

1 **MODELING CRASH PROBABILITY IN LARGE TRAFFIC SIMULATORS**

2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28

**Carlos Lima Azevedo**

Singapore MIT Alliance for Research and Technology  
1 Create way, 138602 Singapore  
Tel: +65 66011634; Fax: +65 6684 2118; Email: [camil@smart.mit.edu](mailto:camil@smart.mit.edu)

**João Lourenço Cardoso, Corresponding Author**

National Laboratory for Civil Engineering  
101 Av.Do Brasil, 1700-066 Lisbon, Portugal  
Tel: +351 218443661; Fax: +351 218443029 Email: [jpcardoso@lnec.pt](mailto:jpcardoso@lnec.pt)

**Moshe E. Ben-Akiva**

Massachusetts Institute of Technology  
Cambridge, Massachusetts02139, United States of America  
Tel: +16172535324; Fax: +16172531130; Email: [mba@mit.edu](mailto:mba@mit.edu)

Word count: 6,881-90-30-155=6606 words text + 5 tables/figures x 250 words (each) = 7856 words

20-03-2015

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30

## **ABSTRACT**

Currently, traffic microscopic simulation is a common tool for road system analysis. However, only recently have attempts been carried out to its application in safety assessment; also, most approaches still ignore causal relationships between different levels of vehicle interactions or/and crash types, lacking a valid representation of the crash phenomena itself. In this paper, a new generic probabilistic safety assessment framework for traffic microscopic simulation tools is proposed, as developed within the context of intelligent urban motorway speed limit management.

The probability of a specific crash occurrence is assumed as estimable by a crash propensity function, with a random component and a deterministic safety score component. This component depends on the type of occurrence, detailed vehicle interactions and manoeuvres, and simulation modelling features. The generic model is specified for no-crash events and three types of crash events (rear-end, lane-changing, and run-off-road) in a nested (logit) structure.

Artificial trajectories from a detailed calibrated microscopic simulation tool were used in the safety model fitting. Improved trajectory replication was obtained by a novel detailed comprehensive calibration effort: real trajectories were extracted from generic scenarios; the simulation tool was calibrated using the collected trajectories; lastly, the simulation model was re-calibrated using aggregate data from each selected replicated event.

The final estimated safety model allowed for the identification and interpretation of several simulated vehicle interactions (only 9% false crash alarms). The fact that these considerations were extracted from simulated runs shows the real potential of traffic microscopic simulation for detailed safety assessments in road design.

*Keywords:* Traffic microscopic simulation, Road safety, Probabilistic modelling, Driving behaviour modelling, Surrogate safety measures, Calibration

## 1 INTRODUCTION

2 Traffic microscopic simulation tools have been widely applied and its development has grown in  
3 recent years. They are now accepted as main tools for designing road infrastructure operation and  
4 for assessing transportation solutions, by researchers and practitioners. These tools incorporate  
5 several driving behaviour models that simulate vehicle movements, drivers' decisions and road  
6 user interactions at a very detailed level (1). The level of detail considered in driving behaviour  
7 models is particularly critical when disaggregated interactions between vehicles are more  
8 important than the aggregate traffic flow characteristics, such as in detailed safety assessments.  
9 Driving behaviour models typically include acceleration, lane-changing, route choice models and  
10 even more detailed features such as courtesy yielding or target gap selection models that were  
11 estimated based on a few (typically just one) sets of trajectory data.

12 Due to the complex nature of traffic systems and to the level of detail aimed at by  
13 several traffic simulation models (and the limited data used for its estimation), the calibration of  
14 detailed traffic variables has gained increasing importance in the application of microscopic  
15 simulation tools (2). The difficulty in replicating detailed traffic variables in traffic microscopic  
16 simulation has also hampered its application to safety analysis, especially when compared with  
17 its widespread use in network traffic performance analysis (17). There is a strong gap between  
18 the solid research on classical accident statistical analysis (3, 4) and the most recent  
19 developments in safety assessment using simulation.

20 Surrogate safety models (5) and a few real-time accident probability models (6, 7 and 8)  
21 are the two main safety research streams that have emerged recently with satisfactory results,  
22 especially as regards intersections, where the importance of vehicle interactions is  
23 straightforwardly recognized. Conflicts are used based on the assumption that the expected  
24 number of accidents occurring on a system is proportional to the number of conflicts, making  
25 them suitable for systems' comparisons (9). One main limitation of using conflicts is the correct  
26 estimation of this proportionality. This difficulty has motivated the development of several  
27 models to estimate accident frequency from traffic conflicts counts (10). For modelling purposes,  
28 another limitation is the lack of standardized practical definitions and measurement procedures  
29 (as it does not estimate the probability of an accident itself). For this purpose several time-based,  
30 deceleration-based and dynamic-based surrogate safety performance indicators were proposed in  
31 the literature (11). Despite this, these models are the most widely used within microscopic  
32 simulation studies (12, 13, 14, 15, 16).

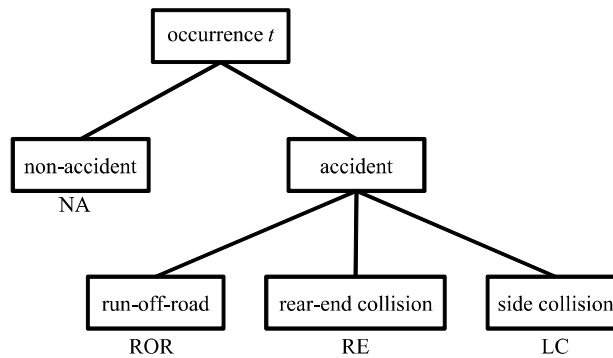
33 Very recently, efforts have been made to integrate drivers' interaction in probabilistic  
34 modelling frameworks (14, 15 and 16). The above mentioned accident probability models try to  
35 link the probability of a specific accident occurrence using a statistical model fitted to aggregated  
36 data. However, probabilistic frameworks try to formally represent cause-effect relationships  
37 between performed driving tasks and traffic scenarios related to typical accident events. Such  
38 approach has a higher potential in replicating the intrinsic nature of an accident mechanism and,  
39 ultimately, would not depend on safety records itself. On the other hand, probabilistic  
40 frameworks depend on much more detailed information as the distributions and relationships  
41 between all variables at stake are needed (e.g.: evasive manoeuvres probabilities for different  
42 situations or pavement conditions for different scenarios).

43 The objective of this study was to understand how microscopic simulation models can  
44 replicate detailed traffic data and how the results from simulated driver interactions may be used  
45 to calculate probabilistic safety scores to assess the influence on safety of key geometric design  
46 features, such as configuration and length of weaving sections, horizontal curve radius and lane  
47 width.

## 1 THE PROBABILISTIC SAFETY MODEL

### 3 Overall formulation

4 A generic framework for modelling cause-effect mechanisms between detailed vehicle  
 5 interactions from simulated motorway environments and the accident occurrence probability was  
 6 developed. It is assumed that the state of a vehicle  $n$  at any given time  $t$  is represented by a  
 7 discrete variable whose state outcome  $k$  may take one of four values: no accident or one of three  
 8 types of accident (rear-end, lane change or run-off-road – the three prevalent accidents on  
 9 motorways). This is represented in Figure 1.



11  
12  
13 **FIGURE 1 Model structure for motorway accident occurrence**

14  
15 An individual outcome  $k$  among all possible outcomes  $K$  is considered to be predicted if  
 16 its probability  $P_{n,t}(k)$  is maximum. This probability is assumed to be a function of specific  
 17 observed variables characterizing the interaction between vehicles 17. Such considerations step  
 18 away from the assumption of a fixed coefficient model converting the surrogate event frequency  
 19 into accident frequency, typically used in the traffic conflicts technique. The probability for a  
 20 specific accident involving vehicle  $n$  to occur at time  $t$  is assumed to be estimable by a specific  
 21 accident propensity (or proximity) measure (21):

$$22 \quad P_{n,t}(k) \sim U_k \quad (1)$$

23  
24  
25 In this model, each accident propensity function  $U_k$ , has a (deterministic) safety score  
 26 ( $V_k$ ) component and a random component ( $\varepsilon$ ):

$$27 \quad U_k = V_k(X, \beta) + \varepsilon \quad (2)$$

28  
29  
30 Where  $X$  is the vector of explanatory variables,  $\beta$  is the vector of unknown parameters  
 31 to be estimated and  $\varepsilon$  is the random term (the terms  $n$  and  $t$  were omitted for simplicity). The  
 32 assumption of the deterministic safety score component agrees with a recent research stream  
 33 where detailed interaction variables directly affect the accident occurrence probability itself (see  
 34 21 and 32). The random component  $\varepsilon$  represents the unobserved effects involved in the  
 35 determination of the outcome; these may be derived from a random process in the occurrence of  
 36 a specific event or correspond to insufficient knowledge of this process.

37 The accident phenomenon results from many different known factors (variables):

1  
 2 
$$V_k(n, t) = f_k(X_{n,t}, X_{n',t}, X_{D,t}, X_S) \tag{3}$$
  
 3

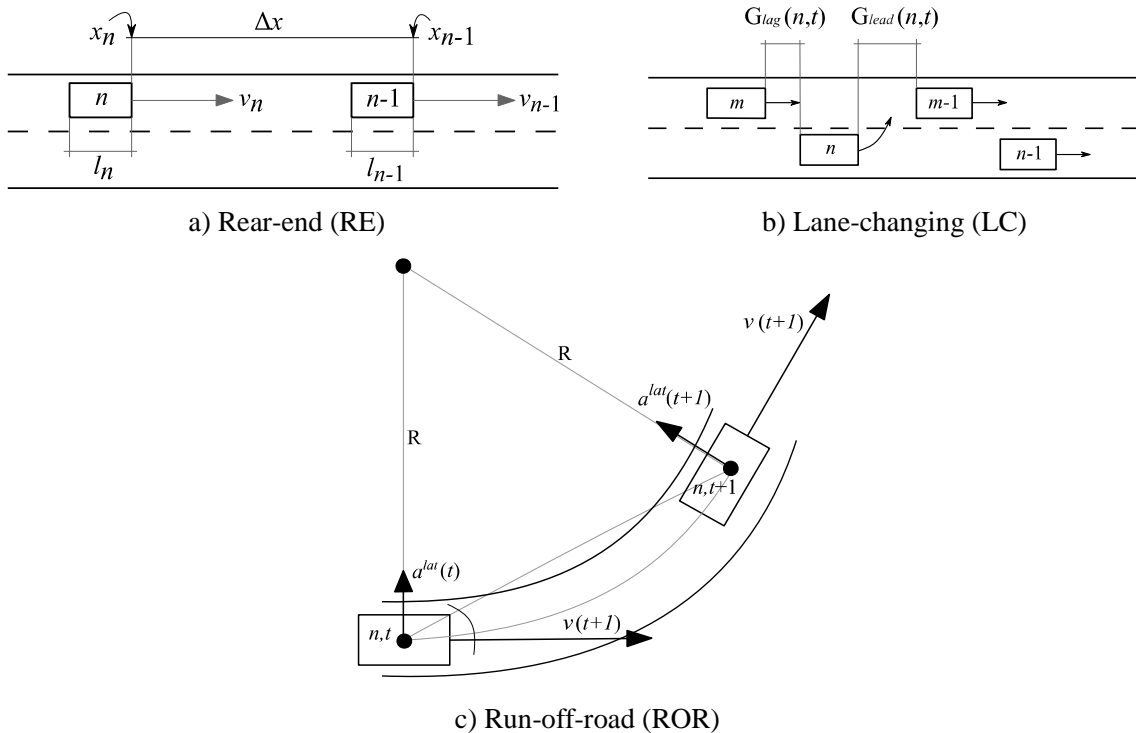
4 Where the  $k$  accident-type specific scoring function  $f_k$  depends on:

- 5  $X_{n,t}$ , the driver-vehicle unit  $n$  specific variables at time  $t$ ;  
 6  $X_{n',t}$ , the variables at time  $t$  for the interaction between  $n$  and a conflicting driver-vehicle  
 7 unit  $n'$ ;  
 8  $X_{D,t}$ , the dynamic environmental variables at time  $t$  (e.g.: weather, variable speed limit,  
 9 lighting conditions, etc.);  
 10  $X_S$ , the static environmental variables (e.g.: geometrics, road signs, etc).

11 Note that driver characteristics are typically not considered in traffic simulation tools,  
 12 which substantially limits the number of available candidate explanatory variables  $X_{n,t}$ .

13 This general formulation was applied to a specific set of accidents that typically occur  
 14 on busy urban motorways: rear-end accidents, side collisions during lane-change manoeuvres  
 15 and run-of-road accidents (Figure 2). These three different outcomes correspond to very distinct  
 16 phenomena. However, it is arguable that these three outcomes are independent, namely if one  
 17 considers accident outcomes following an evasive action from different risky interactions.

18 In the presented model we framed the formulation of each function  $f_k$  to represent a  
 19 cause-effect relationship, to simultaneously deal with different non-independent types of  
 20 accident outcomes and to consider a disaggregated probability for any vehicle state  $(n, t)$   
 21 observation. This is a departure from the existing aggregate formulation used in real-time  
 22 accident probability models.  
 23



24  
 25  
 26 **FIGURE 2 Accident outcome types**

1 General descriptions of the systematic safety score for each accident type are presented  
 2 below. More detailed descriptions of these functions and comprehensive discussion of its  
 3 assumptions are presented in reference (32).  
 4

### 5 **Rear-end (RE) conflicts**

6 The probability of a collision due to in-lane interactions is assumed dependent on: the subject  
 7 vehicle braking requirements to avoid a RE collision; and the maximum available braking power.

8 The first corresponds to the difference between the actual relative acceleration of the  
 9 subject vehicle  $n$  to the leader ( $n - 1$ ) and the deceleration rate required to avoid a crash  
 10 ( $DRAC$ ) estimated using Newtonian physics applied to the speed, longitudinal position and  
 11 length of the subject and precedent vehicles (see Figure 2a).

12 We further split of the needed deceleration rate ( $\Delta a^{need}$ ) into its positive,  
 13  $\Delta a_+^{need}(n, t) \geq 0$ , and negative,  $\Delta a_-^{need}(n, t) \leq 0$ , components, allowing for the consideration  
 14 of separate modelling parameters, with the advantage of considering the current acceleration  
 15 state as well. Component  $\Delta a_-^{need}$  represents safer situations, as the vehicle is already applying a  
 16 deceleration rate greater than  $DRAC$ .

17 We further improve this formulation by dividing the required deceleration by the time-  
 18 to-collision,  $TTC$ , thus considering also how long the driver has before the potential collision.  
 19 The  $RE$  safety score function will then depend on the available time for adjustment, resulting in a  
 20 relative needed deceleration ratio:  $RA^{need}(n, t) = \Delta a^{need}(n, t)/TTC(n, t)$ .

21 The measure of the maximum available deceleration rate is similar to the  $CPI$  surrogate  
 22 safety measure described in 9. The formulation emulates the influence of the vehicle speed itself  
 23 and allows for heterogeneous safety conditions due to different vehicle categories (e.g.  
 24 cars/heavy vehicles) and pavement conditions (e.g.: dry/wet) affecting the deceleration  
 25 performance. This simple formulation of the friction coefficient results from the small number of  
 26 variables typically available in simulated environments. The rate  $RA^{lim}(n, t) =$   
 27  $\Delta a^{lim}(n, t)/TTC(n, t)$  is used in the safety score function to account for  $TTC$ .

28 The systematic safety score for  $RE$  collisions is formulated by the following equation:  
 29

$$30 V_{RE}(n, t) = \beta_0^{RE} + \beta_1^{RE} RA_+^{need}(n, t) + \beta_2^{RE} RA_-^{need}(n, t) + \beta_3^{RE} RA^{lim}(n, t) \quad (4)$$

31  
 32 where  $RA_+^{need}$  and  $RA_-^{need}$  are the positive and negative components of the relative needed  
 33 deceleration ratio computed using  $\Delta a_+^{need}$  and  $\Delta a_-^{need}$  respectively;  $RA^{lim}$  is the maximum  
 34 available deceleration ratio; and  $\beta_0^{RE}$ ,  $\beta_1^{RE}$ ,  $\beta_2^{RE}$  and  $\beta_3^{RE}$  are the estimable parameters.  
 35

### 36 **Lane-changing (LC) conflicts**

37 Lane change decisions are typically modelled by means of gap acceptance models (22) or,  
 38 alternatively, by acceleration variation models (23). One would expect the probability of lane-  
 39 change collisions to be dependent on vehicle lateral movements. However, most current micro-  
 40 simulation tools do not provide this modelling feature. Therefore, surrogate measures depending  
 41 on lateral movements, such as the *Time-To-Lane-Crossing* proposed in (24) and the *Post-  
 42 Encroachment-Time* used in (25), are not easily integrated.

43 The probability of a  $LC$  collision is based on gap acceptance models and formulated in  
 44 terms of gap variation. The gap acceptance is generally modelled separately regarding the lead  
 45 and the lag gaps on the target lane (Figure 2b). This disaggregation is of special interest as  
 46 different parameters may be computed for different gaps (26). It is known that the lane-changing  
 47 process becomes increasingly difficult as the speed differences increase between the subject

1 vehicle and the lead and lag vehicles in the target lanes (27). Thus, in the proposed formulation  
 2 the safety score of the *LC* event is specified in terms of relative gap variations ( $R^{Gap}_{(n,t)}$ ), which  
 3 are a function of both the gap and the speed difference between the subject vehicle and the lead  
 4 or lag vehicles.

5 Two specific variables are used to represent gap variation: when positive,  $RG_+^{gap}$  takes a  
 6 positive value and  $RG_-^{gap}$  is nul; and the reciprocal, when the gap variation is negative. This  
 7 allows for associating specific parameters to different safety conditions, i.e. for gaps that are  
 8 either increasing or decreasing, respectively.

9 Following the above formulation, a gap with a higher relative shrinking rate should have  
 10 a higher impact on the *LC* conflict probability, and therefore its parameter estimate should be  
 11 negative.

12 The following equation represents the systematic component for *LC* collisions:

$$14 V_{LC}(n, t) = \beta_0^{LC} + \beta_1^{LC} RG_+^{lag}(n, t) + \beta_2^{LC} RG_-^{lag}(n, t) + \beta_3^{LC} RG_+^{lead}(n, t) + \beta_4^{LC} RG_-^{lead}(n, t) \quad (5)$$

15 where  $RA_+^{gap}$  and  $RA_-^{gap}$  are the positive and negative components (with gap = {lead, lag}) and  
 16  $\beta_0^{LC}$ ,  $\beta_1^{LC}$ ,  $\beta_2^{LC}$ ,  $\beta_3^{LC}$  and  $\beta_4^{LC}$  are the estimable parameters.

### 18 **Run-off-road (ROR) events**

19 *ROR* events are assumed as being primarily related to individual vehicle dynamics rather than to  
 20 interactions with others. This assumption is especially true under free-flow scenarios. However,  
 21 *ROR* events may also result from evasive manoeuvres due to dangerous lane-changing or car-  
 22 following interactions.

23 Vehicle dynamics in traffic simulation models are represented in a much simplified  
 24 manner, when compared with the detailed movements' description of real events currently  
 25 achieved with accident reconstruction software. Generally, detailed vehicle lateral movement  
 26 descriptions, true road geometric characteristics (such as transition curves), pavement surface  
 27 characteristics features, and detailed vehicle geometric and mechanical specifications are missing  
 28 from micro-simulation tools. This significantly limits the current potential for a *ROR* micro-  
 29 simulation modelling framework. However, relevant variables are already available, such as  
 30 vehicle speed, overall description of road geometrics and generic vehicle type specifications.

31 In the proposed framework, the safety score of *ROR* events is assumed to depend on the  
 32 difference between the current lateral acceleration of a vehicle and a site specific critical lateral  
 33 acceleration. On horizontal curves the vehicle is assumed to follow a simple circular path, its  
 34 yaw being equal to the curve bearing (Figure 2.c). The lateral acceleration of a vehicle,  $a^{lat}$ , is  
 35 therefore derived from its current speed and the curve radius  $R$  using Newton mechanics. As  
 36 mentioned, it is expected that lane change manoeuvres may also affect the *ROR* event  
 37 probability. Using test track data, Chovan et al. (26) considered peak lateral acceleration values  
 38 of 0.4g, 0.55g and 0.7g for mild, moderate, and aggressive steering manoeuvres, respectively.  
 39 Since detailed lane change models are typically not available in microscopic traffic simulation  
 40 platforms, a generic 0.5g peak acceleration add-on for lane change was incorporated in the  
 41 centripetal acceleration equation, to account for a potentially higher *ROR* probability in road  
 42 sections with high lane change frequency.

43 The maximum allowed lateral acceleration ( $a_{cr}^{lat}(n, t)$ ) depends on the critical lateral  
 44 friction coefficient, the road superelevation, the vehicle class and speed, and on the pavement  
 45 condition (wet/dry).

46 The safety score function is formulated in terms of the positive (unsafe) and negative  
 47

1 (safe) components of the difference between the current and the critical lateral accelerations:

$$2$$

$$3 \quad V_{ROR}(n, t) = \beta_0^{ROR} + \beta_1^{ROR} \Delta a_+^{lat}(n, t) + \beta_2^{ROR} \Delta a_-^{lat}(n, t) \quad (6)$$

$$4$$

5 where  $\Delta a_+^{lat}$  and  $\Delta a_-^{lat}$  are the positive and negative components of  $\Delta a^{lat} = a^{lat} - a_{cr}^{lat}$ ,  
6 respectively, and  $\beta_0^{ROR}$ ,  $\beta_1^{ROR}$  and  $\beta_2^{ROR}$  are the estimable parameters.

### 7 **Estimation framework**

8 As previously stated, the explanatory variables of one type of accident may influence the  
9 occurrence of others and evasive manoeuvres may create correlations between different accident  
10 outcomes. When modelling multiple discrete outcomes, the multinomial nested logit model  
11 proposed by Ben-Akiva, (28), has advantages over the simple multinomial logit model, because  
12 it can simultaneously estimate the influence of independent variables while allowing for the error  
13 terms to be correlated – formulation details may be found in (29).

14 To directly estimate the proposed model, a large set of all types of model outcomes and  
15 its vehicle interaction data is needed. Unfortunately, a large data allowing for the direct  
16 association between trajectories and accident occurrence is still not available. Furthermore,  
17 although the proposed model is specified individually for any vehicle  $n$  at every time  $t$ , the  
18 philosophy of microscopic simulation applications is to replicate aggregate measurements (even  
19 for such detailed values as accelerations, headways and *TTC*). Thus, to estimate the above model  
20 the use of artificial (simulated) trajectories from a microscopic simulation tool is proposed. This  
21 artificial data is then used with accident historical data in the model estimation process. Yet, a set  
22 of critical assumptions must be considered:

- 23 • The microscopic simulation model must be appropriately calibrated to replicate statistical  
24 distributions of detailed traffic variables. To achieve appropriate calibration, we assume  
25 that both aggregated data, such as loop sensor-based speeds and traffic counts, and a  
26 sample of disaggregated data, such as detailed vehicle trajectories are needed. Indeed, as  
27 it was concluded in a previous work that disaggregated calibration improves significantly  
28 the accuracy of simulated trajectories and spot-speeds, which are important for adequate  
29 representation of vehicle interactions in safety studies (36).
- 30 • Trajectories extracted in a generic day represent the general driving behaviour of traffic.  
31 Confidence on this assumption depends on the amount and breath of information  
32 available for treatment. Other factors (such as weather) influence general driving  
33 behaviour parameters; part of this variability may be assessed by means of a dedicated  
34 calibration, carried out for each specific event, using readily available data sets (eg.: from  
35 loop sensors).
- 36 • Simulated descriptions of detailed traffic variables can be linked to the accident  
37 probability, even though simulation models are crash-free. This is supported by previous  
38 studies (12, 6).
- 39 • It is typically expected that both the loop-based variables used for calibration and the  
40 accident occurrence reported variables are defined for given time and spatial units.  
41 Occasionally, such aggregated intervals maybe too large to capture short-term variations;  
42 nevertheless several authors (27, 2) have successfully used aggregated periods up to 5  
43 min intervals to perform accident occurrence probability analyses. With the absence of  
44 true trajectory variables for each vehicle involved in each observed event, the  
45 characterization of the detailed traffic variables for a specific accident occurrence must be  
46 linked by means of spatial and temporal aggregation. Additionally, it is well known that  
47



1 safety records have time and spatial errors. Therefore, for estimation one needs to  
 2 aggregate all vehicle state outcome probabilities  $P_{n,t}(k)$  by standardized intervals of  
 3 space,  $s$ , and time periods,  $p$ :

$$4 \quad P_{s,p}(k) = \frac{1}{N} \sum_N P_{n,t}(k) \quad (7)$$

6 where  $P_{n,t}(k)$  is the probability of occurrence  $k$  for any relevant observation of vehicle  $n$   
 7 at time  $t$ , traveling in spatial interval  $s$  and time period  $p$  and defined by the proposed  
 8 nested logit model;  $N$  is the total number of observations for all vehicles that travelled in  
 9 the interval  $s, p$ . According to this formulation, the model is based on mean values and  
 10 not on extreme values. This follows the traffic micro-simulation specification philosophy,  
 11 where the replication of averaged variables is expected. However, it is possible to push  
 12 the use of extreme formulations and rely on detailed calibration methods of extreme  
 13 values. Such formulation was not tested for the present document.

- 14 • Finally, if one considers a large historical observation period typically needed for the  
 15 observation of a significant number of accident occurrences, it is expected that the loop  
 16 sensors will fail for some instances. Furthermore, the computational memory and  
 17 processing resources needed to generate and use the simulated trajectory data for a large  
 18 set of no-accident occurrence units during the “meaningful” observation period is  
 19 impractical. Therefore, an outcome (choice)-based random sampling was assumed to  
 20 select the events to be simulated, and the weighted exogenous sample maximum  
 21 likelihood function proposed in (31) was used, to account for this biased sampling  
 22 process.  
 23

## 24 THE URBAN MOTORWAY CASE AND TESTING DATASET

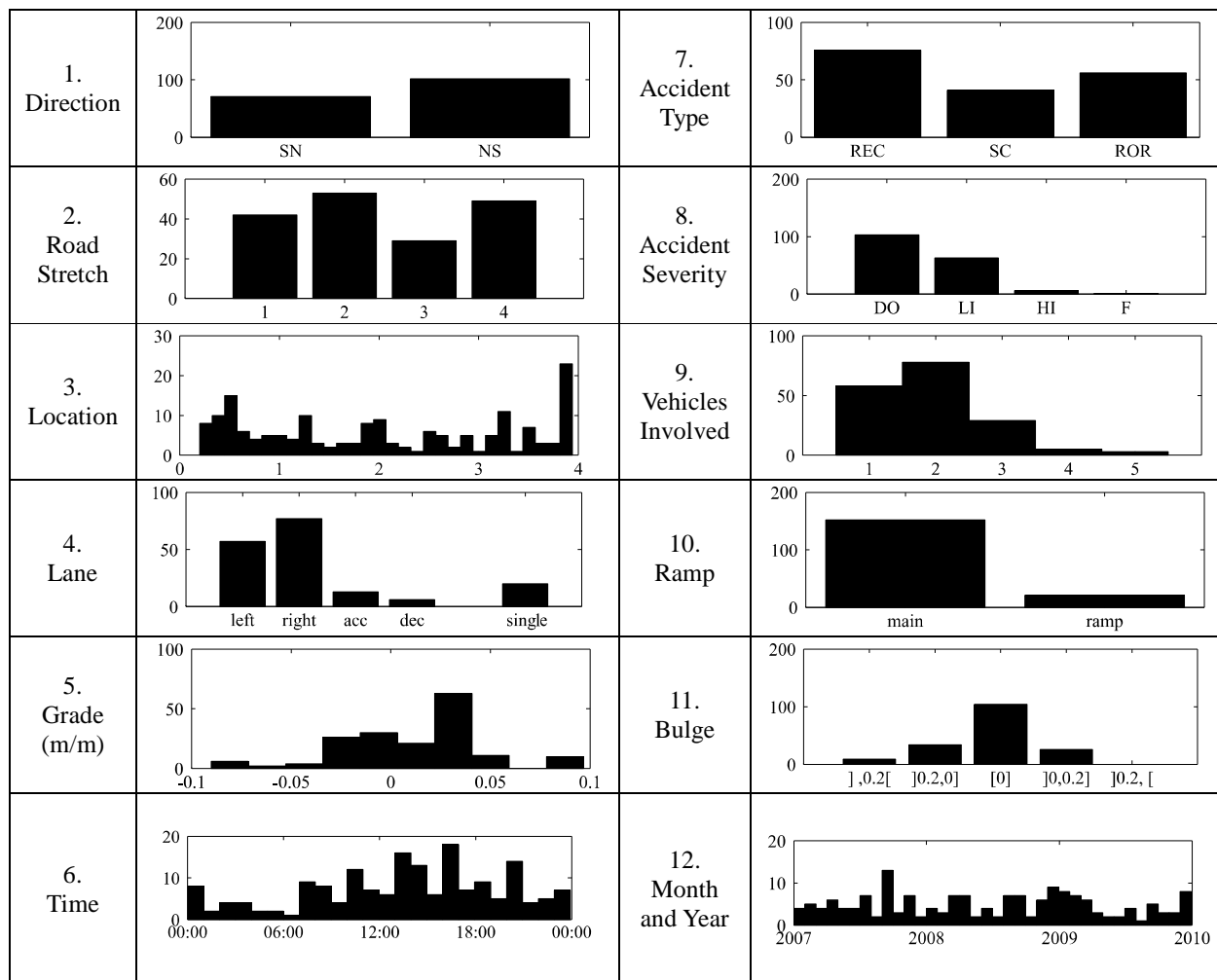
25 The proposed model was estimated with data collected and simulated for the A44 urban  
 26 motorway near Porto, in Portugal. This road was selected as case study due to its dense traffic,  
 27 unusually high number of lane changes, short spacing between interchanges and high percentage  
 28 of heavy goods vehicles. A44 is a 3,940m long dual carriageway urban motorway with 5 major  
 29 interchanges, two 3.50m wide lanes and 2.00m wide paved shoulders in each direction (see  
 30 Figure 4 for overall layout). There are acceleration and deceleration lanes at all interchanges,  
 31 although several as short as 150m. On and off-ramps connect to local roads, which generally  
 32 have tight horizontal curves, intersections or pedestrian crossings, features that tend to impose  
 33 significant reductions in vehicle speeds.  
 34

35 Three different traffic data sets were specifically collected for the present study: a  
 36 dynamic seed origin-destination (OD) matrix based on a sample of license plate matching and  
 37 vehicle counts ; 5 min loop sensor average speeds and counts for the existing eight traffic  
 38 stations (4 in each direction), between 2007 and 2009 32; and vehicle trajectories collected for a  
 39 generic morning (with and without congestion) by aerial remote sensing for the entire length and  
 40 access links of the A44 motorway (33). Finally, incident records were also collected for the same  
 41 period of 2007 to 2009 including a total of 144 side-collisions, rear-end collisions and run-off-  
 42 road accidents.

43 In Table 1 the distribution of the collected variables are presented (ordinates are the  
 44 number of observations). During the period of analysis one fatality, seven serious injuries and 98  
 45 slight injuries were registered in the concessioned area.

- 1 From Table 1 a few particular observations deserve to be mentioned:  
 2 • Variable 3 (event location of occurrence, in kilometers) shows that the edging  
 3 interchanges and adjacent sections have a big share of accident records. This is due to the  
 4 motorway layout, with frequent lane changes and speed variations during dense traffic  
 5 conditions due to route choice manoeuvres.

6 **TABLE 1 Accident Data Statistics.**  
 7



- 8  
 9 • From variable 7 (type of accident), 44% of the accidents are rear-end-collisions (*REC*),  
 10 32% are run-of-road (*ROR*) and 24% are lateral collisions (*SC*). Variable 9 (number of  
 11 vehicles involved in an accident) also indicates that vehicle interactions have a clear  
 12 impact in A44 safety records.  
 13 • Variable 8 (severity) represents the consequences of the occurrence, with '*DO*' as  
 14 damage-only accidents, '*LI*', '*HI*' and '*FI*' as light injury, severe injury and fatality.  
 15 • From variable 10 (type of section), 12% of the accidents occurred on entry or exit links.  
 16 • Variable 11 (bulge) is defined as the tangent of 1/4 of the included angle of the arc  
 17 between the curve vertex edges. A negative bulge value indicates that the arc goes  
 18 clockwise from the selected vertex to the next vertex. A bulge of 0 indicates a straight

1 segment. Bulge is directly linked to the curve radius and is a proxy of spiral curves. It is  
 2 commonly used in simulation software and was kept as such for easier simulation  
 3 modelling. The road curvature affects accident frequency, but the impact of this variable  
 4 is significantly smaller under denser traffic conditions such as the generic A44 daylight  
 5 traffic scenario. This partially explains the lower share of run-of-road accidents, when  
 6 compared with the typical share of run-of-road accidents on dual carriageway.

- 7 • Variable 5 (grade) had an important impact, due to the steep grade on the North-South  
 8 direction near the southern interchange, slowing down all vehicles considerably,  
 9 especially the heavy vehicles, and resulting in local congestion and higher *REC* rates.
- 10 • Both variables 5 and 11 (bulge and grade) were obtained after a manual georeferencing  
 11 of all road accidents.

12 Along with the 5 min intervals for the observed traffic data, the above accident location  
 13 records required spatial observation units of 50 m. These units are those considered for  
 14 aggregation of individual probabilities. Using those units, more than  $180 \times 10^6$  no-accident (*NA*)  
 15 events were observed during the three years period. Excluding the days with bad sensor data, a  
 16 random sampling technique was used to select 6,400 *NA* events, resulting in a total of 6,544  
 17 events to be calibrated and simulated for artificial data generation.

18 The integrated driver behaviour model (22) implemented in *MITSIMLab* (34) was used  
 19 to simulate trajectories for each observed event. The global multi-step sensitivity-analysis based  
 20 calibration proposed in (35) was used for the calibration of every sampled event. The method  
 21 was then coupled with a meta-model based calibration, for calibrating the simulator with  
 22 trajectory data, and with a powerful simultaneous demand-supply calibration method for the  
 23 calibration of the large set of accident and non-accident events using aggregated data. The  
 24 microscopic simulation tool is calibrated once using a pre-estimated seed OD matrix estimated  
 25 from historical license plate matching, and both aggregate (loop sensor-based speeds and counts)  
 26 and disaggregate (observed vehicle trajectories) data collected for a specific generic day. The  
 27 demand and a sub-set of the overall microscopic simulation model parameters are then re-  
 28 calibrated for each of the sampled events using the event specific loop sensor data. After this, the  
 29 calibrated set of parameters for each event is used to generate a set of event specific (artificial)  
 30 detailed traffic variables.

31 The artificial data generated by the calibrated models showed a clear divergence  
 32 between accident and non-accident event simulated outputs typically used in safety assessment -  
 33 see detailed methods and results in (36).

## 34 **RESULTS**

### 35 **Modeling assumptions**

36 Computing the *RE* and *ROR* model components requires that both longitudinal and lateral skid  
 37 resistance are specified. On-site measured values were not available; hence, generic values were  
 38 adopted (37): a linear variation from 0.85 at 0km/h to 0.75 at 130km/h for dry pavements; and  
 39 from 0.70 at 0km/h to 0.20 at 130km/h for wet pavements. An increase factor of 1.10 was  
 40 considered for the lateral skid resistance. Furthermore, heavy vehicles' longitudinal and lateral  
 41 values were decreased by a factor of 0.70 under dry conditions.

42 The availability of each occurrence alternative was included in the specification of the  
 43 likelihood function. For each observation:

- 44 • *RE* conflicts are possible whenever the subject vehicle is in car-following state;

- 1 • *LC* conflicts are possible if the road carriageway has two or more lanes and the subject
- 2 vehicle wants to change lanes;
- 3 • *ROR* event are possible if the road section is a curve or the subject vehicle is changing
- 4 lanes.

5 Multiple replications should be used directly in the estimation phase within a Monte  
 6 Carlo process. The main burden in such an approach is the computer memory and processing  
 7 resources needed during the estimation phase. In the current study, the estimation process was  
 8 carried using only three replications and considering each of them independent. Finally, the  
 9 maximum likelihood estimates of the model parameters are calculated by maximizing this  
 10 function:

$$11 \quad \mathcal{L} = \sum_{s,p} \sum_k y_{k,s,p} w_k \ln[P_{s,p}(k)] \quad (8)$$

12 where  $k$  are all possible outcomes considered for the proposed model,  $P_{s,p}(k)$  is the probability  
 13 of outcome  $k$  for spatial interval  $s$  and time period  $p$  (given by equation 8),  $w_k$  is the outcome  $k$ -  
 14 specific sampling ratio,  $y_{k,s,p}$  is 1 if  $k$  is the observed outcome for the observation pair  $s, p$  and 0  
 15 otherwise. In this study, the *PythonBIOGEME* open source software was used 38. For numerical  
 16 reasons, it is good practice to scale the data so that the absolute values of the parameters are  
 17 between 0 and 1; thus, all relative gap variation variables were divided by 10 and the lateral  
 18 acceleration differences specified in  $0.1\text{m/s}^2$ .

## 21 Model Estimation Results

22 The estimation results are presented in Table 2.

23 When the positive  $RA^{need}$  component is close to zero, the relative deceleration is close  
 24 to *DRAC* and thus closer to a safe situation. When  $RA_+^{need}$  increases the probability for a *RE*  
 25 accident is higher, as the difference between the vehicle relative deceleration rate and its *DRAC*  
 26 gets higher.  $\beta_1^{RE}$  has a higher absolute magnitude than  $\beta_2^{RE}$ , penalizing much more any safety  
 27 decay in the unsafe domain ( $RA^{need} > 0$ ) rather than in the safe one ( $RA^{need} < 0$ ). Regarding the  
 28 negative component, i.e. when the follower has already adjusted its acceleration, lower  $RA_-^{need}$   
 29 results in higher *RE* probabilities, due to lower *TTC*. The positive sign of  $\beta_3^{RE}$  and its statistical  
 30 significance make the consideration of different exogenous safety conditions non-negligible.  
 31 Both the vehicle category (car/truck or bus) and the pavement (wet/dry) conditions were  
 32 considered.

33 The parameters of the negative components of the lead and lag gap variation during *LC*  
 34 events ( $\beta_2^{LC}$  and  $\beta_4^{LC}$ ) are also significant: largest absolute values of its independent variables  
 35 ( $RG_{lag}$  and  $RG_{lead}$ ) represent significantly shrinking gaps. As both parameters are negative, any  
 36  $RG_{lag}$  or  $RG_{lead}$  will increase the probability of *LC* accidents. The lead relative gap variation  
 37 came out as the most statistically significant regarding *LC* events and its higher magnitude is due  
 38 to the much smaller simulated lead gaps during lane-change not only when compared to lag gaps  
 39 but also when comparing accident events with no-accidents.

40  
41  
42  
43  
44  
45  
46

TABLE 2 Estimation results.

| Event Parameter  | Value   | Standard deviation | t-stat | p-val |
|--|---------|--------------------|--------|-------|
| <b>Rear-end conflict</b>                                 |         |                    |        |       |
| RE constant $\beta_0^{RE}$                               | -13.09* | 0.608              | -5.08  | <0.01 |
| Positive relative needed deceleration $\beta_1^{RE}$     | 2.917   | 0.917              | 3.18   | 0.01  |
| Negative relative needed deceleration $\beta_2^{RE}$     | -1.92   | 0.784              | -2.45  | 0.03  |
| Maximum available deceleration ratio $\beta_3^{RE}$      | 2.03    | 1.034              | 1.96   | 0.07  |
| <b>Lane-change conflict</b>                              |         |                    |        |       |
| LC constant $\beta_0^{LC}$                               | -7.08*  | 0.457              | 6.32   | <0.01 |
| Positive relative lag gap variation $\beta_1^{LC}$       | -0.011  | 0.012              | -0.92  | 0.38  |
| Negative relative lag gap variation $\beta_2^{LC}$       | -0.568  | 0.338              | -1.68  | 0.12  |
| Positive relative lead gap variation $\beta_3^{LC}$      | -0.311  | 0.255              | -1.22  | 0.25  |
| Negative relative lead gap variation $\beta_4^{LC}$      | -0.628  | 0.315              | -1.99  | 0.07  |
| <b>Run-off-road event</b>                                |         |                    |        |       |
| ROR constant $\beta_0^{ROR}$                             | -12.45* | 0.367              | -6.68  | <0.01 |
| Positive lateral acceleration difference $\beta_1^{ROR}$ | 0.023   | 0.013              | 1.77   | 0.10  |
| Negative lateral acceleration difference $\beta_2^{ROR}$ | 1.775   | 0.965              | 1.84   | 0.09  |
| Scale parameter for the accident nest $\mu$              | 1.622   | 0.567              | 2.86   | 0.01  |

Number of parameters 13 (\* are parameters affected by weights)

Sample size: 10733084 (3 replications)

Initial log-likelihood: -9636.49

Final log-likelihood: -2047.53

$\rho^2$ : 0.787

$\bar{\rho}^2$ : 0.786

Regarding ROR events, when  $\Delta a^{lat}$  is positive, the simulated lateral acceleration is higher than the critical lateral acceleration and the vehicle is under unsafe conditions. Thus, when  $\beta_1^{ROR} > 0$  there is a higher probability of ROR events. Similarly, when  $\Delta a^{lat}$  is negative, larger absolute values are related to safer conditions, as the simulated lateral acceleration is much smaller than the critical one ( $\beta_2^{ROR} < 0$ ). Yet, one would expect a higher absolute magnitude for  $\beta_1^{ROR}$ ; but these results may be due to the small number of observations with  $\Delta a^{lat} > 0$ .

The estimated scale parameter of the accidents nest  $\mu$  was also significant, revealing a non-negligible effect of shared unobserved attributes of the different types of accident analysed.

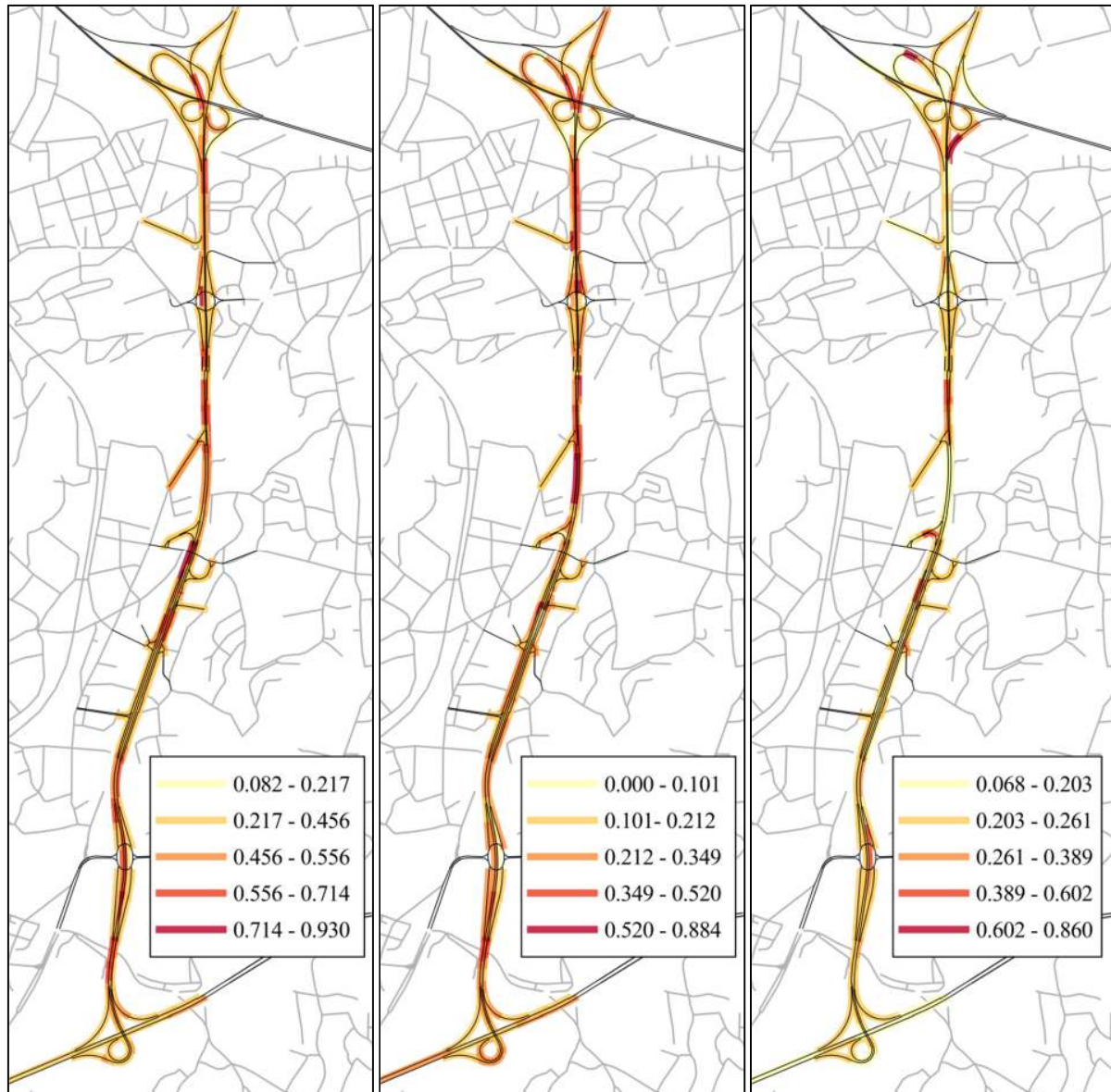
### Model Application

In Figure 4 an example of a simple use of the proposed model is presented. The model was run for a new set of artificial trajectories for the same sampled events on the A44. The values of the estimated probabilities shown in this Figure represent the average probability of observing a specific type of accident for 50m and 5min units:

- The probability of RE and SC events is larger in sections close to weaving areas. This is due to the dense traffic and to short interchange spacing in A44. High accelerations and decelerations along with short lateral gaps are associated with such probabilities.

- 1       • The upper half sections of the A44 close to entry and exit ramps have a higher probability  
 2 of *SC*, mainly due to the absence of dedicated slip roads and sub-standard acceleration  
 3 and deceleration lane lengths. These factors affect drivers gap acceptance, forcing lower  
 4 than normal critical gaps in lane-changing manoeuvres, for vehicles to keep their routes.  
 5       • Similarly, link sections following short entry ramps have higher *RE* event probabilities.  
 6       • A few curved elements clearly show an increased probability of *ROR* events, revealing  
 7 inadequate (simulated) speed choices in those locations.

8



9

10

11 **FIGURE 3 Average probabilities for *RE* (left), *SC* (center) and *ROR* (right) accidents**12 **estimated by 50m and 5 min (in  $10^{-6}$ )**

13

1           The analysis of possible safety levels or thresholds requires further research and is not  
2 discussed in this paper. Yet, the potential of such method is easily envisaged as the simple  
3 application shown in Figure 4 can be extended to simulation integration and used for testing and  
4 comparing alternative design and operational scenarios. That is the case for adding extra lanes to  
5 diverging and weaving sections; locating variable speed limit signs and automatic enforcement  
6 systems; locating additional directional signing; and setting speed limit strategies attending to  
7 both atmospheric and traffic prevailing conditions.

## 8 9 **CONCLUSIONS**

10 A generic framework for modelling cause-effect mechanisms between detailed traffic variables  
11 and accident occurrence probability in traffic microscopic simulation tools was proposed and  
12 tested in a real road environment. Detailed variables of vehicle motion and interactions were  
13 found to be linked to diversity in accident probabilities. The nested structured allowed to capture  
14 existing trade-offs between different types of accidents. The fact that all these considerations  
15 were extracted from simulated analysis shows the real potential of advanced traffic microscopic  
16 simulation regarding detailed safety assessments, as long as detailed calibration is successfully  
17 carried out. The interaction between vehicle gaps and relative motions has been proved as a key  
18 factor for accident occurrence in previous safety related studies. Yet, no probabilistic formulation  
19 accommodating such interaction and integrated in traffic simulation models had previously been  
20 reported in the literature.

21           Several enhancements regarding the specific formulation of the proposed probabilistic  
22 safety model for urban motorways may be introduced. The inclusion of further components in  
23 the safety scoring function (e.g.: driver related variables), the formulation of non-linear safety  
24 score functions, the specification of additional accident types and the definition of more powerful  
25 modelling structures, such as the mixed logit, or estimation methods, such as a panel data  
26 estimation based on multiple replications, may be tested.

27           At this stage, *MITSIMLab* and the presented probabilistic safety models may be used for  
28 testing and benchmarking alternative design layouts and operational scenarios on Portuguese  
29 urban motorways. Safety and mobility consequences of decisions regarding safety  
30 countermeasures (such as those described in 39) may be evaluated at the design stage. Starting  
31 from the detailed calibrated base scenario, the microsimulation may be used for detecting  
32 dangerous sections for each accident type, and afterwards for evaluating the changes in the road  
33 safety scores due to alternative scenarios (ex.: adding an extra lane at a dangerous diverging  
34 sections; changing lane assignment before an exit; changing variable speed limit AADT ranges;  
35 and suiting speed limits to rainfall conditions). The fact that these safety score benchmarkings  
36 are carried out for each type of accident also allows for an indirect consideration of accident  
37 severity in the scenario comparisons.

## 38 39 **REFERENCES**

- 40 1. Barceló, J. (ed.). *Fundamentals of Traffic Simulation*, 1<sup>st</sup> Edition. Springer, 2010.
- 41 2. Punzo, V. and Ciuffo, B. How Parameters of Microscopic Traffic Flow Models  
42 Relate to Traffic Dynamics in Simulation. In *Transportation Research Record:  
43 Journal of the Transportation Research Board*, No. 2124, Transportation  
44 Research Board of the National Academies, Washington, D.C., 2009, pp. 249–  
45 256.
- 46 3. Lord, D., Mannering, F. L., Savolainen, P. T., Quddus, M. A. The statistical  
47 analysis of crash-frequency data: A review and assessment of methodological

- 1 alternatives. *Transportation Research Part A: Policy and Practice* Vol. 44 (5),  
2 2010, pp. 291–305.
- 3 4. Savolainen, P. T., Mannering, F. L., Lord, D., Quddus, M. A.. The statistical  
4 analysis of highway crash-injury severities: A review and assessment of  
5 methodological alternatives. *Accident Analysis & Prevention* Vol. 43 (5), 2011,  
6 pp. 1666–1676.
- 7 5. Hydén, C. *The development of a method for traffic safety evaluation: The*  
8 *Swedish Traffic Conflicts Technique*. Technical Report, Lund University,  
9 Sweden, 1987.
- 10 6. Abdel-Aty, M., Pemmanaboina, R., Hsia, L. Assessing crash occurrence on  
11 urban freeways by applying a system of interrelated equations. In: *Proceedings of*  
12 *the 85th Annual Meeting of the Transportation Research Board*. Washington  
13 D.C., USA, 2006.
- 14 7. Abdel-Aty, M., Pande, A., Lee, C., Gayah, V., 2007. Crash risk assessment using  
15 intelligent transportation systems data and real-time intervention strategies to  
16 improve safety. *Journal of Intelligent Transportation Systems* Vol. 11 (3), 2007,  
17 pp. 107–120.
- 18 8. Abdel-Aty, M., Gayah, V. Real-time crash risk reduction on freeways using  
19 coordinated and uncoordinated ramp metering approaches. *ASCE Journal of*  
20 *Transportation Engineering* Vol. 136 (5), 2010.
- 21 9. Cunto, F. and Saccomanno, F. F. “Calibration and validation of simulated vehicle  
22 safety performance at signalized intersections”. *Accident Analysis & Prevention*  
23 Vol. 40 (3), 2008, pp.1171-1179.
- 24 10. Gettman, D., Sayed, T., Pu, L., Shelby, S. *Surrogate Safety Assessment Model*  
25 *and Validation*. Technical Report. Federal Highway Administration, Virginia,  
26 USA, 2008.
- 27 11. Lareshyn, A., Svensson, A., Hydén, C. Evaluation of traffic safety, based on  
28 microlevel behavioral data: theoretical framework and first implementation.  
29 *Accident Analysis & Prevention* Vol. 42 (6), 2010, pp. 1637–1646.
- 30 12. Archer, J. *Indicators for traffic safety assessment and prediction and their*  
31 *application in micro-simulation modelling: A study of urban and suburban*  
32 *intersections*. Ph.D. thesis, KTH - Royal Institute of Technology, Sweden, 2005.
- 33 13. Ozbay, K., Yang, H., Bartin, B., Mudigonda, S. Derivation and Validation of a  
34 New Simulation-based Surrogate Safety Measure. In *Transportation Research*  
35 *Record: Journal of the Transportation Research Board*, No. 2083,  
36 Transportation Research Board of the National Academies, Washington, D.C.,  
37 2008, pp. 105-113.
- 38 14. Dijkstra, A., Marchesini, P., Bijleveld, F., Kars, V., Drolenga, H., Maarseveen,  
39 M. V.. Do Calculated Conflicts in Microsimulation Model Predict Number of  
40 Crashes?. In *Transportation Research Record: Journal of the Transportation*  
41 *Research Board*, No. 2147, Transportation Research Board of the National  
42 Academies, Washington, D.C., 2010, pp. 105–112.
- 43 15. Okamura, M., Corporation, A., Fukuda, A., Morita, H., Suzuki, H., Nakazawa,  
44 M. Impact evaluation of a driving support system on traffic flow by microscopic  
45 traffic simulation. In: *3rd International Conference on Road Safety and*  
46 *Simulation*. Indianapolis, USA, 2011.
- 47 16. Huang, F., Liu, P., Yu, H., Wang, W., Jan. 2013. Identifying if VISSIM



- 1 simulation model and SSAM provide reasonable estimates for field measured  
2 traffic conflicts at signalized intersections. *Accident; analysis and prevention*  
3 Vol. 50, 2013, pp. 1014–24.
- 4 17. Songchitruksa, P., Tarko, A. P. The extreme value theory approach to safety  
5 estimation. *Accident Analysis & Prevention*, Vol. 38 (4), 2006, pp. 811–822.
- 6 18. Saunier, N., Sayed, T.. Probabilistic Framework for Automated Analysis of  
7 Exposure to Road Collisions. In *Transportation Research Record: Journal of the*  
8 *Transportation Research Board*, No. 2083, Transportation Research Board of the  
9 National Academies, Washington, D.C., 2008, pp. 96–104.
- 10 19. Wang, W., Jiang, X., Xia, S., Cao, Q. Incident tree model and incident tree  
11 analysis method for quantified risk assessment: An in-depth accident study in  
12 traffic operation. *Safety Science* Vol. 48 (10), 2010, pp. 1248–1262.
- 13 20. Young, W., Sobhani, A., Lenné, M. G., Sarvi, M. Simulation of safety: A review  
14 of the state of the art in road safety simulation modelling. *Accident Analysis &*  
15 *Prevention*, Vol. 66 (C), 2014, pp. 89–103.
- 16 21. Tarko, A. P., Davis, G., Saunier, N., Sayed, T., Washington, S. P. *Surrogate*  
17 *Measures of Safety: A White Paper*. Technical Report n° 3, Transportation  
18 Research Board. ANB20 - Committee on Safety Data Evaluation and Analysis,  
19 USA, 2009.
- 20 22. Toledo, T., Koutsopoulos, H., Ben-Akiva, M. E. Integrated driving behavior  
21 modeling. *Transportation Research Part C: Emerging Technologies* Vol. 15 (2),  
22 2007, pp. 96-112.
- 23 23. Kesting, A., Treiber, M., Helbing, D. General Lane-Changing Model MOBIL for  
24 Car-Following Models. In *Transportation Research Record: Journal of the*  
25 *Transportation Research Board*, No. 1999, Transportation Research Board of the  
26 National Academies, Washington, D.C., 2007, pp. 86-94.
- 27 24. vanWinsum, W., de Waard, D., Brookhuis, K. A. Lane change maneuvers and  
28 safety margins. *Transportation Research Part F: Traffic Psychology and*  
29 *Behavior* Vol. 2 (3), 1999, pp. 139–149.
- 30 25. Zheng, L., Ismail, K., Meng, X.. Freeway Safety Estimation using Extreme  
31 Value Theory Approaches: a comparative study. *Accident Analysis & Prevention*,  
32 Vol. 62, 2014, pp. 32–41.
- 33 26. Chovan, J., Tijerina, L., Alexander, G., Hendricks, D. *Examination of Lane*  
34 *Change Crashes and Potential IVHS Countermeasures*. Technical Report. US  
35 DOT, NHTSA, Washington D.C., USA, 1994.
- 36 27. Hidas, P. Modelling vehicle interactions in microscopic simulation of merging  
37 and weaving. *Transportation Research Part C: Emerging Technologies* Vol. 13  
38 (1), 2005, pp. 37–62.
- 39 28. Ben-Akiva, M. E. *Structure of passenger travel demand models*. Ph.D. thesis,  
40 Massachusetts Institute of Technology, Cambridge, USA, 1973.
- 41 29. Ben-Akiva, M. E., Lerman, S. R. *Discrete choice analysis: theory and*  
42 *application to travel demand*. MIT Press, Cambridge, USA, 1985.
- 43 30. Abdel-Aty, M., Uddin, N., Pande, A. Split Models for Predicting Multivehicle  
44 Crashes During High-Speed and Low-Speed Operating Conditions on Freeways.  
45 In *Transportation Research Record: Journal of the Transportation Research*  
46 *Board*, No. 1908, Transportation Research Board of the National Academies,  
47 Washington, D.C., 2005, pp. 51–58.

- 1 31. Manski, C. F., Lerman, S. R. The Estimation of Choice Probabilities from  
2 Choice Based Samples. *Econometrica* Vol. 45 (8), 1977, pp. 1977–88.
- 3 32. Lima Azevedo, C. *Probabilistic safety analysis using traffic microscopic*  
4 *simulation*. PhD Thesis, Instituto Superior Técnico, University of Lisbon, Portugal,  
5 2014.
- 6 33. Lima Azevedo, C., Cardoso, J. L., Ben-Akiva, M. E. Applying Graph Theory to  
7 Automatic Vehicle Tracking by Remote Sensing. *Proceedings of the 93<sup>rd</sup> Annual*  
8 *Meeting of the Transportation Research Board*, Washington D.C., USA, 2014.
- 9 34. Qi, Y., Koutsopoulos, H. N. and Ben-Akiva, M. E. Simulation Laboratory for  
10 Evaluating Dynamic Traffic Management Systems. In *Transportation Research*  
11 *Record: Journal of the Transportation Research Board*, No. 1710,  
12 Transportation Research Board of the National Academies, Washington, D.C.,  
13 2000, pp. 122-130.
- 14 35. Ciuffo, B. and Lima Azevedo, C. A Sensitivity-Analysis-Based Approach for the  
15 Calibration of Traffic Simulation Models. In *IEEE Transactions on Intelligent*  
16 *Transportation Systems* PP (99), 2014, pp.1-12.
- 17 36. Lima Azevedo, C., Ciuffo, B., Cardoso, J. L. and Ben-Akiva, M. E., 2015.  
18 “Dealing with uncertainty in detailed calibration of traffic simulation models for  
19 safety assessment”. . Accepted for publication in *Transportation Research Part*  
20 *C: Emerging Technologies* (Forthcoming).
- 21 37. Inoue, T., Hioki, Y. Skid resistance monitoring in Japan. *Roads*, Vol. 280, 1993.
- 22 38. Bierlaire, M. BIOGEME: A free package for the estimation of discrete choice  
23 models. In: *Proceedings of the 3rd Swiss Transportation Research Conference*.  
24 Ascona, Switzerland, 2003.
- 25 39. Newman, T, Pfefer, R., Slack, K., Council, F. (2003). Guidance for  
26 Implementation of the AASHTO Strategic Highway Safety Plan. Volume 6: A  
27 Guide for Addressing Run-Off-Road Collisions. AASHTO, Washington, DC