2 players in Lemmas 1 and 2:

**PROPOSITION:** In the most efficient equilibria, any jointly operated businesses will be neighbors. Furthermore, entrepreneurs will operate two businesses in equilibrium iff

\[v[1 + \delta(\rho + \{1 - \rho\}q^*_B) - K(1)] > \text{Max}\{v(q_B, (1 + \delta)vq_B - (1 + \delta)\delta K(1), K(S))\}\]
The Proposition also allows us to see why multi-business firms only exist if neighborhoods have both of the two properties summarized below Table 2. Multi-Business firms are dominated by single business Employment or Sequential Contracting if neighboring businesses do not have correlated needs and by the Market if human capital does not degrade more outside than inside business neighborhoods (such that \( q_B^* = q_B \)).

**COROLLARY:** Only if neighboring businesses have correlated needs and \( q_B < q_B^* \), can the net surplus of Multi-Business Firms be higher than the highest of the single business Employment, Sequential Contracting, and the Global market.

**Proof:** See Appendix A

**Extensions**

To apply the model to the large multi-business firms of today’s economy, we need to allow for factors other than individual workers. Fortunately, the model extends quite readily in several directions.

The most immediate generalization is to the case in which production requires several workers such that there are complementarities between them. Consider, for example, the extreme case in which it takes two workers to perform each service. In such a model, mechanisms clear when twice as many workers as businesses enter and we would define payoffs relative to that benchmark. If we use “\( w \)” to denote a pair of workers with adjacent numbers, the model would not need to change – as the two workers would have clear incentives to act in unison. At least in our model, no individual can credibly threaten to leave the team. It is worth noting that this category includes often cited intangibles such as “having a team that is good at” something.

Consider next the case of rival non-human inputs, such as time on machines.\(^\text{11}\) In this case we need to reinterpret the model such that \( w \) is a machine that, if operated for one period by

\(^{11}\) A good example is large earth moving equipment. Construction companies often weigh whether to buy or rent such assets.
a business, can produce output. Each machine is initially owned by a separate “capitalist”, who can rent it out for one period at a time or sell it to an entrepreneur. There are no bargaining costs if the machine is offered to two or more entrepreneurs, but bargaining costs are $K(I)$ if it is offered for rental to a single entrepreneur and $K(2)$ if it is sold. Prior to the start of the game, capitalists can, at no cost, customize their machines to an individual business or a business neighborhood, but they can also just leave it unspecialized in the business domain. At the same time, they can customize the machine to an individual service, customize it to a service neighborhood or leave it as a general purpose machine in the service domain. Together, all these very natural assumptions map the machine interpretation perfectly into the model. So we get an analog of the Proposition with Ownership replacing Employment.\(^\text{12}\)

Consider finally partially rival non-human factors, such as information, other IP, or brand names.\(^\text{13}\) Assume that each of these factors can meet the, possibly different, needs of two businesses per period and that they initially are owned by capitalists who can rent them out on a period-by-period basis or sell them to entrepreneurs. The bargaining costs from rental or sales are twice as high as in the rival case above. That is, it costs $2K(I)$ to rent for one period to two specific businesses and it costs $2K(2)$ to sell the factor. Assume further that each entrepreneur initially operates a “mini neighborhood” consisting of two (for our purposes) very similar businesses and that two of these in turn make up a “mega neighborhoods” consisting of four of slightly less similar businesses. Suppose further that a factor can be customized to a mini neighborhood or left unspecialized in the business domain. Similarly, we assume that a factor can be customized to perform a single service or be left unspecialized. This is now perfectly analogous to our worker model except that entry of factors will stop when $W/2 \geq B$.

So multi-business firms are created to more efficiently utilize a broad range of productive factors which satisfy two conditions; one having to do with bargaining cum transactions costs and the other having to do with factor capacity and business neighborhoods. The two conditions are: \((C1)\) The factor is statically and dynamically indivisible in the sense that it is uneconomical

\(^{12}\) Note that a worker can be seen as the owner of his labor and that the advantage of employment is that it gives control rights to the entrepreneur such that all adaptations can be costlessly made by fiat.

\(^{13}\) In spite of their common use as examples of the term, not even these factors are truly non-rival since each of them only can be used in a possibly large, but still finite set of businesses.
to trade a fraction of it or subject it to part time rentals (i. e. for less than a full period).\textsuperscript{14} (C2)
When put to its best use on a full-time basis, it can serve more than one business, either simultaneously or by frequently switching back and forth. The intuition for the general case is the same as with labor; the firm leverages specialized capacity by entering a neighboring business because it is uneconomical to sell or rent out part of this capacity.

The fact that the model extends to non-human factors has important implications for the definition of the scope of the firm. If labor is the only factor, it is natural to define the scope of the firm by its employees. But since ownership plays the same role for other productive assets, we will use the following

\textbf{Definition: }The scope of the firm is given by the human and non-human factors of production it controls by virtue of employment contracts or ownership.

IV. Predictions

Let us now look at several predictions of our theory and discuss a bit of anecdotal evidence. (Some large sample tests of these are presented in Section V.)

(i) \textit{If a firm with excess capacity of a specialized factor acquires a target, the input mix of the latter will change in the direction of the former.}

This prediction depends on two innocuous extensions of the model (which we wanted to keep as uncluttered as possible). First, the model would not change if we assumed that some services require costless non-labor complementary inputs that vary with productivity, while others do not. As an example, doing more carpentry requires more wood, while digging a ditch requires one shovel regardless of how many yards are dug in a day. Second, we could generalize the model to allow for some services to be substitutes, such that the value of one would decrease if more is done of the other. For example, a firm might value welding less if it has a very productive carpenter.

\textsuperscript{14} As is known from the literature, the bargaining cum transactions costs may be very high in particular for some non-rival inputs as well as for teams of employees.
Before starting to think about mergers, we hasten to admit that our model leaves no room for them. Multi-business firms are created at the start of the game and there are no changes in scope during either period. However, if an exogenous change caused a disequilibrium situation in which firms found themselves with excess capacity of a specialized factor, they would expand their scope to utilize it. (Since we observe mergers and changes in scope in the “real world”, it is hard to avoid appeals to such shocks.)

Given this preamble, we will now derive Prediction (i) in the context of two examples. First, and very close to the model, suppose that a two-business firm has a specialized carpenter and uses a generalist for welding. Compared to other two-business firm that use generalists to perform both services, the would-be acquirer will consume more wood and less metal. We assume that these workers perform services, such as ditch digging, that do not require quantity dependent complementary inputs, in those pre-merger periods when the business needs neither carpentry nor welding. We further assume that the target, another two-business firm, uses generalists for both carpentry and welding, and therefore consumes less wood and more metal than the would-be acquirer. A merger changes nothing for the acquirer, it still uses a specialist carpenter, a generalist welder, more wood, and less metal. The target, however, gets access to the specialist and will therefore use more wood and less metal, changing its input mix to more closely resemble that of the acquirer. Second, and closer to the empirical study, if a firm is good at advertising, it will spend more money on advertising and less on alternative ways to enhance revenues, such as R&D. After an acquisition, the target firm will want to take advantage of the advertising skills and do more advertising and less R&D.

The prediction is consistent with the finding of Atalay, Hortacsu, and Syverson (2014) who report that newly acquired firms change their product-market mixes to more closely resemble those of their acquirers.

It is easy to find anecdotal examples of cross-industry combinations that could be interpreted as leverage of (often intangible) factors with excess capacity. Brand names: Fender Musical Instruments Corporation making Fender electric guitars and Fender amplifiers, and Coca Cola makes Coke and Diet Coke. Relationships: Procter and Gamble making a host of consumer goods sold by the same retailers, and large pharmaceutical firms leverage their access to MD’s
over several drugs, *Know how*: Emerson Electric makes many products with small electric motors, and Novo makes a number of insulin products.

In Section V, we will look at a sample of acquisitions to offer a rather direct test of the prediction *(i)*.

**(ii) Multi-business firms experience decreasing profit rates as they diversify more.**

Multi-business firms in our model use labor that is specialized at the neighborhood level. However, if a firm, prior to one of the shocks discussed above, was active in a single business, its labor was specialized to that business. Getting access to this factor benefits the target, but it is a bit less valuable for the target than for the acquirer \( q_B^* \text{ vs } 1 \text{ in the model} \). The post-merger per-business profit rate of the combined firm will therefore be lower than the acquirer’s pre-merger profit rate.

It is important to make clear that we are predicting falling per-business profit rates as a firm diversifies more. This is not inconsistent with total profits going up. However, it is inconsistent with the often proposed idea that the profits of both businesses should go up because fixed costs can be spread over both more units.

The prediction is consistent with the “diversification discount” found by Lang and Stulz (1994), Montgomery and Wernerfelt (1988), and Wernerfelt and Montgomery (1988). While those tests are cross-sectional, we will, in Section V, test it more directly by looking at time-series data on a sample of mergers. Specifically, we will show that the post-merger profit rates remain below the pre-acquisition profit rates of the acquirers.

**(iii) Multi-business firms combine similar businesses.**

This follows from the result that targets and acquirers come from the same neighborhood. On the input side, Alfaro, Antras, Chor, and Conconi (2019), Montgomery and Hariharan (1991), and Neffke and Henning (2013) report strong supporting evidence from three different angles. Tests from the output side have, to the best of our knowledge, this has not yet been undertaken. This is important if some mergers are motivated by factors not captured in the input-output
tables. We will therefore, in Section V, show that acquirers systematically select targets that compete in “nearby” SIC codes.

(iv) Multi-business firms can hire specialized employees to perform services that single-business firms get from the market or employees with broader job descriptions.

We are not aware of any systematic studies of this, but the prediction is consistent with many stylized facts: Only bigger firms hire corporate counsels, landlords only hire their own maintenance crews (plumbers, electricians, etc.) once they have a large number of properties, firms transition from independent sales reps to their own sales force when sales are sufficiently strong, and the marketing and sales functions are one and the same in the typical start-up.

As promised, we will now report on new and more direct tests of (i), (ii), and (iii).

V. Tests

Hypotheses

As promised in Section IV, we test Prediction (i) - that the input mix of targets change to closer resemble that of their acquirers - by looking at changes in advertising and R&D spending following acquisitions. As briefly explained above, we assume that advertising and R&D skills are substitutes\(^\text{15}\), that advertising spending and R&D spending are complements to them, and that only the acquirer has one or more skills in excess capacity.\(^\text{16}\) If the acquirer, prior to a merger, used an input more intensely than a target, we could conjecture that it had excess capacity and that the target would increase its use after the merger. Conversely, if the acquirer used a factor less intensely, the conjecture would be that another, perhaps not measured, factor is present in excess capacity. In such cases, we would expect the target to start using more of the latter input as well, thus reducing its use of the former. So we do not need to know which factor is in excess capacity, and we do not need to measure it. Using advertising-to-sales and R&D-to-sales ratios to measure input intensities, we arrive at

\(^{15}\) The predictions for complementary factors would be different.

\(^{16}\) We just need to assume that the excess capacity more often rests with the acquirer than with the target. We provide evidence consistent with this in Table 6.
**Hypothesis (i):** Post-acquisition, targets change their advertising-to-sales and R&D-to-sales ratios in the direction of their acquirers.\(^{17}\)

We test also test Prediction (ii) – that firms, when they expand their scope, experience decreasing profit rates - on the same sample of acquisitions. Once again, the idea is that the factor in excess capacity is more valuable in the original business of the acquirer and thus adds less value to the target. So we have

**Hypothesis (ii):** The post-acquisition profit per business is below the pre-acquisition profit rate of the acquirer.

We finally test Prediction (iii) by looking at a measure of the distance between targets and acquirers. In our model, the “distance” between two businesses varies with factors. However, for a given factor, distance depends on the extent to which the factor is needed in both businesses as well as the degree to which it can be productive in both of them. There is no reason to believe that the distance between two businesses should be the same for all factors and certainly no reason to believe that it should be equal to the distance between the SIC codes of these businesses. However, SIC distance is not a terrible measure. In particular, SIC proximity may capture relatedness based on certain intangible factors that are excluded from input-output tables. Businesses in very distant SIC codes (e.g. different 2-digit codes) seem to have very few factors in common, while businesses in neighboring codes often seem to be very “similar” in the everyday language use of the term.

**Hypothesis (iii):** The SIC codes of targets are closer to those of acquirers than would happen if acquirers and targets were matched randomly

**Data**

Almost all previous empirical studies of the various theories of multi-business firms (e.g. Lang and Stulz, 1994; Montgomery and Wernerfelt, 1988; and Wernerfelt and Montgomery, 1988) have been focused on past decisions – cross-sectional studies working off the profiles of firms that are more or less diversified. We will look at acquisitions and thus current decisions.

\(^{17}\) It might be better to measure input intensities by the corresponding output elasticities. We do, however, not have enough data to do so.
To the extent that things happen after a firm has diversified, our data should be less polluted than those in the earlier studies.

The *SDC Platinum Mergers & Acquisitions* database gave us all transactions announced in 2012 and 2013. Since we intend the theory to explain mergers motivated by operational “synergies”, we required both parties to be US operating firms. This also led us to exclude mergers between pharmaceutical firms, since those often are motivated by patents, rather than operational concerns. In order to make sure that the effect sizes are measurable, we finally excluded acquisitions in which the acquirer’s sales were more than 100x those of the target, and retained only cases for which the *Capital IQ* database gave us financial statements both firms prior to and after the acquisition. These conditions were met by a total of 86 mergers or acquisitions.\(^{18}\) Many of these acquisitions have received extensive press coverage. The sample includes, for example, Avis’ acquisition of Zipcar, Office Depot’s acquisition of Office Max, Hanes’ acquisition of Maidenform, and Starbucks’ acquisition of Teavana.

For each of the 172 original firms, we classified them as a target (t) or an acquirer (ac) based on information from the merger announcements. We then collected SIC codes, sales \((S_t, S_{ac})\), growth rates, advertising expenses \((A_t, A_{ac})\), R&D expenses \((R_t, R_{ac})\), and various measures of performance, including gross profits \((P_t, P_{ac})\), in the year immediately prior to the announcement. For the 86 merged (m) firms, we collected data on sales \((S_{m\tau})\), advertising \((A_{m\tau})\), R&D \((R_{m\tau})\), and gross profits \((P_{m\tau})\), in the first two years \((\tau = 1, 2)\) after the merger.\(^{19}\) Descriptive statistics and correlations are given in Tables 3 and 4 below.

\(^{18}\) In all 86 transactions, at least one party was publicly traded.

\(^{19}\) So for 2012 (2013) mergers, \(\tau = 1\) denotes 2013 (2014), etc.
Table 3

Descriptive Statistics

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Table 4

Correlation Matrix

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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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</tr>
<tr>
<td>$P_t$</td>
<td>.992</td>
<td>.987</td>
<td>.705</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
<tr>
<td>$S_{ac}$</td>
<td>.723</td>
<td>.692</td>
<td>.248</td>
<td>.687</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$A_{ac}$</td>
<td>.839</td>
<td>.846</td>
<td>-.145</td>
<td>.825</td>
<td>.820</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$R_{ac}$</td>
<td>.020</td>
<td>-.062</td>
<td>.194</td>
<td>.091</td>
<td>.904</td>
<td>.671</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$P_{ac}$</td>
<td>.924</td>
<td>.921</td>
<td>.262</td>
<td>.917</td>
<td>.899</td>
<td>.926</td>
<td>.834</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$S_{m}^1$</td>
<td>.720</td>
<td>.683</td>
<td>.3296</td>
<td>.679</td>
<td>.998</td>
<td>.808</td>
<td>.883</td>
<td>.890</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$A_{m}^1$</td>
<td>.823</td>
<td>.837</td>
<td>-.134</td>
<td>.812</td>
<td>.816</td>
<td>.992</td>
<td>.597</td>
<td>.993</td>
<td>.806</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$R_{m}^1$</td>
<td>.047</td>
<td>-.031</td>
<td>.275</td>
<td>.158</td>
<td>.904</td>
<td>.657</td>
<td>.993</td>
<td>.845</td>
<td>.888</td>
<td>.570</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$P_{m}^1$</td>
<td>.925</td>
<td>.922</td>
<td>.347</td>
<td>.916</td>
<td>.903</td>
<td>.922</td>
<td>.834</td>
<td>.999</td>
<td>.896</td>
<td>.911</td>
<td>.852</td>
<td>1</td>
</tr>
</tbody>
</table>

The fact that the correlation between the pre-acquisition advertising intensities (.846) is so large, is not surprising in light of Montgomery and Hariharan (1991). Rather, the surprising result is that the correlation between the pre-acquisition R&D intensities (.194) is so low.
Tests

To test Hypothesis \((i)\), we define the “predicted” post-acquisition advertising-to-sales and R&D-to-sales ratios as the sales weighted averages of the firms’ pre-acquisition ratios. So if the target and the acquirer had 3% and 6% ratios, respectively, and the acquirer’s sales were twice that of the target, the predicted ratio is \((1/3)3\% + (2/3)6\% = 5\%\). Since the hypothesis is that the acquirer’s actual weight is larger than that, we try to estimate \(\beta_A\) and \(\beta_R\) in the additive models

\[
(1) \quad (T - \beta_A)A_t/S_t + (1 - T + \beta_A)A_{ac}/S_{ac} = A_m^{\dagger}/S_m^{\dagger} + \text{error}, \quad \text{where} \quad T \equiv S_t/[S_t + S_{ac}]
\]

and

\[
(2) \quad (T - \beta_R)R_t/S_t + (1 - T + \beta_R)R_{ac}/S_{ac} = R_m^{\dagger}/S_m^{\dagger} + \text{error}
\]

Being agnostic about the functional form, we also estimate \(\gamma_A\) and \(\gamma_R\) in the multiplicative models

\[
(3) \quad (1 - \gamma_A)T_{A_t}/S_t + (1 - (1 - \gamma_A)T)A_{ac}/S_{ac} = A_m^{\dagger}/S_m^{\dagger} + \text{error}
\]

\[
(4) \quad (1 - \gamma_R)T_{R_t}/S_t + (1 - (1 - \gamma_R)T)R_{ac}/S_{ac} = R_m^{\dagger}/S_m^{\dagger} + \text{error}
\]

The hypothesis is that the \(\beta\)’s and the \(\gamma\)’s are positive and we will rewrite \((1), (2), (3)\) and \((4)\) to estimate them in the forms

\[
(5) \quad T_{A_t}/S_t + (1 - T)A_{ac}/S_{ac} - A_m^{\dagger}/S_m^{\dagger} = \beta_A(A_t/S_t - A_{ac}/S_{ac}) + \text{error}
\]

\[
(6) \quad T_{A_t}/S_t + (1 - T)A_{ac}/S_{ac} - A_m^{\dagger}/S_m^{\dagger} = \beta_A(T_{A_t}/S_t - A_{ac}/S_{ac}) + \text{error},
\]

\[
(7) \quad T_{R_t}/S_t + (1 - T)R_{ac}/S_{ac} - R_m^{\dagger}/S_m^{\dagger} = \beta_R(R_t/S_t - R_{ac}/S_{ac}) + \text{error}
\]

\[
(8) \quad T_{R_t}/S_t + (1 - T)R_{ac}/S_{ac} - R_m^{\dagger}/S_m^{\dagger} = \gamma_R(T_{R_t}/S_t - R_{ac}/S_{ac}) + \text{error}
\]

The hypothesis is that the slopes, \(\beta\) and \(\gamma\), are positive, and that none of the intercepts are significantly different from zero. To interpret the regressions, note that when the target’s ratio is larger than the acquirer’s, positive slopes in \((5)-(8)\) imply that the “predicted” post-acquisition ratios are bigger than the actuals. Conversely, if the target’s ratio is smaller, positive slopes imply that the “predicted” ratios are smaller than the actuals.
The results for $\tau = 1$ are given in Table 5.\(^{20}\)

### Table 5

**OLS Regressions**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$N = 86$</td>
<td>$N = 86$</td>
<td>$N = 86$</td>
<td>$N = 86$</td>
</tr>
<tr>
<td>Intercept</td>
<td>-.00 (s.d.)</td>
<td>-.00 (s.d.)</td>
<td>-.02 (s.d.)</td>
<td>-.01 (s.d.)</td>
</tr>
<tr>
<td>Slope</td>
<td>.54*** (s.d.)</td>
<td>.79*** (s.d.)</td>
<td>.57*** (s.d.)</td>
<td>2.36*** (s.d.)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>.46</td>
<td>.61</td>
<td>.29</td>
<td>.44</td>
</tr>
</tbody>
</table>

**\(*** \) \(p < .01\).**

As can be seen from Table 5, the multiplicative model, estimated as (6) and (8), fits best. Most importantly, however, all four columns show strong support for Hypothesis (i) in the sense that the slopes are significantly positive. (With $t$-statistics that are very large in light of our relatively modest sample size.)\(^{21}\) In addition, it is gratifying to see that none of the intercepts are significant. (The underlying data are plotted in Appendix B.)

One possible alternative interpretation is that the acquirers businesses grow faster such that their weights are too low if estimated from two or three year old data. However, since acquirers grew by only 1% more per year (13% vs 12%), this is not nearly enough to explain the

\(^{20}\) The regressions for the second year after the merger ($\tau = 2$) are qualitatively similar with very significant positive slopes. We here show the first year since changes to advertising and R&D budgets can be implemented quickly.\(^{21}\) Harrison, Hitt, Hoskisson, and Ireland (1991) suggest that post acquisition performance in conglomerate acquisitions (when target and acquirer were in different 2-digit SIC codes) was higher when the differences between pre acquisition R&D-to-sales ratios were larger. If this mean that acquirers look for targets with low R&D intensity when theirs is high, it could constitute an alternative explanation for the results.
results. For example, if we make the indicated changes in the weights, there is only a .007 change in the slope in the additive advertising model in Table 5. As an extra check, we also ran OLS regressions with target and acquirer growth rates as controls, and found no material differences in the results.

There are, as always, many alternative explanations and we cannot claim that the changes in input intensities happen because targets move towards their acquirer. All we can say is that the theory was not falsified by the above test. So it is important to “triangulate” further, and we do so by testing two more predictions.

Coming then to Hypothesis (ii), we measure the profit rate by profits over sales. The hypothesis is that the post-merger performance should be below that of the acquirer (since the factor in excess capacity should be worth more in its original use). This implies that

\[(9) \frac{P_{ac}}{S_{ac}} > \frac{P_{m^\tau}}{S_{m^\tau}}\]

The results of pairwise tests are given in second and third columns of Table 6 for \(\tau = 1\) and \(\tau = 2\) using several measures of profits.\(^{22,23}\)

\(^{22}\) Since the test involves ratios between contemporaneous variables, we do not have to adjust for inflation. To control for business cycle effects on profitability, we scale post-merger profits by the ratio between the EPS of the S&P 500 in the year before each merger and years \(\tau = 1\) and \(\tau = 2\). (http://www.multpl.com/s-p-500-earnings/table and https://data.bls.gov/cgi-bin/cpicalc.pl)

\(^{23}\) Since many of the acquirers themselves get acquired later, the number of observations fall as we get further from the merger date. After two years we have lost four, but after four years, more than a third (31) are gone, (and these firms are presumably not selected randomly).


Table 6

Means and Standard Deviations

<table>
<thead>
<tr>
<th>Performance Measure</th>
<th>$\frac{P_{ac}/S_{ac}}{} - \frac{P_{t}/S_{t}}{}$</th>
<th>$\frac{P_{ac}/S_{ac}}{} - \frac{P_{m1}/S_{m1}}{}$</th>
<th>$\frac{P_{ac}/S_{ac}}{} - \frac{P_{m2}/S_{m2}}{}$</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross Profit/Sales</td>
<td>.017</td>
<td>.100***</td>
<td>.017*</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>(. s. d.) .020</td>
<td>.011</td>
<td>.011</td>
<td></td>
</tr>
<tr>
<td>EBITDA/sales</td>
<td>.031</td>
<td>.092**</td>
<td>.009</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>(. s. d.) .038</td>
<td>.040</td>
<td>.014</td>
<td></td>
</tr>
<tr>
<td>EBIT/Sales</td>
<td>.032</td>
<td>.106**</td>
<td>.030**</td>
<td>64</td>
</tr>
<tr>
<td></td>
<td>(. s. d.) .039</td>
<td>.047</td>
<td>.014</td>
<td></td>
</tr>
<tr>
<td>EBT/Sales</td>
<td>.130***</td>
<td>.014</td>
<td>-.043</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>(. s. d.) .030</td>
<td>.012</td>
<td>.013</td>
<td></td>
</tr>
<tr>
<td>Net Income/Sales</td>
<td>.053</td>
<td>.160**</td>
<td>.064**</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>(. s. d.) .042</td>
<td>.075</td>
<td>.032</td>
<td></td>
</tr>
</tbody>
</table>

*** p < .01, ** p < .05, * p < .1, one-sided tests

The pattern in the second and third columns is consistent with Hypothesis (ii). The merged entity does not do as well as the acquirer, though performance seems to be picking up in the second year. The first column shows that $\frac{P_{ac}/S_{ac}}{} > \frac{P_{t}/S_{t}}{}$, such that the acquirers on the average are more profitable than the targets. If we interpret profits as indicating the presence of

---

24 Gross profits equal revenues minus cost of goods sold. Subtracting selling, administrative and research expenses gives you EBITDA (earnings before interest, taxes, depreciation, and amortization), and adding back in amortization and depreciation yields EBIT. Finally, EBIT minus interest and taxes gives net income. For our 19 banks, EBT is reported instead of Gross Profits, EBIT, and EBITDA.
excess capacity, the pattern validates our assumption, used in the test of Hypothesis (i), that it is the initial excess capacity rests with the acquirers.

As promised, we finally test Prediction (iii) by looking at the relationship between the SIC codes of targets and acquirers. From nasdaq.com and manta.com, we have the primary 4-digit SIC codes for both target and acquirer in 84 of the mergers. To judge whether the 84 pairs of SIC codes are more or less similar than could be expected from random matching, we take the set of acquirers as a given and assume that the 84 firms that ultimately were acquired are the only possible targets. We first calculate the expected number of 4-digit matches when 84 acquirers and 84 targets are paired up randomly. We then eliminate the 34 mergers that actually were between firms in the same 4-digit SIC codes and look for the expected number of 3-digit matches resulting from random pairings of the remaining $84 - 34 = 50$ acquirers and 50 targets. After that we proceed analogously to look for the expected number of 2-digit matches when $50 - 16 = 34$ acquirers and 34 targets are paired up randomly, etc.

The results are given in Table 7 below. For example, 12 is the number of actual mergers between two firms with the same 2-digit SIC codes, but different 3-digit codes. Further, 2.47 is the expected number of such mergers in random pairings between the 34 acquirers and 34 targets that were not part of mergers with the same 3-digit code.

Table 7

Proximity Between the SIC Codes of Targets and Acquirers, $N = 84$

<table>
<thead>
<tr>
<th></th>
<th>4-digit</th>
<th>3-digit</th>
<th>2-digit</th>
<th>1-digit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Highest SIC code</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>match</strong></td>
<td>34</td>
<td>16</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td><strong>With random</strong></td>
<td>2.36</td>
<td>3.46</td>
<td>2.47</td>
<td>4.91</td>
</tr>
<tr>
<td><strong>matching (out of)</strong></td>
<td>(84)</td>
<td>(50)</td>
<td>(34)</td>
<td>(22)</td>
</tr>
</tbody>
</table>
As can be seen, there is very strong evidence that mergers combine similar businesses, at least in the sense measured by SIC codes. We hasten to admit that this, to an even higher degree than the other findings, can have many alternative explanations. In particular, mergers that are motivated by market power concerns would show the same patterns.

VI. Discussion

We have developed a theory of multi-business firms that can be used to make sense of the dominant perspective in management research. We provide new empirical evidence on three predictions, and report on others that have been tested in already published work.

The theory suggests that the scope of a firm be defined by the factors of production it controls through employment contracts or ownership. It makes no distinction between vertical, horizontal, and diversifying expansions. To the extent that the first two types are more common, it could be that they offer more opportunities to share factors and that the services they need more frequently are correlated.

The theory is consistent with the findings reported in Atalay et al. (2014), as well as the authors’ interpretation of them. Reflecting on the paucity of product flows between plants in vertically integrated firms, they propose that an important reason for vertical integration is that it “promotes efficient intrafirm transfers of intangible inputs”. They mention the “organizational capabilities theory” (aka the resource-based view) and the “equilibrium assignment view” of Lucas (1978) and Rosen (1982) as the two existing theories that could be consistent with their findings. The latter theory is based on the assumption that factors differ in their quality and posits that mergers, through assortative matching, bring together high quality complementary factors. In contrast to our Hypotheses (i) and (ii), equilibrium assignment would therefore predict that post-merger use intensities decrease for both acquirer and target, and that both improve their performance. This is obviously not what we found.

Beyond the specifics, the present paper contributes more broadly to the theory of the firm. First, because multi-business activities are an empirically important aspect of the scope of the firm. Second, because it is part of an emerging dialogue between economists’ theories of the firm and those that are prominent in the management literature.
APPENDIX A: PROOFS

Proof of Lemma 1:

We first show, in part I of the proof, that the five proposed equilibria are subgame perfect and that no others are. In part II, we show that each of the five equilibria may dominate the other four. Throughout this proof and that of Lemma 2, we will refer to the first \( B/2 \) workers as the “early” movers, while the second \( B/2 \) workers will be the “late” movers. We also label two representative neighboring businesses such that \( N(b) = \{b, b'\} \) and \( s'^b = s^b \).

Preliminaries:

This is a big game with many stages. However, our task can be eased by using a combination of arguments based on dominated strategies, symmetry, and branch and bound. First, entry decisions are made in dominant strategies: Both in stages 4 and 8, only the first \( B \) workers will enter a mechanism, since all later entrants will end up on the long side of any mechanism they might enter. Second, workers will weakly prefer to be matched up with businesses that need their neighborhood’s common service in period 1, implying that the early movers will choose their human capital accordingly. The late movers will then have to make their decisions in light of the choices made by early movers. Third, in stages 8 and 4, since workers’ payoffs increase with their productivity, those that enter will all choose that mechanism which gives the highest joint surplus given the history (human capital and all earlier mechanism choices by all players). Because entrepreneurs move first, this means that workers’ aggregate mechanism choices will match those of entrepreneurs, such that we can focus on just one choice. Fourth, in stages 7 and 3, since also entrepreneurs get payoffs that increase with productivity, all businesses will enter those mechanisms in which workers, given the history, can provide them with needed service in return for a share of the highest joint surplus.

To facilitate reading, we start by listing the five proposed equilibria and their payoffs.

**Employment:** \((h_{wB}, h_{wS}) = (b, \Omega_S), (m^1_B, m^1_S) = (b, \Omega_S), \text{ and } (m^2_B, m^2_S) = (\emptyset, \emptyset)\), with (total two period per worker) net surplus \((1 + \delta)v(1 + q_S) - K(S)\)
**Sequential Contracting:** \((h_{wB}, h_{wS}) = (b, \Omega_S), (m^l_B, m^l_S) = (b, s^l_b)\) and \((m^2_B, m^2_S) = (b, s^2_b)\) if \(s^2_b \neq s^l_b\) and \((m^2_B, m^2_S) = (\Omega, \Omega)\) if \(s^2_b = s^l_b\). The net expected surplus is therefore \((1 + \delta)v(1 + q_S) - [1 + (1 - \rho)\delta]K(1)\).

**Global Market:** \((h_{wB}, h_{wS}) = (m^l_B, m^l_S) = (\Omega_B, s)\), with net surplus \((1 + \delta)v(q_B + 1)\)

**Local Market:** Label two neighboring businesses such that \(s^l_b = s^*_{b}\) and \(N(b) = \{b, b'\}\). One of the first \(B/2\) workers sets \((h_{wB}, h_{wS}) = (b, s^*_{b}), (m^l_B, m^l_S) = (b, s^*_{b}), (m^2_B, m^2_S) = (b', s^*_{b})\) if \(s^2_b' = s^*_b\) and \((m^2_B, m^2_S) = (\Omega, \Omega)\) if \(s^2_b = s^*_{b}\), while one of the second \(B/2\) workers sets \((h_{wB}, h_{wS}) = (b', \Omega_S), (m^l_B, m^l_S) = (b', s^l_{b'}), (m^2_B, m^2_S) = (b, s^2_{b'})\) if \(s^2_{b'} = s^*_{b}\) and \((m^2_B, m^2_S) = (\Omega, \Omega)\) if \(s^2_b = s^*_{b}\). Expected net surplus for the first worker will be \(v[2 + \delta(1 + \rho + \{1 - \rho\}q_B^*)] - [1 + (1 - \rho)\delta]K(1)\), while that for the second worker will be \(v[1 + q_S + \delta(\rho + \{1 - \rho\}q_B^* + q_S)] - [1 + (1 - \rho)\delta]K(1)\).

**Hybrid Market:** Label two neighboring businesses such that \(s^l_b = s^*_{b}\) and \(N(b) = \{b, b'\}\). One of the first \(B/2\) workers sets \((h_{wB}, h_{wS}) = (b, s^*_{b}), (m^l_B, m^l_S) = (b, s^*_{b}), (m^2_B, m^2_S) = (b', s^*_{b})\) if \(s^2_b' = s^*_b\) and \((m^2_B, m^2_S) = (\Omega, \Omega)\) if \(s^2_b = s^*_b\). One of the second \(B/2\) workers becomes a market specialist, setting \((h_{wB}, h_{wS}) = (m^l_B, m^l_S) = (\Omega_B, s^l_{b'}), (m^2_B, m^2_S) = (\Omega, \Omega)\), serving \(b'\) in period 1, and working for another business needing \(s^l_{b'}\) in period 2. Expected net surplus for the first worker is \(v[2 + \delta(1 + \rho + \{1 - \rho\}q_B^*)] - [1 + (1 - \rho)\delta]K(1)\), while that for the second worker is \((1 + \delta)v(q_B + 1)\).

**Part I:**

We here check the postulated equilibria against deviations and at the same time establish that no other equilibria are subgame perfect. We do this by going through all four possible human capital choices for the \(B/2\) workers moving first:

1. Suppose first that the first \(B/2\) workers set \((h_{wB}, h_{wS}) = (b, \Omega_S)\).
   a. Assume that the late movers set \((h_{wB}, h_{wS}) = (b', \Omega_S)\). If the early movers enter \((m^l_B, m^l_S) = (b, \Omega_S)\), they can set \((m^2_B, m^2_S) = (\Omega, \Omega)\), and their (two period) net surplus is \((1 + \delta)v(1 + q_S) - K(S)\). Later movers can get the same surplus by entering the analog mechanisms, and they will prefer that when the early movers do. This is the Employment equilibrium.
(ii) Assume again that the late movers set \((h_{WB}, h_{WS}) = (b', Q_S)\). If the first movers enter \((m'_B, m'_S) = (b, s^*_b)\), they can set \((m^2_B, m^2_S) = (b, s^*_b)\) if \(s^2_b \neq s^*_b\) and \((m^2_B, m^2_S) = (\emptyset, \emptyset)\) if \(s^2_b = s^*_b\). Their expected net surplus is therefore \((1 + \delta)v(1 + q_S) - [1 + (1 - \rho)\delta]K(1)\). Later movers can get the same surplus if the enter the analog mechanisms, and they will prefer that when the early movers do. This is the Sequential Contracting equilibrium.

(iii) Continue to assume that the late movers set \((h_{WB}, h_{WS}) = (b', Q_S)\). If two pairs enter the same mechanism in order to save on bargaining costs, \((m'_B, m'_S) = (\Omega_B, Q_S)\), net surplus is \((1 + \delta)v(q_B + q_S)\). This is dominated by Employment or Sequential Contracting by Assumption 8. More generally, mechanisms in which \(m'_B\) has more than two elements are less efficient since there the worker is more likely to work outside his area of expertise. Finally, if \(m'_S \subset Q_S\) there is a chance he will be asked to perform services for which no price has been negotiated (outside his job description).

(iv) Suppose next that late movers select \((h_{WB}, h_{WS}) = (b', s'_b)\). The early movers will proceed as in Employment or Sequential Contracting. For the late movers, the most attractive mechanism choice is \((m'_B, m'_S) = (b', s'_b)\), followed by \((m^2_B, m^2_S) = (b', s^2_b)\) if \(s^2_b = s'_b\) and \((m^2_B, m^2_S) = (\emptyset, \emptyset)\) if \(s^2_b \neq s'_b\). The expected payoff from this, 
\[v(2 + \delta[2\rho + (1 - \rho)(1 + q_S)] - [1 + (1 - \rho)\delta]K(1))\], is dominated by that from Sequential Contracting by Assumption 9.

(v) Finally, if late movers select \((h_{WB}, h_{WS}) = (\Omega_B, s'_b)\), they may be unable to work in the second period, and if they select \((h_{WB}, h_{WS}) = (\Omega_B, Q_S)\), they do not take advantage of any specialization advantages.

(2) Suppose next that the first \(B/2\) workers set \((h_{WB}, h_{WS}) = (\Omega_B, Q_S)\).

If \((m'_B, m'_S) = (\Omega_B, Q_S)\), net surplus is \((1 + \delta)v(q_B + q_S)\), which is dominated by Global Markets. There is no gain from using \(m'_B \subset \Omega_B\) and it is less efficient to use \(m'_S \subset Q_S\), since there is a chance a worker will be asked to perform services outside his job description.

(3) Suppose instead that the first \(B/2\) workers set \((h_{WB}, h_{WS}) = (b, s)\) and assume wlog that they choose such that \((h_{WB}, h_{WS}) = (b, s'_b)\).

(i) Suppose first that the late movers set \((h_{WB}, h_{WS}) = (b', Q_S)\). If the early movers enter \((m'_B, m'_S) = (b, s^*_b)\), \((m^2_B, m^2_S) = (b', s^*_b)\) if \(s^2_b = s^*_b\) and \((m^2_B, m^2_S) = (\emptyset, \emptyset)\) if \(s^2_b \neq s^*_b\). The expected payoff from this, 
\[v(2 + \delta[2\rho + (1 - \rho)(1 + q_S)] - [1 + (1 - \rho)\delta]K(1))\], is dominated by that from Sequential Contracting by Assumption 9.
If $s^2_b = s^*_b$, their expected net surplus is $v_2 + v_\delta [1 + \rho + (1 - \rho)qB^*] - [1 + (1 - \rho)\delta]K(1)$. If the later movers enter $(m^1_B, m^1_S) = (b', s^1_B')$, $(m^2_B, m^2_S) = (b, s^2_B)$ if $s^2_B = s^*_b$ and $(m^2_B, m^2_S) = (\emptyset, \emptyset)$ if $s^2_B = s^*_b$, their expected surplus is $v_2 + v_\delta [1 + qS + \delta (1 - \rho)qB^* + qS^*] - [1 + (1 - \rho)\delta]K(1)$. This is the Local Market. As long as the late movers have set $(h_wB, h_wS) = (b', \Omega_S)$, any mechanisms other than those specified above will cost them a loss of specialization advantages.

If the later movers set $(h_wB, h_wS) = (m^1_B, m^1_S) = (\Omega_B, s)$, their net surplus will be $(1 + \delta)v(qB + 1)$. This is the hybrid Market. As long as the late movers have set $(h_wB, h_wS) = (\Omega_B, s)$, any mechanisms other than $(\Omega_B, s)$ will cost them a loss of specialization advantages.

If the later movers set $(h_wB, h_wS) = (m^1_B, m^1_S) = (\Omega_B, \Omega_S)$, they are unspecialized and do very poorly.

If the late movers set $(h_wB, h_wS) = (m^1_B, m^1_S) = (b, s^*_b)$, their most attractive mechanism choices are $(m^1_B, m^1_S) = (b', s^1_B')$, followed by $(m^2_B, m^2_S) = (b, s^2_B)$ if $s^2_B = s^*_b$ and $(m^2_B, m^2_S) = (\emptyset, \emptyset)$ if $s^2_B = s^*_b$. The expected payoff from this, $v_2 + v_\delta [2\rho + (1 - \rho)(qB^* + qS^*)] - [1 + (1 - \rho)\delta]K(1)$, is dominated by that from the Local Market by Assumption 9.

(ii) For any human capital choices by the late movers, if the early movers set $(m^1_B, m^1_S) = (b, s^*_b)$, they can expect $v_2 + v_\delta (1 + \rho + \{1 - \rho\}qS^*) - [1 + (1 - \rho)\delta]K(1)$, which is less than what they get in the Local and Hybrid Markets. More generally, if the early movers enter a mechanism with $|m^1_B| > 1$ they do very poorly since they then have a big chance of working for a business other than $b$.

(iii) For any human capital choices by the late movers, if early movers enter a mechanism with $m^1_B = b$, and $|m^1_S| > 1$ they get the same result as in (i) above as long as $m^1_S$ includes $b$. (if not, they do really, really poorly.)

(4) Suppose finally that the first $B/2$ workers set $(h_wB, h_wS) = (\Omega_B, s)$.

If the early movers enter $(m^1_B, m^1_S) = (\Omega_B, s)$, their net surplus is $(1 + \delta)v(qB + 1)$. Later movers can get the same surplus by also setting $(h_wB, h_wS) = (\Omega_B, s)$ and they will prefer that when the early movers do. This is the Global Market equilibrium.
Part II:

We here show that each of the five equilibria may dominate the other four. To this end, we assume throughout that \( v \) is large and that \( q_s^* \) and \( \rho \) are close to zero. We start by looking at the \( B/2 \) workers who move first and show that any of the five equilibria may dominate the other four. First, Employment and Sequential Contracting dominate the others if \( q_B^* \approx q_B \approx 0, q_s \approx 1/2 \), and \( \delta > 1 \) because payoffs then are close to \( v(1 + \delta)^{3/2} - \min\{K(S), (1 + (1 - \rho)\delta)K(1)\} \), while the other equilibria have payoffs close to \( v(1 + \delta) \) or \( v(2 + \delta) \), possibly less bargaining costs. Second, Global Markets dominate the others if \( q_B \approx 1, q_B^* \approx 1/2, \) and \( q_S \approx 1/2 \), because payoffs in that case are close to \( v(1 + \delta)2 \), while Local and Hybrid Markets have payoffs close to \( v(2 + \delta^3/2) \) and Employment and Sequential contracting have payoffs close to \( v(1 + \delta^3/2) \). Third, the Local and Hybrid markets dominate the others if \( q_B^* \approx 1 \), and \( q_B \approx q_S \approx 1/2 \) because payoffs in that case are close to \( v(1 + \delta)2 \), while the other equilibria have payoffs close to \( v(1 + \delta^3/2) \). (Note that the first \( B/2 \) workers have the same payoffs in these two equilibria.)

Consider finally the \( B/2 \) workers who move next in the Local and hybrid Markets. We have seen that the first movers will select these equilibria if \( q_B^* \approx 1 \) and \( q_B \approx q_S \approx 1/2 \). The second group of workers will prefer the Local Market if \( v(1 + \delta)(q_S - q_B) > [1 + (1 - \rho)\delta]K(1) \), and the Hybrid Market if not. So each of the five equilibria may dominate the other four.

So Employment, Sequential Contracting, Global Markets, Local Markets, and Hybrid Markets are un-dominated in the set of subgame perfect equilibria when each entrepreneur owns a single business.

Q.ED

Proof of Lemma 2:

The new equilibria, which dominate the Local and Hybrid Markets, are:

**Multi-business with Dual Employment:** One of the first \( B/2 \) workers sets \((h_{wB}, h_{wS}) = (b, s^*_b), (m^*_B, m^*_S) = (N(b), s^*_b), (m^*_B, m^*_S) = (\emptyset, \emptyset)\), and one of the second \( B/2 \) workers sets \((h_{wB}, h_{wS}) = (b^*, \Omega_S), (m^*_B, m^*_S) = (N(b), \Omega_S), (m^*_B, m^*_S) = (\emptyset, \emptyset)\). Expected net surplus for the first
worker is \(v[2 + \delta(\rho + \{1 - \rho\}qB^* + 1)] - K(1)\), while that for the second worker is \(v[1 + qS + \delta(\rho + \{1 - \rho\}qB^* + qS)] - K(S)\).

**Multi-business Employment with a Contractor:** One of the first \(B/2\) workers sets \((h_{wB}, h_{wS}) = (b, s^*_b), (m^1_B, m^1_S) = (N(b), s^*_b), (m^2_B, m^2_S) = (\emptyset, \emptyset)\), and one of the second \(B/2\) workers sets \((h_{wB}, h_{wS}) = (b', \Omega_S), (m^1_B, m^1_S) = (b', s'_{b'}), (m^2_B, m^2_S) = (b, s^2_b)\) if \(s^2_b = s^*_b\) and \((m^2_B, m^2_S) = (b', s^2_b)\) if \(s^2_b = s^*_b\). Expected net surplus for the first worker is \(v[2 + \delta(\rho + \{1 - \rho\}qB^* + 1)] - K(1)\), while that for the second worker is \(v[1 + qS + \delta(\rho + \{1 - \rho\}qB^* + qS)] - [1 + (1 - \rho)\delta]K(1)\).

**Multi-business Employment with Market Workers:** One of the first \(B/2\) workers sets \((h_{wB}, h_{wS}) = (b, s^*_b), (m^1_B, m^1_S) = (N(b), s^*_b), (m^2_B, m^2_S) = (\emptyset, \emptyset)\). One of the second \(B/2\) workers becomes a market specialist, setting \((h_{wB}, h_{wS}) = (m^1_B, m^1_S) = (\Omega_B, s'_{b'})\), serving \(b'\) in period 1, and working for another business needing \(s'_{b'}\) in period 2. Expected net surplus for the first worker is \(v[2 + \delta(\rho + \{1 - \rho\}qB^* + 1)] - K(1)\), while that for the second worker is \((1 + \delta)v(qB + I)\).

We proceed as in the proof of Lemma 1. In part I of the proof we check that three postulated equilibria are subgame perfect and show that no other equilibria are. In part II, we show that each of the five equilibria may dominate the other four.

Part I.

1. If the early movers set \((h_{wB}, h_{wS}) = (b, s^*_b), (m^1_B, m^1_S) = (N(b), s^*_b), (m^2_B, m^2_S) = (\emptyset, \emptyset)\), their expected net surplus is \(v[2 + \delta(\rho + \{1 - \rho\}qB^* + 1)] - K(1)\).

i. If the later movers set \((h_{wB}, h_{wS}) = (b', \Omega_S), (m^1_B, m^1_S) = (b', s'_{b'}), (m^2_B, m^2_S) = (b, s^2_b)\) if \(s^2_b = s^*_b\) and \((m^2_B, m^2_S) = (\emptyset, \emptyset)\) if \(s^2_b = s'_{b'}\), their expected surplus is \(v[1 + qS + \delta(\rho + \{1 - \rho\}qB^* + qS)] - [1 + (1 - \rho)\delta]K(1)\). This is Multi-business Employment with a Contractor. If the late movers set \((h_{wB}, h_{wS}) = (b', \Omega_S), (m^1_B, m^1_S) = (N(b), \Omega_S), (m^2_B, m^2_S) = (\emptyset, \emptyset)\). Their expected surplus is \(v[1 + qS + \delta(\rho + \{1 - \rho\}qB^* + qS)] - K(S)\). This is Multi-business with Dual Employment. As long as the late movers have set \((h_{wB}, h_{wS}) = (b', \Omega_S)\), any mechanisms other than those specified above will cost them a loss of specialization advantages.
(i) If the later movers set \((h_{wB}, h_{wS}) = (Ω_B, s'_b), (m'_B, m'_S) = (b', s'_1b')\). Their expected net surplus will be \((1 + δ)v(q_B + 1)\). This is Multi-business Employment with Market Workers. As long as the late movers have set \((h_{wB}, h_{wS}) = (Ω_B, s)\), any mechanisms other than \((Ω_B, s)\) will cost them a loss of specialization advantages.

(ii) If the later movers set \((h_{wB}, h_{wS}) = (m'_B, m'_S) = (Ω_B, Ω_S)\), they are unspecialized and do very poorly.

(iii) If the late movers set \((h_{wB}, h_{wS}) = (m'_B, m'_S) = (b', s'_1b')\), their most attractive mechanism choices are \((m_1^B, m_1^S) = (b', s'_1b')\), followed by \((m_2^B, m_2^S) = (b, s_2b)\) if \(s_2b = s_*b\) and \((m_2^B, m_2^S) = (Ø, Ø)\) if \(s_2b = s'_1b\). The expected payoff from this \(v/2 + δ[2ρ + (1 − ρ)(q_B^* + q_S^*)] − [1 + δ(1 − ρ)]K(1)\), is dominated by that from the Multi-business Employment with a Contractor by Assumption 9.

(2) Multi-business firms are not attractive after any other human capital choices by the first movers such that we can apply the analysis from the proof of Lemma 1.

Part II.
Compared to the Local and Hybrid Markets, the Multi-business equilibria take advantage of the fact that a single entrepreneur can own both businesses in a neighborhood, such that the bargaining costs of the first \(B/2\) workers can be unambiguously reduced. Other than that, the two equilibria are identical. So for the first \(B/2\) workers, the Multi-business equilibria dominate the Local and Hybrid Markets because \(K(1) < [1 + (1 − ρ)δ]K(1)\). Therefore, at least in the regions where the Local and Hybrid Markets dominate the three other single–business equilibria (Employment, Sequential Contracting, and Global Markets), the three Multi-business equilibria dominate them as well.

QED

Proof of Corollary:
We will prove the Corollary for Multi-business with a Contractor. (Recall that the first \(B/2\) workers have the same payoffs in all three Multi-business equilibria.)

If there is no correlation in needs, such that \(s^2_b\) and \(s^2_{b'}\) are random draws from \(\Omega_s\), each of the \(B/2\) early movers’ surplus from Multi-business with a Contractor is \(v/2 + \delta(1 + q_s) - (1 + \delta)K(1)\), but this is smaller than the per worker surplus from Sequential Contracting, \((1 + \delta)v(1 + q_s) - [1 + \delta]K(1)\), by Assumption 9.

If \(q^*_B = q^*_B\), per worker surplus in Multi-business with a Contractor is \(v/2 + \delta(\rho + (1 - \rho)q^*_B + 1)) - (1 + \delta)K(1)\). This is smaller than per worker surplus in the Market, \((1 + \delta)v(q^*_B + 1)\), by Assumption 9.

QED
APPENDIX B: PLOTS FOR EQUATIONS (5)-(8)
REFERENCES


