Probabilistic Safety Analysis using Traffic Microscopic Simulation

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ABSTRACT

Traffic microscopic simulation applications are currently a common tool in road system analysis and several application attempts to safety performance assessment have been recently carried out. However, current approaches still ignore causal relationships between different levels of vehicle interactions or/and accident types, lacking a physical representation of the accident phenomena itself. In this paper, a new generic probabilistic safety assessment framework for traffic microscopic simulation tools is proposed. The probability of a specific accident occurrence is assumed to be estimable by an accident propensity function, consisting in a deterministic safety score component and a random component. The formulation of the safety score component may be specified as dependent on the type of occurrence, detailed vehicle interactions and maneuvers, and on selected key simulation modelling features. This generic model was applied to the case of urban motorways and specified to four types of events: non-accident events and three types of accidents in a nested logit structure: rear-end and lane-changing conflicts, and run-off-road events.

As there is still no available large disaggregated data set linking trajectories to accident occurrence, artificial trajectories from a detailed calibrated microscopic simulation tool were used. These trajectories were obtained following a comprehensive calibration effort: extracting trajectories for a generic scenario, calibration of the simulation tool using the collected trajectories, and re-calibration of the simulation model using aggregate data for each event selected for replication and used in the safety model estimation phase.

The final estimated safety model allowed for the identification and interpretation of several simulated vehicle interactions at stake. The fact that these considerations were extracted from simulated analysis shows the real potential of well (detailed) calibrated traffic microscopic simulation for detailed safety assessments and their potential as a lay-out design tool.

KEYWORDS

Traffic microscopic simulation; road safety; probabilistic modeling; driving behavior modeling; surrogate safety measures; calibration.
1 INTRODUCTION

Traffic microscopic simulation applications are currently becoming a common tool in both the transportation practitioners and researchers communities. The original purpose for developing such tools was network efficiency assessment. The need for simultaneously assessing safety impacts of transportation systems soon arose. However, despite several enhancements at the driving behaviour modelling level (for a detailed review see (1)), dedicated safety modelling in simulation has been frequently neglected due to the existing limited applied model formulations on driver’s perception, decision and error mechanisms and to the lack of data for its development.

With the development of many infrastructure-based Intelligent Transportation Systems, research efforts have been dedicated to identify, traffic scenarios that might be used as accident precursors. Models developed with this aim are referred as (real-time) accident probability models and, typically, make use of aggregated real-time traffic data collected by sensing technologies (generally from loop detectors), road geometric characteristics and, in some cases, weather conditions to statistically predict changes in the accident occurrence probability. Some researchers opted for the use of these accident probability models to perform the safety assessment in microscopic simulation experiments (2), (3) and (4). These modelling streams rely on the availability of historical accident records and depend on some level of aggregation regarding the traffic operations data used as explanatory variables. As accidents are considered rare events and it is hard to isolate the effect of many factors affecting its occurrence, conflicts have also been used as an alternative estimator of system safety (5). The use of conflicts is based on the assumption that the expected number of accidents occurring on a system is proportional to the number of conflicts making suitable for systems’ comparisons (6). One of the main limitations of using conflicts is the correct estimation of this proportionality. This difficulty has motivated the research community to develop several models to estimate accident frequency from traffic conflicts counts (7). Another difficulty in using conflicts for modelling purposes is the lack of practical definitions and measurement standards (as it does not estimate the probability of an accident itself). For this purpose several time-based, deceleration-based and dynamic-based surrogate safety performance indicators were proposed in the literature (8). Despite this, these models are the most widely used within microscopic simulation studies ((9), (10), (11), (12), (13)).

Very recently, efforts have been made to integrate interaction in probabilistic modelling frameworks. While the above mentioned accident probability models try to link the probability of a specific accident occurrence using a statistical model fitted to aggregated data, probabilistic frameworks try to formally represent cause-effect relationships between performed driving tasks and traffic scenarios that may lead to typical accident events. Such approach has a higher potential in replicating the intrinsic nature of an accident mechanism and, ultimately, would not depend on safety records itself. On the other hand, probabilistic frameworks depend on much more detailed information as the distributions and relationships between all variables at stake are needed (e.g.: evasive manoeuvres probabilities for different situations or pavement conditions for different scenarios). Songchitrucks and Tarko (14) proposed an Extreme Value (EV) approach to build up relationships between occurrence of right-angle accidents at urban intersections and frequency of traffic conflicts measured by using PET as accident proximity variable. Saunier and Sayed (15) developed a comprehensive probabilistic framework for automated road safety analysis based on motion prediction. Wang et al. (16) propose an incident tree model and an incident tree analysis method for the identification of potential characteristics of accident occurrence in a quantified risk assessment framework. These efforts step forward in a more comprehensive formulation of the accident phenomena, but still haven’t been widely validated or integrated in simulation tools for practical application. Several simulation-based safety studies were also documented in a very recent and comprehensive review by Young et al. (17). In summary, the authors clearly pointed out the need for analysing the probabilistic nature of the link between the vehicle interaction and the accident itself and for generalising the models to accommodate for different types of accidents. Furthermore, the need was recognized for differentiating distinct cause-effect relationships for diverse types of accidents and for a probabilistic formulation without the limitations resulting from the
aggregation of both traffic data and safety records.

2 GENERIC MODEL FORMULATION

A generic framework for modelling cause-effect mechanisms between detailed vehicle interactions from simulated environments and the accident occurrence probability is proposed. It is first assumed that the state of a vehicle \( n \) at any given time \( t \) can be viewed as a discrete variable whose state outcome \( k \) can be one of different types of accident or no accident at all. An individual outcome \( k \) among all possible outcomes \( K \) is considered to be predicted if its probability \( P_{n,t}(k) \) is maximum. As in previous research studies, the main difficulty is how to estimate \( P_{n,t}(k) \). This probability should be a function of specific observed variables characterizing the interaction between vehicles (14). Such considerations step away from the assumption of a fixed coefficient model converting the surrogate event frequency into accident frequency, typically used in the traffic conflicts technique. The probability for a specific accident involving vehicle \( n \) to occur at time \( t \) is assumed to be estimable by a specific accident propensity (or proximity) measure (18):

\[
P_{n,t}(k) \sim U_k
\]

In the proposed model, each accident propensity function \( U_k \), is considered to have a (deterministic) safety score \( V_k \) component and a random component \( \varepsilon \):

\[
U_k = V_k(X, \beta) + \varepsilon
\]

where \( X \) is the vector of explanatory variables, \( \beta \) is the vector of unknown parameters to be estimated and \( \varepsilon \) is the random term (the terms \( n \) and \( t \) were omitted for simplicity). The assumption of the deterministic safety score component agrees with the recent research stream where detailed interaction variables directly affect the accident occurrence probability itself. The random component \( \varepsilon \) is assumed to represent the unobserved effects involved in the determination of the outcome; these may be derived from a random process in the occurrence of a specific event or caused by a lack of knowledge of this process.

As it is common in safety modeling research, the accident phenomenon relies on many different variables:

\[
V_k(n,t) = f_k(X_{n,t}, X_{n',t}, X_{D,t}, X_S)
\]

where the \( k \) accident-type specific scoring function \( f_k \) depends on: \( X_{n,t} \), the driver-vehicle unit \( n \) specific variables at time \( t \); \( X_{n',t} \), the variables at time \( t \) for the interaction between \( n \) and a conflicting driver-vehicle unit \( n' \); \( X_{D,t} \), the dynamic environmental variables at time \( t \) (e.g.: weather, variable speed limit, lighting conditions, etc); and \( X_S \), the static environmental variables (e.g.: geometrics, road signs, etc). Note that driver characteristics are typically not considered in traffic simulation tools, which substantially limits the number of available candidate explanatory variables \( X_{n,t} \).

In the presented model we framed the formulation of each function \( f_k \) to represent a cause-effect relationship, to simultaneously deal with different non-independent types of accident outcomes and to consider a disaggregated probability for any vehicle state \((n,t)\) observation (instead of the existing aggregate formulation used in real-time accident probability models).

3 MODELLING DIFFERENT ACCIDENT TYPES

The above general formulation is now detailed to a specific set of accidents that typically occur on busy urban motorways: rear-end accidents, side collisions during lane-change maneuvers and run-off-road accidents. It is clear that these three different outcomes correspond to very distinct phenomena.
However, it is also known that these three outcomes may be related, namely if one considers accident outcomes following an evasive action from different risky interactions (see FIGURE 1):

![Diagram](image)

**FIGURE 1** Model structure for motorway accident occurrence

### 3.1 Rear-end (RE) conflicts

When facing rear-end interactions the probability of a collision is formulated in terms of: the subject vehicle braking requirements to avoid a RE collision and the maximum available braking power. The first is represented by the difference between the actual relative acceleration between the subject vehicle and its leader \((n - 1)\), \(\Delta a (m/s^2)\), and the deceleration rate required to avoid crash, \(\text{DRAC} (m/s^2)\), estimated using Newtonian physics:

\[
\Delta a_{\text{need}}(n, t) = \text{DRAC}(n, t) + \Delta a(n, t)
\]

\[
\text{DRAC}(n, t) = \frac{[v(n - 1, t) - v(n, t)]^2}{2[x(n - 1, t) - x(n, t) - l(n)]}
\]

where \(v(n, t), x(n, t)\) and \(l(n)\) are the speed, longitudinal position and length of vehicle \(n\) (FIGURE 2.a).

We further split of the needed deceleration rate into its positive, \(\Delta a_{\text{need}}^+(n, t) \geq 0\), and negative, \(\Delta a_{\text{need}}^-(n, t) \leq 0\), components, allowing for the consideration of different parameters. The advantage of using \(\Delta a_{\text{need}}\) instead of just the DRAC is the consideration of the current acceleration state. The value of \(\Delta a_{\text{need}}\) is easily interpreted: the negative values represent safer values, for which the vehicle is already applying a deceleration rate greater than DRAC.

![Diagram](image)

a. Rear-end (RE)

b. Lane-changing (LC)
We further improve this formulation by dividing the required deceleration by the time-to-collision, TTC, thus considering also how long the driver has before the potential collision. The RE safety score function will then depend on the available time for adjustment, resulting in a relative needed deceleration ratio $RA_{\text{need}}(n, t)$:

$$RA_{\text{need}}(n, t) = \frac{\Delta a_{\text{need}}(n, t)}{TTC(n, t)}$$  \hspace{1cm} (6)

$$TTC(n, t) = \frac{x(n - 1, t) - x(n, t) - l(n)}{(v(n, t) - v(n - 1, t))}$$  \hspace{1cm} (7)

Finally, similarly to the CPI surrogate safety measure described in (6), a measure of the maximum available deceleration rate is also considered. It allows considering heterogeneous safety conditions regarding different vehicle categories and different pavement conditions (e.g.: dry/wet) that are expected to influence the deceleration performance:

$$\Delta a_{\text{lim}}(n, t) = DRAC(n, t) - (\mu_{\text{long}}(n, t) + d)g$$  \hspace{1cm} (8)

$$\mu_{\text{long}}(n, t) = f_{\text{long}}(v(n, t), \alpha_{\text{type}}, \alpha_{\text{wet}})$$  \hspace{1cm} (9)

where $\Delta a_{\text{lim}}(n, t)$ is the maximum available deceleration for vehicle $n$ at time $t$, $d$ is the grade rate (m/m), $g$ is the gravitational acceleration of 9.81 m/s$^2$ and $\mu_{\text{long}}(n, t)$ is the maximum available longitudinal friction coefficient, which depends on the speed of the vehicle itself $v(n, t)$ and on two factors that account for the vehicle type, $\alpha_{\text{type}}$, and the pavement condition, $\alpha_{\text{wet}}$. This simplified formulation of the friction coefficient is due to the small number of variables typically available in simulated environments. Similarly to the previous variables, the rate $RA_{\text{lim}}(n, t) = \Delta a_{\text{lim}}(n, t)/TTC(n, t)$ is used in the safety score function to account for the TTC.

The systematic safety score for RE collisions may now be formulated as:

$$V_{\text{RE}}(n, t) = \rho_{0}^{\text{RE}} + \rho_{1}^{\text{RE}}RA_{\text{+need}}(n, t) + \rho_{2}^{\text{RE}}RA_{-\text{need}}(n, t) + \rho_{3}^{\text{RE}}RA_{\text{lim}}(n, t)$$  \hspace{1cm} (10)

where $RA_{\text{+\text{need}}}$ and $RA_{\text{-\text{need}}}$ are the positive and negative components of the relative needed deceleration.
ratio computed using $\Delta a^\text{need}_+^{\text{a}}$ and $\Delta a^\text{need}_-^{\text{a}}$ respectively; RA$^\text{lim}$ is the maximum available deceleration ratio; and $\beta^\text{RE}_0$, $\beta^\text{RE}_1$, $\beta^\text{RE}_2$ and $\beta^\text{RE}_3$ are the estimable parameters.

### 3.2 Lane-changing (LC) conflicts

The lane change action decision is typically modelled by means of gap acceptance models (19) or, alternatively, by acceleration variation models (20). One would expect the probability of lane-change collisions to be function of vehicles lateral movements. However, the large majority of the current micro-simulation tools do not provide this modelling feature. Therefore, surrogate measures depending on lateral movements, such as the time-to-lane-crossing proposed in (21) and the Post-Encroachment-Time used in (22), are not easily integrated.

The probability of a LC collision is based on gap acceptance models and formulated in terms of gap variation. The gap acceptance is generally modelled separately regarding the lead and the lag gaps on the target lane (FIGURE 2.b). This disaggregation is of special interest as different parameters may be computed for different gaps (23). It is known that the lane-changing process becomes increasingly difficult as the speed differences between the subject vehicle and the lead and lag vehicles in the target lane increases (24). Thus, in the proposed formulation, the safety score of the LC event is specified in terms of relative gap variation:

$$RG^{\text{gap}}(n, t) = \frac{\Delta v^{\text{gap}}(n, t)}{G^{\text{gap}}(n, t)}$$

where $G^{\text{gap}}$ is the gap in meters and $\Delta v^{\text{gap}}$ represents the speed difference between the subject vehicle and the lead (or lag) vehicle on the target lane in m/s:

$$\Delta v^{\text{lead}}(n, t) = v(m - 1, t) - v(n, t)$$

$$\Delta v^{\text{lag}}(n, t) = v(n, t) - v(m, t)$$

Where $v(m - 1, t)$ and $v(m, t)$ are the speed of the lead vehicle $m - 1$ and the lag vehicle $m$ in the target lane, respectively. Again, the split of the relative gap variation into its positive, $RG^{\text{gap}}_+$, and negative, $RG^{\text{gap}}_-$ values allows for the consideration of different parameters associated with different safety conditions, i.e. for gaps that are either increasing or decreasing, respectively.

$$RG^{\text{gap}}_+(n, t) = \max(0, RG^{\text{gap}}(n, t)) \rightarrow RG^{\text{gap}}_+(n, t) \geq 0$$

$$RG^{\text{gap}}_-(n, t) = \min(0, RG^{\text{gap}}(n, t)) \rightarrow RG^{\text{gap}}_-(n, t) \leq 0$$

Following the above formulation a gap with a higher relative shrinking rate ($RG^{\text{gap}}_-(n_1, t_1) < RG^{\text{gap}}_-(n_2, t_2) \leq 0$), for example, should have a higher impact on the LC conflict probability, $P_{n_1,t_1}(\text{LC}) > P_{n_2,t_2}(\text{LC})$, and therefore, its parameter estimate should be negative.

The systematic component for LC collisions may now be formulated as:

$$V_{\text{LC}}(n, t) = \beta^\text{LC}_0 + \beta^\text{LC}_1 RG^{\text{lag}}_+(n, t) + \beta^\text{LC}_2 RG^{\text{lag}}_-(n, t) + \beta^\text{LC}_3 RG^{\text{lead}}_+(n, t) + \beta^\text{LC}_4 RG^{\text{lead}}_-(n, t)$$

where $RA^\text{gap}_+$ and $RA^\text{gap}_-$ are the positive and negative components (with gap = {lead, lag}) and $\beta^\text{LC}_0$, $\beta^\text{LC}_1$, $\beta^\text{LC}_2$, $\beta^\text{LC}_3$ and $\beta^\text{LC}_4$ are the estimable parameters.
3.3 Run-off-road (ROR) events

ROR events are assumed as being primarily related to individual vehicle dynamics rather than interaction with others. This assumption is especially true under free-flow scenarios. However, it may also result from evasive manoeuvres due to risky lane-changing or car-following decisions.

Vehicle dynamics in traffic simulation models are represented in a much simplified manner when compared with the detailed movements’ description of real events and its representation currently achieved with accident reconstruction models. This significantly limits the current potential for a ROR micro-simulation modelling framework. The vehicle lateral movement, the true road geometric characteristics (such as transition curves), the pavement surface characteristics, and the vehicle detailed physical and mechanical attributes are generally not available. However, some relevant variables that may potentially be useful for the analysis of ROR events are already available in micro-simulation tools, such as vehicle speed, general road geometrics and the generic vehicle type.

In the proposed framework, the safety score of ROR events is assumed to be linked to the difference between the current lateral acceleration of vehicle \( n \) and a site specific critical lateral acceleration. First, as vehicle lateral movements and the true road geometrics are not modelled, the vehicle path on curve elements is assumed as a simple circular path and the vehicle yaw equal to the curve bearing (FIGURE 2.c). The lateral acceleration of vehicle \( n \), \( a_{\text{lat}} \), is therefore derived from its current speed and the curve radius \( R \) (m):

\[
a_{\text{lat}}(n,t) = \frac{[v(n,t)]^2}{R}
\]

(17)

Although the majority of the simulation tools do not provide information on lateral movement during a lane change, it is expected that this type of manoeuvres will also affect the ROR event probability. Using test track data, Chovan et al. (23) considered peak lateral acceleration values of 0.4g, 0.55g and 0.7g for mild, moderate, and aggressive steering manoeuvres, respectively. As detailed lane change models are typically not available in microscopic traffic simulation platforms, a generic peak acceleration add-on for lane change of 0.5g was adopted and integrated in eq. 17 to account for a potential increased ROR probability in road sections with high frequency of lane:

\[
a_{\text{lat}}(n,t) = \frac{[v(n,t)]^2}{R} + 0.5g\delta_{\text{lc}}(n,t)
\]

(18)

where \( \delta_{\text{lc}}(n,t) \) is a dummy variable to account for lane change (1 if the vehicle is performing a lane change, 0 otherwise).

The maximum allowed lateral acceleration \( a_{\text{lat}}^{\text{cr}}(n,t) \) directly depends on the critical lateral friction coefficient \( \mu_{\text{lat}} \) and the road super-elevation \( e \) (m/m):

\[
a_{\text{lat}}^{\text{cr}}(n,t) = (\mu_{\text{lat}}(n,t) + e)g
\]

(19)

Similarly to its longitudinal counterpart, the values of the maximum lateral friction coefficient, \( \mu_{\text{lat}} \), also depend on the vehicle speed itself \( v \), on the pavement condition (wet/dry) and on the type of vehicle:

\[
\mu_{\text{lat}}(n,t) = f_{\text{lat}}(v(n,t), \alpha_{\text{type}}, \alpha_{\text{wet}})
\]

(20)

The safety score function may now be formulated in terms of the positive (unsafe) and negative (safe) components of the difference between the current and the critical lateral accelerations:
where $\Delta a^+_{lat}$ and $\Delta a^-_{lat}$ are the positive and negative components of $\Delta a_{lat} = a_{lat} - a_{cr}$, respectively.

### 3.4 Estimation framework

As previously stated, the explanatory variables of one type of accident may influence the occurrence of others and evasive manoeuvres may create correlations between different accident outcomes. When modelling multiple discrete outcomes, the multinomial nested logit model proposed by Ben-Akiva (25) has advantages over the simple multinomial logit model, because it can simultaneously estimate the influence of independent variables while allowing for the error terms to be correlated and, therefore, allowing for the violation of the independence of irrelevant alternatives (IIA) property (the reader is referred to (26), for its derivation and formulation details).

To directly estimate the proposed model, a large set of all types of model outcomes and its vehicle interaction data is needed. Unfortunately, a large data allowing for the direct association between trajectories and accident occurrence is still not available. Furthermore, although the proposed model is specified individually for any vehicle $n$ at every time $t$, the philosophy of microscopic simulation applications is to replicate as close as possible real aggregate measurements, even at such detailed level as aggregated accelerations, headways or TTC. Thus, to estimate the above model the use of artificial (simulated) trajectories is proposed. Yet, a set of critical assumptions must be considered:

1. A well calibrated microscopic simulation model must be calibrated appropriately to replicate statistical distributions of detailed traffic variables.
2. Trajectories extracted in a generic day represent the general driving behaviour of traffic. Confidence on this assumption depends on the amount and breath of information available for treatment. Other factors (such as weather) influence general driving behaviour parameters; part of this variability will be assessed by means of a dedicated calibration, carried out for each specific event, using readily available data sets (eg.: from loop sensors).
3. Although simulation models are accident free, their description of detailed traffic variables can be linked to the accident probability. This is supported by previous studies (9), (2).

The microscopic simulation tool is then calibrated once using the pre-estimated seed Origin-Destination (OD) matrix, and both aggregate (loop sensor based) and disaggregate (observed vehicle trajectories) data collected for a specific generic day $d_0$. The optimum sets of the microscopic simulation model parameters $\beta_0$ are then used as initial parameters in an aggregate calibration process using the aggregated data available for each event observation $i$. After this, the optimum set of parameters for each event $i$, $\beta_i$, is used to generate a set of (artificial) detailed traffic variables. Finally, this set of detailed traffic variables is used jointly with its associated outcome of event $i$ to estimate the proposed safety model.

It is typically expected that both the loop-based variables used for calibration and the accident occurrence reported variables are defined for a pre-defined time and spatial units. In some cases, such aggregated intervals maybe too large to capture short-term variations; nevertheless several authors (27), (2) have successfully used aggregated periods up to 5 min intervals to perform accident occurrence probability analyses. With the absence of true trajectory variables for the vehicle $n$ involved in each observed event $i$, the characterization of the detailed traffic variables for a specific accident occurrence must be linked by means of spatial and temporal aggregation. Additionally, it is well known that safety records have time and spatial errors. Therefore, for estimation one needs to aggregate all vehicle state outcome probabilities $P_{n,t}(k)$ by standardized intervals of space, $s$, and time periods, $p$:

$$P_{s,p}(k) = \frac{1}{N} \sum_{N} P_{n,t}(k)$$  (22)
where $P_{nt}(k)$ is the probability of occurrence $k$ for any relevant observation of vehicle $n$ at time $t$, traveling in spatial interval $s$ and time period $p$ and defined by the proposed nested logit model; $N$ is the total number of observations for all vehicles that travelled in the interval $s, p$. It is important to point out that, following this formulation, the model is based on mean values and not on extreme values. This follows the traffic micro-simulation specification philosophy, where the replication of averaged variables is expected. However, one may want to push the use of extreme formulations and rely on detailed calibration methods of extreme values, or by extending the specification of the driver behaviour to better model such scenarios. Such formulation was not tested for the present document.

Finally, if one considers a large observation period, typically needed to have a relevant number of accident occurrences, it is expected that the loop sensors will fail for some instances. Furthermore, the computational memory and processing resources needed to generate and use the simulated trajectory data for a large set of no-accident occurrence units is impractical. For this purpose an outcome (choice)-based random sampling was assumed. Then, to account for this biased sampling process the weighted exogenous sample maximum likelihood function (WESML) proposed in (29) was used.

4 THE URBAN MOTORWAY CASE AND TESTING DATASET

The proposed model was estimated using collected and simulated data for the A44 urban motorway near Porto, Portugal. This road was selected as case study due to its dense traffic, unusually high number of lane changes, short spacing between interchanges and high percentage of heavy goods vehicles. A44 is a 3,940m long dual carriageway urban motorway with 5 major interchanges, two 3.50m wide lanes and 2.00m wide shoulders in each direction (FIGURE 3). There are acceleration and deceleration lanes at all interchanges, although several as short as 150m. On and off-ramps connect to local roads, which generally have tight horizontal curves, intersections or pedestrian crossings, features that tend to impose significant reductions in vehicle speeds.

Three different traffic data sets were specifically collected for the present study: a dynamic seed OD based on a sample of license plate matching and vehicle counts (Lima Azevedo, 2014); 5 min loop sensor average speeds and counts for the existing eight traffic stations (4 in each direction), between 2007 and 2009 (30); and vehicle trajectories collected for a generic morning (with and without congestion) by aerial remote sensing for the entire length and access links of the A44 motorway (31). Finally, incident records were also collected for the same period of 2007 to 2009 including a total of 144 side-collisions rear-end collisions and run-off-road accidents.

Along with the 5 min temporal units for the observed traffic data, the nature of the accident location record required a spatial observation unit of 50 m. These units are the ones to be considered for the aggregation of individual probabilities. Using such units, a very large number of no-accident (NA) events were observed during this three years period (more than 180 $\times 10^3$). After excluding the days with bad sensor data, a random sampling technique was used to select 6,400 no-accident events, resulting in a total of 6,544 events to be calibrated and simulated for artificial data generation.

The integrated driver behaviour model (19) implemented in MITSIMLab (32) was used to simulate trajectories for each observed event. For the calibration, the global multi-step sensitivity-analysis based calibration proposed in (33) was used. The method was then coupled with a meta-model based
calibration for calibrating the simulator with trajectory data and with a powerful simultaneous demand-
334 supply calibration method for the calibration of the large set of accident and non-accident events using
335 aggregated data (34). This procedure was selected, as it was concluded in previous work (34) that
disaggregate calibration improves significantly the accuracy of simulated trajectories and spot-speeds,
which are important for adequate representation of vehicle interactions in safety studies.

The artificial data generated by the calibrated models showed a clear divergence between accident
and non-accident event simulated outputs typically used in safety assessment (see detailed statistics in
330 (30)).

5 ESTIMATION RESULTS

5.1 Modeling assumptions

For the computation of the RE and ROR model components, both \( \mu_{\text{long}} \) and \( \mu_{\text{lat}} \) must be
344 specified. Unfortunately, on-site measured values were not available. Hence, generic \( \mu_{0} \) values were
345 adopted based on measurements from other urban freeways found in the literature (Inoue and Hioki,
346 1993): a direct variation from 0.85 at 0km/h to 0.75 at 130km/h for dry pavements and from 0.70 at
0km/h to 0.20 at 130km/h for wet pavements. An increase factor of 1.10 was considered for the lateral
348 coefficient \( \mu_{\text{lat}} \). Furthermore, both \( \mu_{\text{long}} \) and \( \mu_{\text{lat}} \) were decreased by a factor of 0.70 for heavy vehicles in
349 dry conditions.

The availability of each occurrence alternative was included in the specification of the likelihood
350 function. For each observation:

- a rear-end conflict was considered as possible whenever the subject vehicle is in a car-following
state;
- a lane change conflict was considered as possible if the road carriageway has two or more lanes
and if the subject vehicle wants to perform a lane change;
- a run-of-road event was considered as possible if the road section is a curve or if the subject
vehicle is performing a lane-change.

Finally, multiple replications should be used directly in the estimation phase within a Monte
359 Carlo process, similar to panel data estimation. With this approach, several observations for the same
360 event are available and directly included in the safety score function with an additional event specific
361 component. The main burden in such an approach is the computer memory and processing resources
362 needed during the estimation phase. In the current study, the estimation process was carried out
363 considering each replication as independent.

The maximum likelihood estimates of the model parameters are calculated by maximizing this
365 function:

\[
\mathcal{L} = \sum_{s,p} \sum_{k} y_{k,s,p} w_{k} \ln[p_{s,p}(k)]
\]  

(23)

where \( k \) are all possible outcomes considered for the proposed model \( p_{s,p}(k) \) is the probability of outcome
371 k for spatial interval \( s \) and time period \( p \) (given by equation 22), \( w_{k} \) is the outcome \( k \)-specific sampling
373 ratio, \( y_{k,s,p} \) is 1 if \( k \) is the observed outcome for the observation pair \( s,p \) and 0 otherwise. In this study,
374 the PythonBIOGEME open source software was used (36).

Finally, for numerical reasons, it is good practice to scale the data so that the absolute values of
the parameters are between zero and 1; thus, all relative gap variation variables were divided by 10 and
372 the lateral acceleration difference specified in 0.1m/s².

5.2 Results

The estimation results are presented in TABLE 1.
When the positive $R_{A}^{\text{need}}$ component is close to zero, the relative deceleration is close to the DRAC and thus closer to a safe situation. When $R_{A}^{\text{need}}$ increases the probability for a RE accident is higher, as the difference between the vehicle relative deceleration rate and its DRAC gets higher. $R_{1}^{\text{RE}}$ has a higher absolute magnitude than $R_{2}^{\text{RE}}$, penalizing much more any safety decay in the unsafe domain ($R_{A}^{\text{need}} > 0$) rather than in the safe one ($R_{A}^{\text{need}} < 0$). Regarding the negative component, i.e. when the follower has already adjusted its acceleration, lower $R_{A}^{\text{need}}$ will result in an increased RE probability due to lower TTC. The positive sign of $R_{3}^{\text{RE}}$ and its statistical significance makes the consideration of different exogenous safety conditions non-negligible. It is worth pointing out that both the vehicle category (car/truck or bus) and the pavement (wet/dry) conditions were considered.

The parameters of the negative components of the lead and lag gaps variation during LC events ($R_{2}^{\text{LC}}$ and $R_{4}^{\text{LC}}$) are also significant: largest absolute values of its independent variables ($R_{G}^{\text{lag}}$ and $R_{G}^{\text{lead}}$) represent significantly shrinking gaps. As both parameters are negative, any $R_{G}^{\text{lag}}$ or $R_{G}^{\text{lead}}$ will increase the probability of LC accident events. The lead relative gap variation came out as the most statistically significant regarding LC events and its higher magnitude is due to the much smaller simulated lead gaps during lane-change not only when compared to lag gaps but also when comparing accident events with no-accidents.

**TABLE 1** Estimation results.

<table>
<thead>
<tr>
<th>Event</th>
<th>Parameter</th>
<th>value</th>
<th>st. dev.</th>
<th>t-stat</th>
<th>p-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rear-end conflict</td>
<td>RE constant $R_{0}^{\text{RE}}$</td>
<td>-13.09*</td>
<td>0.608</td>
<td>-5.08</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>Positive relative needed dec. $R_{1}^{\text{RE}}$</td>
<td>2.917</td>
<td>0.917</td>
<td>3.18</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>Negative relative needed dec. $R_{2}^{\text{RE}}$</td>
<td>-1.92</td>
<td>0.784</td>
<td>-2.45</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Maximum available dec. ratio $R_{3}^{\text{RE}}$</td>
<td>2.03</td>
<td>1.034</td>
<td>1.96</td>
<td>0.07</td>
</tr>
<tr>
<td>Lane-change conflict</td>
<td>LC constant $R_{0}^{\text{LC}}$</td>
<td>-7.08*</td>
<td>0.457</td>
<td>6.32</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>Positive relative lag gap variation $R_{1}^{\text{LC}}$</td>
<td>-0.011</td>
<td>0.012</td>
<td>-0.92</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>Negative relative lag gap variation $R_{2}^{\text{LC}}$</td>
<td>-0.568</td>
<td>0.338</td>
<td>-1.68</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>Positive relative lead gap variation $R_{3}^{\text{LC}}$</td>
<td>-0.311</td>
<td>0.255</td>
<td>-1.22</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>Negative relative lead gap variation $R_{4}^{\text{LC}}$</td>
<td>-0.628</td>
<td>0.315</td>
<td>-1.99</td>
<td>0.07</td>
</tr>
<tr>
<td>Run-off-road event</td>
<td>ROR constant $R_{0}^{\text{ROR}}$</td>
<td>-12.45*</td>
<td>0.367</td>
<td>-6.68</td>
<td>&lt;0.01</td>
</tr>
<tr>
<td></td>
<td>Positive lateral acc. difference $R_{1}^{\text{ROR}}$</td>
<td>0.023</td>
<td>0.013</td>
<td>1.77</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>Negative lateral acc. difference $R_{2}^{\text{ROR}}$</td>
<td>1.775</td>
<td>0.965</td>
<td>1.84</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>Scale parameter for the accident nest $\mu$</td>
<td>1.622</td>
<td>0.567</td>
<td>2.86</td>
<td>0.01</td>
</tr>
</tbody>
</table>

| Nº of parameters | 13 (* are the parameters affected by weights) |
| Sample size:     | 10733084 (3 replications)                  |
| Initial log-likelihood: | -9636.49                                  |
| Final log-likelihood:  | -2047.53                                  |
| $\rho^2$:          | 0.787                                     |
| $\bar{\rho}^2$:    | 0.786                                     |

Regarding ROR events, when $\Delta a^{\text{lat}}$ is positive, the simulated lateral acceleration is higher than the critical lateral acceleration and the vehicle is under unsafe conditions. Thus, when $R_{4}^{\text{ROR}} > 0$ there is a higher probability of ROR events. Similarly, when $\Delta a^{\text{lat}}$ is negative, larger absolute values are related to safer conditions, as the simulated lateral acceleration is much smaller than the critical one ($R_{2}^{\text{ROR}} < 0$). Yet, one would expect a higher absolute magnitude for $R_{1}^{\text{ROR}}$, but these results may be justified with the small
number of observations with $\Delta a^{\text{lat}} > 0$.

The estimated scale parameter of the accidents nest $\mu$ was also significant, revealing a non-negligible effect of shared unobserved attributes of the different types of accident under analysis.

6 VALIDATION

As no other accident data set was available, the validation was performed using two new sets of artificial data, generated by MITSIMLab for the same sample of events.

In TABLE 2 the averaged ratios of the probabilities between a specific type of accident and the no-accident events are presented for both the estimation and validation data sets. The range of both input variables and estimated probabilities for the validation data set are similar to the estimation ones. The trade-offs (correlations) captured by the model are also visible, especially between RE and LC conflicts.

<table>
<thead>
<tr>
<th>TABLE 2 Validation probability ratios regarding $P(\text{NA})$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(RE)</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Estimation</td>
</tr>
<tr>
<td>RE</td>
</tr>
<tr>
<td>LC</td>
</tr>
<tr>
<td>ROR</td>
</tr>
<tr>
<td>Validation</td>
</tr>
<tr>
<td>RE</td>
</tr>
<tr>
<td>LC</td>
</tr>
<tr>
<td>ROR</td>
</tr>
</tbody>
</table>

The accuracy rates for all accident events considered was 38.6% using the validation data set. The accuracy for non-accident events was 92.1% while false alarms reached 7.9%. In a previous model using real loop sensor data, Oh et al. (2001) estimated the prediction accuracy for accidents and non-accidents as 55.8% and 72.1%, and a false alarm rate of 27.9%. Xu et al. (37) estimated the same rates as 61.0%, 80.0% and 20.0%, respectively. The rates obtained with the proposed model with artificial data still remain below the values found in the literature for aggregated accident probability models using real data. The small sample used for estimation may have affected this number. Yet, the false alarm rate is considerably lower than values reported in other studies, indicating a high specificity of the proposed model.

7 CONCLUSIONS

A generic framework for modelling cause effect mechanisms between detailed traffic variables and accident occurrence probability in traffic microscopic simulation tools was proposed and tested in a real road environment. Detailed variables of vehicle motion and interactions were found to be linked to different accident increased probabilities. The nested structured allowed to capture existing trade-offs between different types of accidents. The fact that all these considerations were extracted from simulated analysis shows the real potential of advanced traffic microscopic simulation regarding detailed safety assessments, as long as detailed calibration is successfully carried out. The interaction between vehicle gaps and relative motions has been proved as a key factor for accident occurrence in previous safety related studies. Yet, no probabilistic formulation accommodating such interaction and integrated in traffic simulation models had previously been reported in the literature.

Several enhancements regarding the specific formulation of the proposed probabilistic safety model for urban motorways may be introduced. The inclusion of further components in the safety scoring function (e.g.: driver related variables), the formulation of non-linear safety score functions, the specification of additional accident types and the definition of more powerful modelling structures, such
as the mixed logit, or estimation methods, such as a panel data estimation based on multiple replications,
should be tested. Also, both the validation using other sets of data and traffic scenarios and a benchmark
against alternative non-probabilistic safety assessment tools would be valuable. The availability of large
detailed trajectory data sets from naturalistic studies will be also a key source for potential improvements.
Furthermore, the integration of conceptual perception and error modelling frameworks and more detailed
motion descriptions in microscopic simulation tools may mitigate some of the modelling constraints.
Finally, it is worth remembering that the modelling and estimation structures were formulated in terms of
expected behavioural considerations but constrained by the driving behaviour simulation model
limitations. In fact, when a safety assessment model (probabilistic or not) is integrated into a simulation
tool, the safety formulation should also consider the modelling assumptions and limitations of the traffic
simulator itself.

BIBLIOGRAPHY

by applying a system of interrelated equations. In: *Proceedings of the 85th Annual Meeting of the*
*Transportation Research Board*. Washington D.C., USA.
transportation systems data and real-time intervention strategies to improve safety”. *Journal of*
*Intelligent Transportation Systems* 11 (3), 107–120.
(4) Abdel-Aty, M., Gayah, V., 2010. “Real-time crash risk reduction on freeways using coordinated and
uncoordinated ramp metering approaches”. *ASCE Journal of Transportation Engineering* 136 (5).
performance at signalized intersections”. *Accident Analysis & Prevention* 40 (3), 1171-1179.
Technical Report. Federal Highway Administration, Virginia, USA
behavioral data: theoretical framework and first implementation”. *Accident Analysis & Prevention*
42 (6), 1637–1646.
(9) Archer, J., 2005. *Indicators for traffic safety assessment and prediction and their application in*
Royal Institute of Technology, Sweden.
Simulation-based Surrogate Safety Measure”. *Transportation Research Record: Journal of*
*Transportation Research Board*, 2083, 105-113.
Calculated Conflicts in Microsimulation Model Predict Number of Crashes?”. *Transportation*
*Research Record: Journal of the Transportation Research Board* 2147, 105–112.
evaluation of a driving support system on traffic flow by microscopic traffic simulation”. In: *3rd*
*International Conference on Road Safety and Simulation*. Indianapolis, USA.


