SAFETY ANALYSIS OF PASSING MANEUVERS USING EXTREME VALUE THEORY

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ABSTRACT

The increased availability of detailed trajectory data sets from naturalistic, observational and simulation-based studies are a key source for potential improvements in the development of detailed safety models that explicitly account for vehicle conflict interactions and the various driving maneuvers.

Despite the well-recognized research findings on both crash frequency estimation and traffic conflicts analysis carried out over the last decades, only recently researchers have started to study and model the link between the two. This link is typically made by statistical association between aggregated conflicts and crashes, which still relies on crash data and ignores heterogeneity in the estimation procedure. More recently, an Extreme Value (EV) approach has been used to link the probability of crash occurrence to the frequency of conflicts estimated from observed variability of crash proximity, using a probabilistic framework and without using crash records.

In this on-going study the Generalized Extreme Value (GEV) distribution and the Generalized Pareto Distribution (GPD)-based estimation, in the peak over threshold approach, are tested and compared as EV methods using the minimum time-to-collision with the opposing vehicle during passing maneuvers. Detailed trajectory data of the passing, passed and opposite vehicles from a fixed-based driving simulator experiment was used in this study. One hundred experienced drivers from different demographic strata participated in this experiment on a voluntary base. Several two-lane rural highway layouts and traffic conditions were also considered in the design of the simulator environment. Raw data was collected at a resolution of 0.1 s and included the longitudinal and lateral position, speed and acceleration of all vehicles in the scenario. From this raw data, the minimum time-to-collision with the opposing vehicle at the end of the passing, maneuver was calculated. GEV distributions based on the Block Maxima approach and GPD distributions under the POT approach were tested for the estimation of head-on collision probabilities in passing maneuvers with different results. While the GEV approach achieved satisfactory fitting results, the tested POT underestimated the expected number of head-on collisions. Finally, the estimated GEV distributions were validated using a second set of data extracted from an additional driving simulator experiment.

The results indicate that this is a promising approach for safety evaluation. On-going work of the authors will attempt to generalize this method to other safety measures related to passing maneuvers, test it for the detailed analysis of the effect of demographic factors on passing maneuvers' crash probability and for its usefulness in a traffic simulation environment.

KEYWORDS

Road Safety; Probabilistic Model; Extreme Value; Driving Behavior; Minimum Time-to-Collision

INTRODUCTION

The literature has frequently addressed the advantages of using surrogate safety measures over crash data *(Tarko, Davis, Saunier, Sayed, & Washington, 2009)*, especially nowadays when advanced sensing technologies which facilitate the collection of detailed data on vehicles' trajectories are becoming readily available *(Tarko, 2012)*. Crash data suffer from underreporting and frequently poor quality. Furthermore, the use of crash data is a reactive approach while using surrogate safety measures is a proactive and time-efficient approach *(Archer, 2004)*. The use of aggregate crash data to develop safety models does not provide insights on the crash causations or details on the driver crash avoidance behavior. The use of surrogate safety measures for modeling and estimating safety is considered as a promising approach to achieve those targets. Crashes are also infrequent, the ratio between conflicts and actual crash frequencies, according to *Gettman, Pu, Sayed, and Shelby (2008)*, is 20000 to 1. Thus, there is a clear advantage of using surrogate safety measures over crash data. *Zheng, Ismail, and Meng (2014)* indicate that the validity of a surrogate safety measure is usually determined by its correlation with crash frequency which is usually assessed using regression analysis. For example, *Sayed and Zein (1999)* found a statistically significant relationship between crashes and conflicts with an R^2 in the range of 0.70 - 0.77 at signalized junctions. However, the regression analysis still incorporates the use of crash counts which are known to suffer from availability and quality issues, and thus this approach is limited. Besides, it is difficult to insure the stability of the crash-to-surrogate ratio and this relationship also hardly reflects the physical nature of crash occurrence (*Zheng et al., 2014*). *Jonasson and Rootzén (2014)* concluded that comprehensive and generalized answer to the question "are near-crashes representative for crashes?" may be less useful. Instead careful separate analyses for different types of situations are needed. Recently *Songchitruksa and Tarko (2006)* developed a new and more sophisticated approach based on the Extreme Value (EV) theory to estimate the frequency of crashes based on measured crash proximity. The field of EV theory was pioneered by *Fisher and Tippett (1928)*. It is a commonly applied theory in many fields, such as in meteorology, hydrology, and finance *(Zheng et al., 2014)*. However, *Songchitruksa and Tarko (2006)* indicate that its application in the field of transportation engineering is still limited. According to *Tarko et al. (2009)* the EV approach has three considerable advantages over the traffic conflict technique in the detailed analysis of safety: (1) The EV theory abandons the assumption of a fixed coefficient converting the surrogate event frequency into the crash frequency; (2) the risk of crash given the surrogate event is estimated for any condition based on the observed variability of crash proximity without using crash data; (3) the crash proximity measure precisely defines the surrogate event. This method has the potential to estimate the probability of extreme events from relatively short period of observations and it proposes a single dimension to measure the severity of surrogate events and to identify crashes. The implicit assumption of the EV theory is that the stochastic behavior of the process being modeled is sufficiently smooth to enable extrapolation to unobserved levels *(Songchitruksa & Tarko, 2006)*. In the context of road safety, the more observable traffic events are used to predict the less frequent crashes, which are often unobservable in a short time period *(Zheng et al., 2014)*. More recently, *Songchitruksa and Tarko (2006)* used an EV approach to build up relationships between occurrence of right-angle crashes at urban intersections and frequency of traffic conflicts measured by using post-encroachment time. A major improvement of this study is that it links the probability of crash occurrence to the frequency of conflicts estimated from observed variability of crash proximity, using a probabilistic framework and without using crash records. The generic formulation of the application of EV to road safety analysis was then proposed by *Tarko (2012)* and it was only very recently applied to other crash types and data sets *(Jonasson & Rootzén, 2014; Zheng et al., 2014)*.

In this study the time-to-collision or TTC *(Svensson & Hydén, 2006)* will be used as a surrogate safety measure of the risk to be involved in a head-on collision with the opposite vehicle while passing on two-lane rural highways, using the EV approach. According to *NHTSA (2003)* head-on collisions constitute 2.3% of the total crashes on two-lane highways, but they are responsible for 10.4% of the total fatal crashes. Not many studies have focused on the detailed analysis of the link between passing maneuvers and head-on-collisions. The TTC was previously used by *Farah, Bekhor, and Polus (2009)* to evaluate the risk of passing behavior on two-lane rural highways. The authors defined the minimum TTC, as the remaining gap between the passing vehicle and the opposing vehicle at the end of the passing process. This measure expresses the risk involved in the passing maneuver. The authors developed a Tobit regression model that explains the minimum TTC. Traffic related explanatory variables were found to have the most important effect on the minimum TTC, but also the road geometric design and the driver characteristics were also found to have a significant contribution. Other studies *(Shariat-Mohaymany, Tavakoli-Kashani, Nosrati, and Ranjbari, 2011; Hegeman, 2008)* as well used the TTC to as a measure for head-on conflicts.

There are two families of EV distributions which follows two different approaches to sample extreme events: (1) the Generalized Extreme Value (GEV) distribution which is used in the block maxima or minima (BM) approach, in which maxima over blocks of time (or space) are considered; (2) the Generalized Pareto Distribution (GPD) which is used in the peak over threshold approach *(Fuller & Poter, 2011)*, where all values above some high level are used. Previous studies suggested that the POT approach is more effective in conditions of short-time observations and from the aspect of estimate accuracy and reliability *(Songchitruksa & Tarko, 2006; Zheng et al., 2014)*. In this study both distributions will be examined and compared.

RESEARCH METHOD

This section discusses the modeling approach, the laboratory experiments designed to collect the data, the characteristics of the participants in the study, and a preliminary statistics of the collected data.

Modeling Details

In this study two families of extreme value distributions are used to sample extreme events: (1) Block Maxima (BM) approach using the GEV distribution; and (2) Peak Over Threshold (POT) approach using the GPD. The following paragraphs describe those two approaches in detail.

Block Maxima (BM) Approach Using the Generalized Extreme Value (GEV)

In the GEV distribution the extreme events are sampled based on the block maxima (BM) approach. Following this approach the observations are aggregated into fixed intervals over time and space, and then the extremes are extracted from each block by identifying the maxima in each single block. Mathematically, the standard GEV function is as follows (*Zheng et cl., 2014*):

$$G(x) = \exp\left(-\left[1+\xi\left(\frac{x-\mu}{\sigma}\right)\right]^{\frac{-1}{\xi}}\right) \qquad (eq.\ 1)$$

where, $\{X_1, X_2, \ldots, X_n\}$ is a set of independently and identically distributed random observations with unknown distribution function $F(x) = \Pr(X_i \leq x)$, the maximum $M_n = max\{X_1, X_2, \ldots, X_n\}$ will converge to a GEV distribution when $n \to \infty$. Three parameters identify this distribution: the location parameter, $-\infty < \mu < \infty$; the scale parameter, $\sigma > 0$; and the shape parameter, $-\infty < \xi < \infty$. If the shape parameter, ξ, is positive, then his would yield the Frechet cdf with a finite lower endpoint, $(\mu - \sigma/\xi)$, if ξ is negative, this will yield the (reversed) Weibull cdf with finite upper endpoint $(\mu + \sigma/|\xi|)$, and if $\xi = 0$ this yields the Gumbel cdf.

The BM method can also be used to study minima by considering the maxima of the negated values instead of minima of the original values. This is how the minimum TTC will be handled in this study.

For the BM approach, and in the case that most blocks have enough observations, the r-largest order statistics is recommended, it enables the incorporation of more than one extreme from each interval in order to increase the confidence of parameter estimates. It is usually recommended to have at least a sample of 30 maxima (or minima). The size of the chosen interval should be large enough so that there are enough observations from which a maxima is chosen in which it is truly an extreme value, and small enough to provide a sample larger than 30.

Peak Over Threshold Using the Generalized Pareto Distribution (GPD)

According to the GPD an observation is identified as an extreme if it exceeds a predetermined threshold. The distribution function of exceedances X over a threshold μ for a set of independently and identically distributed random observations $\{X_1, X_2, \ldots, X_n\}$ is: $F_u(x) = \Pr(X - u \leq x | X > u)$. With a high enough threshold u, the conditional distribution $F_u(x)$ can be approximated by a GPD. The function of GPD is given as follows:

$$G(x) = 1 - \left[1+\left(\frac{\xi \cdot x}{\sigma}\right)\right]^{\frac{-1}{\xi}} \qquad (eq.\ 2)$$

where $\sigma > 0$ is the scale and $-\infty < \xi < \infty$ is the shape parameter, respectively.

Similarly to the BM approach, the determination of the threshold in the POT approach determines the sample size. Therefore, an optimal threshold should be chosen so that the observations that exceed the threshold are real extremes, but still constitute a reasonable sample with relatively small variance. Choosing a small threshold will bias the results by considering normal observations as extremes, while choosing a high threshold would result with a few observations as extremes and thus large variability which also would bias the estimation results of the distribution.

In this study, both models' parameters were estimated using the maximum likelihood method (ML) in R (v3.0.3) using the `exTremes` and `evd` packages *(Gilleland & Katz, 2011)*. Details on the statistical properties of the GEV and GPD can be found in *Coles (2001)* and on the theoretical background of its applicability for surrogate safety analysis in *Tarko (2012)*.

Examination of the Criteria for Using EV Theory

When using the EV approach there are three main criteria that should be examined and addressed. These are: sample size, serial dependency, and non-stationarity *(Zheng et al., 2014)*. With respect to the sample size, in the BM approach the interval size determines the sample size while in the POT approach, the chosen threshold determines the sample size. In both approaches the target is to achieve a balance between bias and variance as discussed above. In the case of passing maneuvers, it is possible to assume that the different TTCs resulting from each passing maneuver is independent from each other since the dataset included cases where a single vehicle overtake another single vehicle. However, since these maneuvers are non-stationary, and various factors (road design, traffic conditions, driver characteristics) might affect the measured TTCs and increase the heterogeneity, it does not hold to assume that the TTCs are identically distributed. To solve this problem several covariates will be considered in the estimation procedure.

Estimation of the Risk of Passing Maneuvers

A passing maneuver is considered to be a risky maneuver as it requires from a fast driver who wants to pass a slow driver to search and decide on an appropriate gap in the traffic on the opposite direction and execute this maneuver while maintaining safe distances from all the surrounding vehicles. Therefore, a driver failure to correctly estimate the safe distances from the surrounding vehicles might lead for several potential types of collisions, such as a collision with the opposing vehicle, the passed vehicle, or run of the way crashes. This paper will focus on the risk of head-on collisions.

A quite often used measure for estimating the risk of a head-on collision is the TTC. The TTC is defined by *Hayward (1972)* as the time left to collision between two vehicles if they remain on their paths and continue with constant speeds. *Minderhoud and Bovy (2001)* defined two TTC indicators for risk. The first is the Time Exposed Time to Collision which is the total sum of the times that a driver spent with sub-critical TTC. The second is the Time integrated Time to Collision which is the time integration of the difference between the critical and actual TTC during the time spent with sub-critical TTC. In this study, the minimum TTC to the front vehicle in the opposite lane at the end of the passing maneuver will be used as a head-on collision proximity measure *(Svensson & Hydén, 2006)*. This is actually the most critical time-to-collision in a passing maneuver. This measure has been used by several previous studies *(Farah et al., 2009; Hegeman, Tapani, & Hoogendoorn, 2009; Kiefer, Flannagan, & Jerome, 2006)*, and proved to be a valuable measure for risk of head-on collisions.

Laboratory experiment

A laboratory experiment using a driving simulator previously developed by *Farah et al. (2009)* for modelling drivers' passing behavior on two-lane highways was used in order to collect data on the time-to-collision with the opposing vehicle. The simulator used in this experiment, STISIM *(Rosenthal, 1999)*, is a fixed-base interactive driving simulator, which has a 60 horizontal and 40 vertical display. The driving scene was projected onto a screen in front of the driver. The simulator updates the images at a rate of 30 frames per second. The situations that participants encountered were defined by the vehicles shown in

FIGURE 1. The subject vehicle is passing an impeding vehicle (front vehicle) while another vehicle is approaching from the opposite direction. This paper focuses on the minimum TTC surrogate safety measure while passing on two-lane rural highways. Mathematically, the TTC is calculated by the division of the distance between the fronts of the subject vehicle and the opposing vehicle by the sum of their speeds. The minimum TTC is the TTC value at the end of a successful passing maneuver.

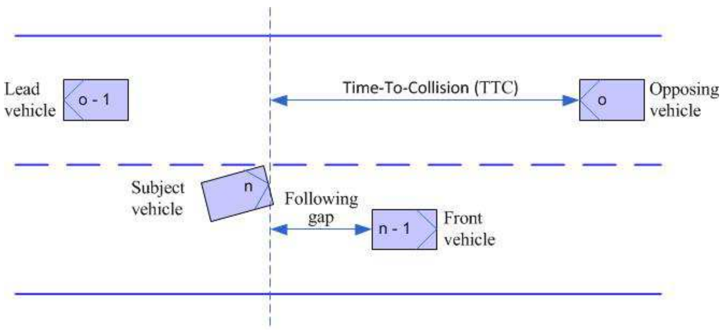

FIGURE 1 - TTC with the Opposing Vehicle

To understand how various infrastructure and traffic factors affect the TTC when passing, a number of simulator scenarios were designed. Each scenario included 7.5 km of two-lane rural highway section with no intersections. The road sections were on a level terrain and with daytime and good weather conditions, which allowed good visibility. However, each scenario design varied according to four main factors of two levels each. The choice of these factors was based on previous studies that showed their impact on passing decisions. Two levels were used for each factor. These factors are: speed of the front vehicle (60 or 80 km/h); speed of the opposite vehicle (65 or 85 km/h); opposite lane traffic volume (200 or 400 veh/h); and road curvature (300-400 m or 1500-2500 m). This produces (2^4)16 different scenarios. The partial confounding method *(Hicks and Turner, 1999)* was used to allocate for each driver 4 scenarios out of the 16 scenarios. Detailed information on this experiment can be found in *Farah et al. (2009)*.

Participants

One hundred drivers (64 males and 36 females) with at least 5 years of driving experience participated in the driving simulator experiment on a voluntary base. The drivers' age ranged between 22 and 70 years old. Drivers were instructed to drive as they would normally do in the real world. An advertisement on the experiment was published at the Technion campus in Israel and drivers who were interested to participate contacted the researchers.

The data

The data set from the driving simulator experiment resulted in 1287 completed passing maneuvers, in which 9 ended with a front-front collision (these observations were removed from the estimation data sets). Table 1 below present a summary statistics of passing maneuvers related variables.

TABLE 1- Data summary statistics

Variable	**mean**	**median**	**15th percentile**	**85th percentile**
Accepted passing gap (s)	21.47	20.75	17.39	28.79
Passing duration (s)	4.98	4.83	3.50	6.48
Passing vehicle speed (m/s)	22.21	21.29	17.27	27.39
Front vehicle speed (m/s)	66.20	60.00	60.00	80.00
Opposing vehicle speed (m/s)	76.28	85.00	65.00	85.00
Following distance from front vehicle when starting to pass (m)	15.47	12.80	8.39	22.92
Minimum TTC (s)	2.37	1.98	0.76	4.10
Gap from passed front vehicle at end of the passing maneuver (s)	2.44	2.24	1.49	3.42

Passing gaps were defined as the gap between two successive opposite vehicles at the time the lead vehicle on the opposite lane is at the same line with the subject vehicle. The passing duration is measured from the moment the subject vehicle left front wheel crosses the center line (as shown in Figure 1) until the passing maneuver ends when the rear left wheel crosses the centerline. Vehicles' speeds as summarized in the Table 1 are measured at the beginning of the passing maneuver. The following distance from front vehicle when starting to pass is measured as the distance between the front of the subject vehicle and the end of the front vehicle as illustrated in Figure 1. Finally, the minimum TTC and the gap from passed front vehicle are both measured at the end of the passing maneuver and reflect the risk to collide with the opposing vehicle, and the front vehicle, respectively.

RESULTS AND ANALYSIS

This section presents the results of the analysis following the research method described above. First, the estimation results of the BM using the GEV model is presented followed by the estimation results according to the POT using the GPD, and a comparison and discussion of the results.

Block Maxima Approach (BM) Results

A Generalized Extreme Value (GEV) distribution is fitted to the 1287 passing maneuvers and the respective minimum TTC measurements. For the block intervals we use the annotated time that contain the entire passing maneuver. Both the chosen block interval and the resulting number of observations in each block are variable. In this case, the calculated probability represents the probability of a head-on collision for a single passing maneuver. Furthermore, past studies concluded that with minimum TTC smaller than a low limit (typically, 1 to 1.5 s) are useful as crash surrogates *(Hýden, 1987; Jonasson and Rootzén, 2014)*. As a first test, the filtered data according to this approach, and choosing a limit of 1.5 s, resulted in 463 maxima. FIGURE 2 (*left*) presents the Cumulative Distribution Function (CDF) of the minimum TTC (*min*{TTC}) for the full data set, while FIGURE 2 (*right*) presents the CDF of the *min*{TTC} for the filtered data. For the full data set, 50% of the observations were less than a TTC of about 2 s, while in the filtered data, 50% of the observations were less than a TTC of about 0.9 s.

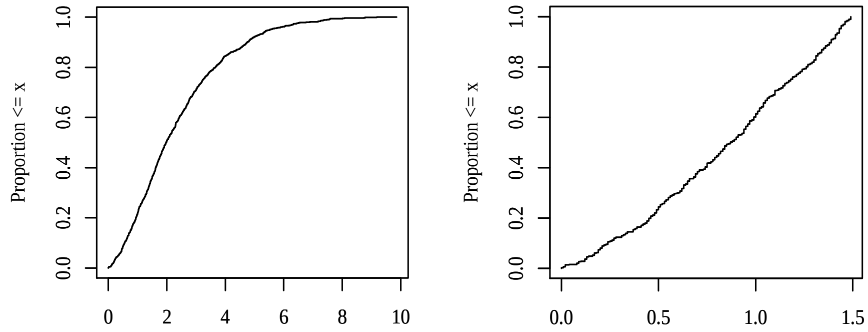

FIGURE 2 – CDF of minTTC (s) for the full dataset (left) and filtered data (right)

Similarly to the approach proposed by *Jonasson and Rootzén (2014)*, we first estimate a stationary block maxima model for the maxima of the negated values instead of minima of the original values, i.e. *max*{-TTC}. The fitted distribution resulted in the following parameters of the GEV cumulative distribution function: $\hat{\mu} = -0.993\ (0.0212)$, $\hat{\sigma} = 0.383\ (0.0163)$ and $\hat{\xi} = -0.236\ (0.0500)$. FIGURE 3 (*left*) presents the probability density function of the empirical and modeled negated TTC, and FIGURE 3 (*right*) presents the simulated QQ plot. From these figures it can be concluded that the modeled GEV distribution has satisfactory fitting results to the empirical data since the points fall close to the 45° line in the simulated QQ plot.

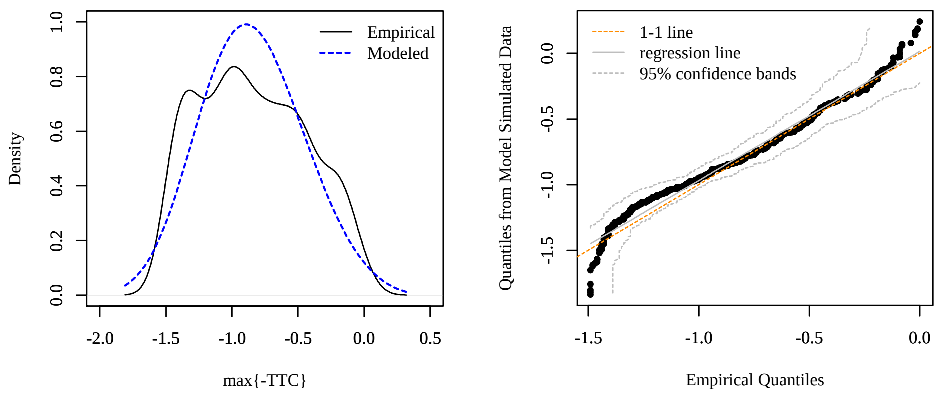

FIGURE 3 – Probability Density plot (left) and simulated QQ plot (right) for the stationary Block Maxima model

With this stationary model using the fitted GEV distribution, the estimated probability of *max*{-TTC}≥0 is 0.0179 with 95% confidence interval (0.0177,0.0182). The confidence intervals of estimations were computed assuming the normal distribution under regularity conditions of the parameters, a simulation experiment size of 1×10^6 and its simulated distribution quantiles. Out of the 463 near head-on collisions in the driving simulator (using the threshold of 1.5 s), 9 maneuvers ended with actual collisions. In other words, the probability for a head-on collision assuming a near head-on collision in a passing maneuver is 9/463= 0.0194, with a 95% binomial confidence interval (0.00893, 0.0366). This value is comparable to the estimate resulting from the fitted GEV distribution.

However, the process of passing maneuver may be affected by the detailed conditions of each specific passing, such as the relative gaps and speeds between the vehicles surrounding the subject vehicle. To account for the fact that the TTCs at the end of the passing maneuvers are non-stationary observations and are affected by several factors, we tested the inclusion of different covariates that were collected during the driving simulation experiment in the location parameter of the BM model:

Several linear combinations of these variables were tested during model estimation task. To test reduced model structures and the inclusion of variables, the likelihood ratio test was used *(Coles, 2001)*. The final model was also tested against the stationary one, resulting in a p-value (3.741×10^{-8}) significantly smaller than *alpha = 0.05*.

TABLE 2 - List of covariates considered in the Block Maxima (BM) Approach

Acronym	**Description**
passinggap	The time gap between two opposite vehicles at the time the subject meet the lead opposite vehicle (s)
speedopposing	The speed of the opposite vehicle at the moment of start passing (m/s)
speedfront	The speed of the front vehicle at the moment of start passing (m/s)
tailgatetp	The time gap between the subject vehicle and the front vehicle at the moment of start passing (s)
passduration	The passing duration (s)
curvature	The road curvature (1/m)

TABLE 3 - Estimation results of best models for BM approach

	Parameter	Estimated value	Standard error
$\hat{\mu}$	$\hat{\mu}_0$	-1.06	0.139
	$\hat{\mu}_1$ *(speedfront)*	0.0245	0.00644
	$\hat{\mu}_2$ *(tailgatetp)*	0.00274	0.00179
	$\hat{\mu}_3$ *(passinggap)*	-0.0212	0.00445
	$\hat{\mu}_4$ *(curvature)*	-38.1	13.5
	$\hat{\sigma}$	0.369	0.0145
	$\hat{\xi}$	-0.225	0.0412
	N	463	
	Neg. loglikelihood	215.54	

The results in TABLE 3 indicate that as the speed of the front vehicle increases the negated TTC increases, and the TTC decreases which is logical since it is easier for the subject vehicle to pass the front vehicle. This is also according to the conclusions by previous studies *(Farah et al., 2009; Llorca and Garcia, 2011)*. Similarly, as the passing gap that is accepted is larger, the negated TTC decreases, and the TTC increases. On the other hand, as drivers start their passing maneuver from a larger gap from the front vehicle, the negated TTC increases and the TTC decreases. This is because it take the drivers longer to pass the front vehicle, and during this time the opposing vehicle has become closer, resulting in a shorter TTC. The road design as well impacts the TTC. As the road curvature is higher, the negated TTC is lower, and the TTC is higher. This indicates an adaptation behavior by drivers who compensate for the difficulty of the passing maneuver on complex roads by increasing their safety margin. Previous results by *Farah and Toledo (2010)* found that on roads with larger curvature, drivers accept larger critical gaps, which supports the results and conclusion of this study. The speed of the opposing vehicle was not found to be significant at the 95% confidence level, however, this variable is indirectly included through the passing gap which is measured in time.

FIGURE 4 (*left*) presents the probability density function of the empirical and modeled standardized[1] maximum negated TTC, and FIGURE 4 (*right*) presents the simulated QQ plot for the non-stationary model. The results

[1] For non-stationary models, it is common practice to transform the data to a density function that does not depend on the covariates, using the following function $Z_i = -log(1 + (\xi/\sigma * (X_i - \mu_i)))^\wedge(-1/\xi)$ (*Gilleland and Katz, 2011*)

indicate a good fit between the modeled GEV distribution and the empirical data, and a better fit compared to the results of the stationary model presented in FIGURE 3. Also, the negative log-likelihood has improved from 229.5 to 215.5, maintaining a $\xi > -0.5$ that assures the regular asymptotic properties of the maximum likelihood estimators *(Coles, 2001)*.

To estimate the probability of a head-on-collision during the passing maneuver (*max*{-TTC}≥0) for the non-stationary model, simulated covariates or directly location parameters have to be generated. From the estimated location parameters for the estimation dataset, a normal distribution was fitted with satisfactory results with mean of -0.996, standard deviation of 0.115 and a Kolmogorov-Smirnov test statistic of 0.0452. The simulated probability of *max*{-TTC}≥0 is 0.0190 with 95% confidence interval (0.0188,0.0193), resulting in a better estimate than the stationary model.

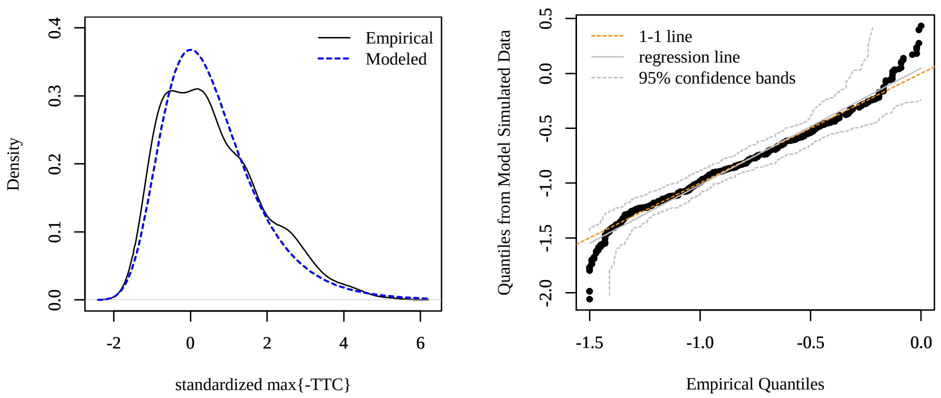

FIGURE 4 – Probability density function (left) and simulated QQ-Plot (right) for the non-stationary Block Maxima model

Peak Over Threshold Results

In this section the estimation results of the GPD following the POT approach are presented. This analysis was conducted in order to compare with the BM approach results, as previous studies concluded that the POT approach performs better than the BM approach especially in situations of short-time observations *(Zheng et al., 2014)*. As a first step for estimating the GPD, a threshold needs to be determined and selected from the observed maximum negated TTC. To determine the optimal threshold an assessment of mean residual life and stability plots were carried out following *Coles (2001)*. A threshold can be determined when the mean residual life plot is almost linear and the modified scale and shape estimates become constant. In FIGURE 5 (*left*) the mean residual life plot of the maximum negated TTC thresholds is linear from a threshold of -2.0 seconds, where the line becomes more stable. FIGURE 5 (*right*) presents the mean residual life plot of the negated TTC thresholds larger than -2.0 seconds. This figure clearly shows the linearity of the plot.

The stability of GPD modified scale and shape parameters were also analyzed. FIGURE 6 shows stability plots considering a range between -2.5 and -0.25 seconds. Both parameters seem to be relatively stable in the range between -1.1 and -0.6 seconds, suggesting a threshold around -1.0 seconds. Considering the low magnitudes of the variability of the modified scale parameter over the full range of tested threshold values, two stationary models were fitted using the full dataset and a threshold of $u = -1.0$ s and $u = -1.5$ s, both with the ML method.

TABLE 4 - Estimation results for two stationary POT models

Parameter	$u = -1.0s$	$u = -1.5s$
$\hat{\sigma}$	0.757 (0.0495)	1.164 (0.00430)
$\hat{\xi}$	-0.753 (0.0520)	-0.774 (2×10^{-8})
N	278	463
Neg. log likelihood	-8.78	175

Since the estimated shape parameter is $\hat{\xi} < -0.5$ the estimators from the MLE are not reliable *(Smith, 1985)*. FIGURE 7 *(left)* presents the probability density function of the empirical and modeled negated TTC, while FIGURE 7 *(right)* presents the simulated QQ plot. Both figures indicate a good fit between the modeled GPD distribution and the empirical data.

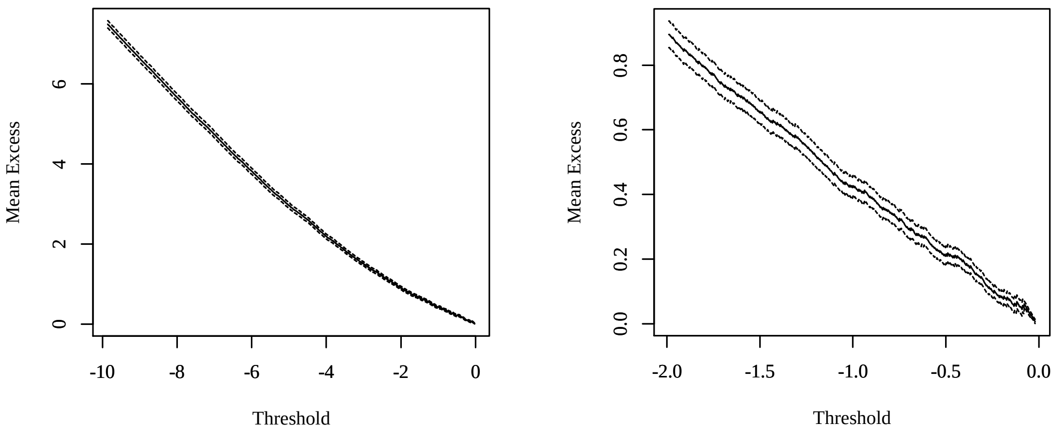

FIGURE 5 – Mean residual life for the full data set (left) and for negated TTC > -2.0 s (right)

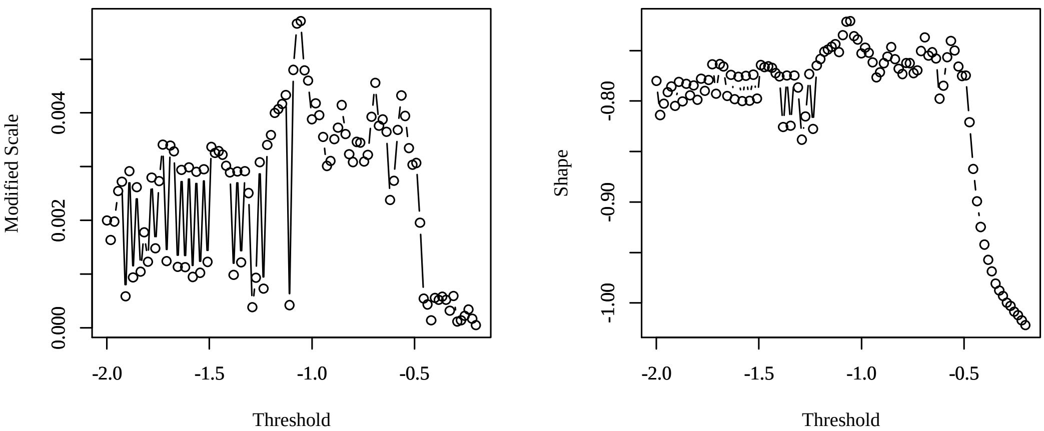

FIGURE 6 – Stability plot for GPD model modified[2] scale parameter (left) and shape parameter (right) for different TTC thresholds

With these stationary models using the fitted GPD distribution, the estimated probability of head-on collision given that a near crash (*min*{TTC}<=1s) happened is 0.000916 with 95% confidence interval (0.000858, 0.000977) and 0.000453 (0.000412, 0.000497) for a 1.5 threshold near-crash. If we account for the ratio of near-crashes the estimated frequency of head-on collisions for the passing maneuvers empirical set can be estimated as: 0.000195 and 0.000158 for $u = -1.0$ and $u = -1.5$, respectively. Both values are far from the empirical 0.00699 (with a 95% binomial confidence of 0.00320, 0.0132). This significantly lower probability results from the short upper tail for the estimated distribution of excesses and its low estimated upper bound ($u - \hat{\sigma}/\hat{\xi}$). Furthermore, the test proposed by *Coles (2001)* for the GPD distribution where both the excesses of a threshold u_0 and those of a higher threshold u should follow a generalized Pareto distribution with similar re-parameterized shape parameter, also failed. A possible explanation for this lower performance might be the fact that the entire time series of continuous observations of TTC was not used in the analysis (but just the *min*{TTC}). This data will be used in subsequent studies by the authors.

Since in the estimation of the stationary model the shape parameter was $\hat{\xi} < -0.5$ which indicates that the estimators from the MLE are not reliable, a non-stationary model was not expected to results with a more significant results. Indeed, the attempt to estimate such model did not result in significant results.

[2] Modified by subtracting the shape multiplied by the threshold.

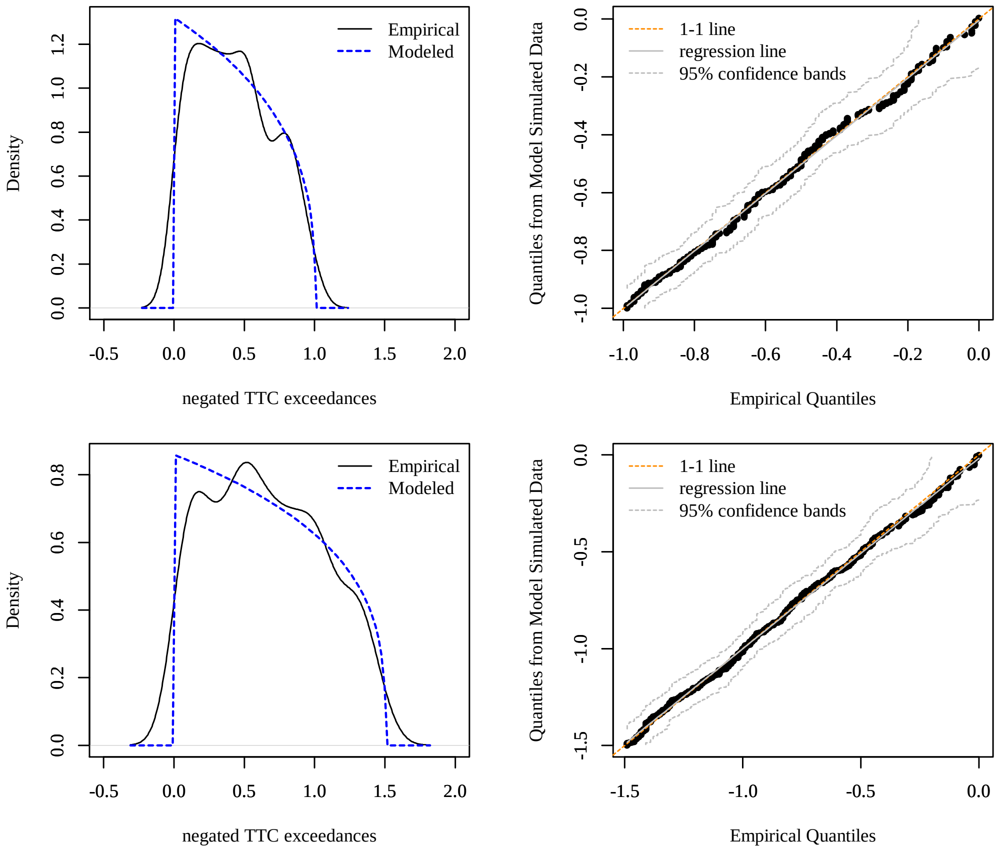

FIGURE 7 – Probability Density plot (left) and simulated QQ plot (right) for the stationary GPD model with threshold $u = -1.0s$ (top) and $u = -1.5s$ (bottom)

Validation

This section compares the results from the GEV approach and its fitness to be used for the specific case of estimating the probability for a head-one collisions.

Data for the validation was obtained from a second experiment. In this experiment different 100 drivers (69 males and 31 females) participated. Their age ranged between 21 and 61 years old. The instructions and experimental conditions were identical to the first experiment. The simulator scenarios included as well rural two-lane road sections each with a total length of 7.5 km. The same two-level four factors as in the first experiment were used to generate the scenarios. However, the values in each level were not fixed but randomly drawn from a specified distribution. Speeds were drawn from truncated uniform distributions, while the passing gaps were drawn from truncated negative exponential distributions. More details on the design of the scenarios can be found in *Farah and Toledo (2010)*. To check the consistency among covariate data sets, the CDF for each of the variables considered in the non-stationary BM model were computed (see FIGURE 8). The data was filtered again for min{TTC}<1.5s. A can be noticed the resulting CDFs are similar except for the front speed which largely differ. This stems from the fact that driving speeds in the first experiment were fixed to certain values while in the second experiment were drawn randomly from truncated uniform distributions.

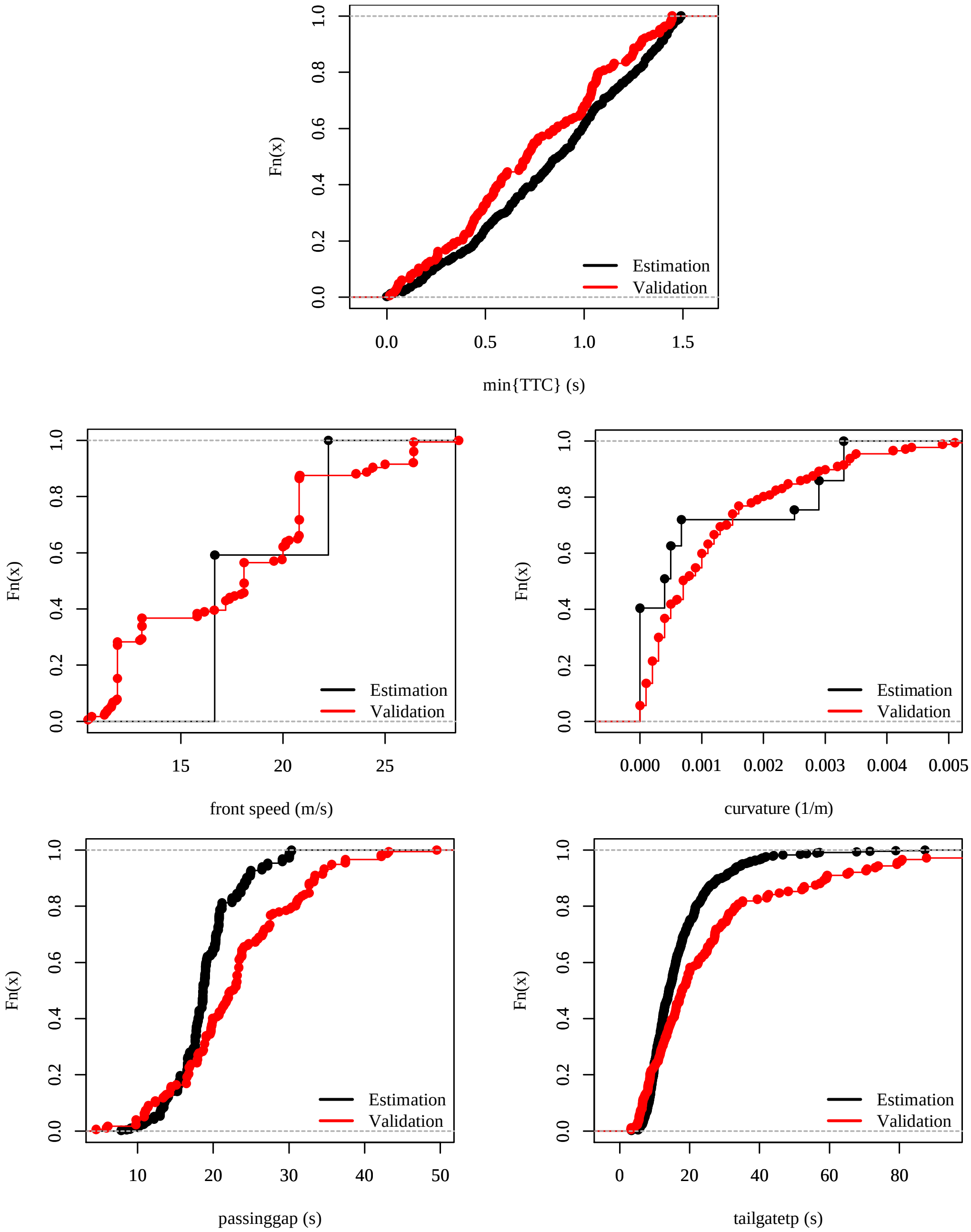

FIGURE 8 – CDF of the minimum TTC and the covariates considered in the non-stationary BM model for both the estimation and the validation data sets

From the estimated stationary model, the probability of *max*{-TTC}≥0 is 0.0179 (0.0177,0.0182). For the validation data set, out of the 562 maneuvers, 166 were considered near head-on collisions (using the threshold of 1.5 seconds) and 8 maneuvers ended with actual collisions. In other words, the probability for a head-on collision in a near head-on collision in a passing maneuver is 8/166= 0.0482, with a 95% binomial confidence interval (0.0210, 0.0927). This value is significantly higher than the one resulting from the fitted GEV distribution. The resulting expected number of head-on collisions is 2.98. FIGURE 9 presents the probability density function and QQ plot of the validation and previously (stationary) modeled negated TTCs.

The same test was carried out for the non-stationary model. The simulated probability of *max*{-TTC}≥0 is 0.0201 with 95% confidence interval (0.0199,0.0204), resulting in an estimated number of head-on collisions of 3.344 – still far from the observed frequency (see FIGURE 10).

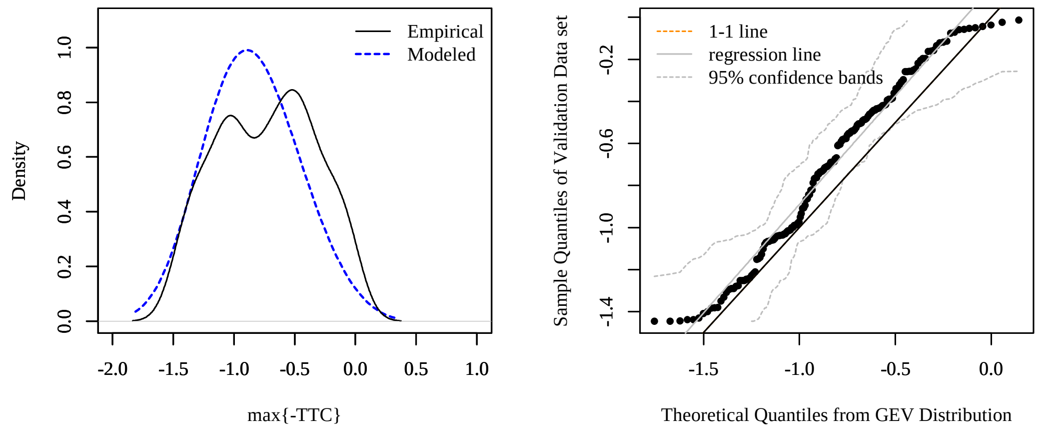

FIGURE 9 – Probability Density plot (left) and QQ plot (right) for the Validation Set and the stationary Block Maxima model

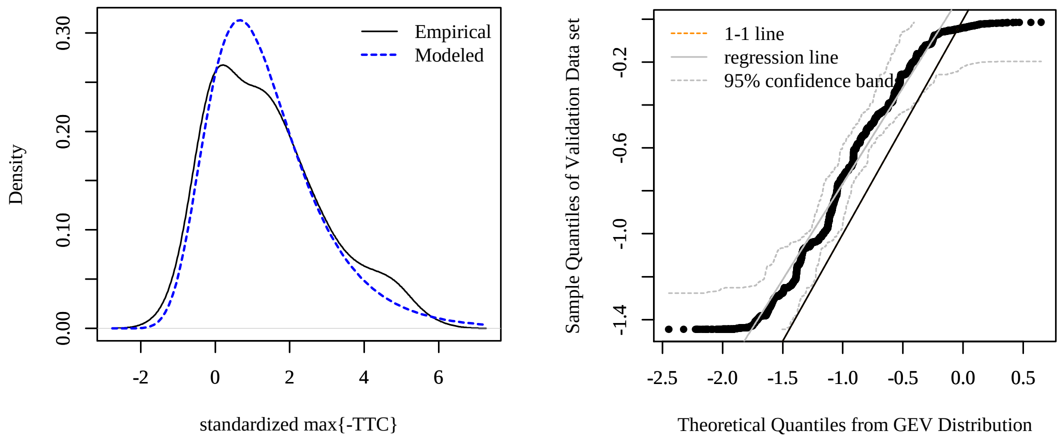

FIGURE 10 – Probability Density plot (left) and QQ plot (right) for the Validation Set and the non-stationary Block Maxima model

No validation attempt was conducted for the POT approach as estimation results did not yield reliable parameters.

SUMMARY AND CONCLUSIONS

In this on-going study an Extreme Value (EV) approach was applied for the estimation of the probability of head-on collisions that result from unsuccessful passing maneuvers on two-lane rural highways. Both, the Block Maxima (BM) approach using the Generalized Extreme Value (GEV) distribution and the Peak Over Threshold (POT) using Generalized Pareto Distribution (GPD), were tested and compared using the minimum time-to-collision with the opposing vehicle during passing maneuvers.

The estimation showed that the BM approach yielded better results compared to the POT approach. *Zheng et al. (2014)* who conducted a comparative study for the case of using post encroachment time measure for predicting lane-changing maneuver related crashes, reached an opposite findings, that the POT approach performed better than the BM approach. This difference might stem from the fact that the data set in the study by *Zheng et al. (2014)* was relatively limited, and for limited data sets the POT is known to be a more efficient approach than the BM approach. *Zheng et al. (2014)* site two studies *(Caires, 2009; Jarušková & Hanek, 2006)* who concluded that “the BM approach would work well if the number of observations is large, while the POT approach would have a poor performance”. However, definitive conclusion regarding which method is supreme can not yet be made and further comparative studies are needed in order to reach a firm conclusion. It can also be that the poor performance of the POT approach resulted from the fact that we did not use the full range of the TTC data but only the min{TTC}. Nevertheless, it was found that the non-stationary BM model performed better than the stationary BM model, but still, both resulted in a satisfactory level of fit to the empirical data. This is expected since the introduced covariates significantly affect the TTC and were found to be important explanatory variables in previous studies *(Farah et al., 2009; Llorca & Garcia, 2011)*. Furthermore, the predicted probability of head-on collisions based on the BM approach was sufficiently close to the probability of head-on collisions based on the empirical data from the driving simulator. This also indicates that for passing maneuvers the TTC is a good surrogate safety measure for near-crashes of head-on collisions. This is different from the conclusion reached by *Jonasson and Rootzén (2014)* who found severe discrepancy between the rear-striking near-crashes (using the TTC) and rear-striking crashes. However, this can be explained by the mechanism of crash occurrence and the state of the driver. In passing maneuvers drivers are completely aware and conscious of their actions and therefore head-on collisions usually result from an error in drivers' judgment of the suitability of the passing gap. On the other hand, in rear-striking collisions, the state of the driver in these collisions might vary a lot. It can result, similarly to passing collisions, from drivers' errors in judging their gap and speed from the front vehicle but can also result from the driver being distracted. In the first case, it is most likely to observe an evasive action of the driver to prevent the collision but in the second case no evasive action will be observed. This causes, as *Jonasson and Rootzén (2014)* indicate, a selection bias, and therefore, careful selection of near-crashes is a crucial issue in preventing this to occur.

Despite these promising results, future research by the authors will attempt to expand this work in several possible directions as follows: (1) testing alternative surrogate measures of head-on collisions such as the Time Exposed Time to Collision or Time integrated Time to Collision *(Minderhoud and Bovy, 2001)*; (2) developing a more sophisticated measure of risk which accounts for the complexity of the passing maneuver and considers the probability to collide not only with the opposite vehicle but also with the passed vehicle. One possibility is, similarly to *Jonasson and Rootzén (2014)*, to use a bivariate GEV which is built from two components of the Block Maxima vectors and which considers the TTC and the headway between the passing and passed vehicle at the end of the passing maneuver; (3) extending the non-stationary models by including other covariates related to road design (this study accounted for only the road curvature) and drivers' characteristics, such as socio-demographic and driving styles; (4) examining the transferability of such model and validation of the results with other datasets especially from field studies; (6) applying the developed models in traffic microscopic simulation environments for safety assessment *(Lima de Azevedo et al., 2014; Gettman and Head, 2003)*.

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